6 PRACTICE PAPERS
JEE Main & Advanced

SAMPLE PAPER
CBSE

www.mtg.in | March 2014 | ₹30

MATHEMATICS today

India's #1 MATHEMATICS MONTHLY for IIT-JEE & AIEEE

20 CHALLENGING PROBLEMS
BRAIN @ WORK

YOU ASKED
WE ANSWERED

PRACTICE PAPER
BITSAT

MATHS MUSING
10 GREAT PROBLEMS
To Stimulate Creative & Critical Thinking

MtG Learning Media
Trust of more than 1 Crore Readers Since 1982

MATHEMATICS today

Vol. XXXII No. 3 March 2014

Corporate Office
Plot 99, Sector 44 Institutional Area, Gurgaon, (HR).
Tel : 0124-4951200
e-mail : info@mtg.in website : www.mtg.in

Regd. Office
406, Taj Apartment, Near Safdarjung Hospital,
Ring Road, New Delhi - 110 029.

Managing Editor : Mahabir Singh
Editor : Anil Ahlawat (BE, MBA)

CONTENTS

Maths Musing Problem Set - 135	**4**
Practice Paper JEE Main	**6**
Challenging Problems	**15**
Maths Musing - Solutions	**19**
Practice Paper JEE (Main & Advanced)	**20**
You Asked, We Answered	**25**
Mock Test Paper JEE Main	**27**
Mock Test Paper JEE Advanced	**34**
Mock Test Paper WB JEE	**50**
Mock Test Paper JEE Main	**59**
Practice Paper : BITSAT	**67**
CBSE Board - 2014 Sample Paper	**75**
Mock Test Paper JEE (Main & Advanced)	**85**

Owned, Printed and Published by Mahabir Singh from 406, Taj Apartment, New Delhi - 29 and printed by Personal Graphics and Advertisers (P) Ltd., Okhla Industrial Area, Phase-II, New Delhi. Readers are adviced to make appropriate thorough enquiries before acting upon any advertisements published in this magazine. Focus/Infocus features are marketing incentives MTG does not vouch or subscribe to the claims and representations made by advertisers. All disputes are subject to Delhi jurisdiction only.
Editor : Anil Ahlawat

Copyright© MTG Learning Media (P) Ltd.
All rights reserved. Reproduction in any form is prohibited.

editorial

Man and Machine-their relationship

Man created the machine. However their relationship changed. The earlier tools of work, such as those of the carpenter, sculptor or mason, were only extensions of the hands of man. Then came the mechanical age. Machinery in factories which produced many products such as spinning machines, cars and trains are of a different type. These machines have to made or assembled from the products of other machines.

If there are problems, specialised engineers have to rectify them. When calculators were discovered, the supremacy of man became shaky. The reliance of man on calculators dulled the ability of man to calculate complicated relations by understanding the process. The calculations were performed mechanically. With the development of computers in the last century, many analytical instruments such as X-ray spectrometers, optical spectrometers, automatic analytical spectrometers using U.V. and other ranges for excitation, X-ray diffraction, etc were all automatic with the computer to take care of calculations.

This automatic equipments put back the development of spectroscopy by a hundred years. Unless one has matching standards, solid for solid, liquid for liquid for every type of sample, one could not do the analysis. Chemical analysis was the only solution. It was only 1980's that mathematical methods could get over these problems of matrices and concentration ranges. However, for most of the persons, automatic methods are easier. It is less taxing!

Now another serious problem is cropping. U.S.A. gets world's first bookless public library. With this online library, one can consult any material or book directly in the computer. Your time is saved and also it is less costly. The pleasure of finding something in the journal which one had not expected or some data which is very important for one's research could be in another unexpected article. Browsing through a book at leisure or journals for similar articles, can be more effective to locate articles for your research.

BOOKS CAN NEVER BE SUBSTITUTED.

A computer is your good friend. Never allow this to be your master.

Anil Ahlawat
Editor

Subscribe online at www.mtg.in

Individual Subscription Rates	1 yr.	2 yrs.	3 yrs.	Combined Subscription Rates	1 yr.	2 yrs.	3 yrs.
Mathematics Today	300	500	675	PCM	800	1200	1700
Chemistry Today	300	500	675	PCB	800	1200	1700
Physics For You	300	500	675	PCMB	900	1500	2100
Biology Today	300	500	675				

Send D.D/M.O in favour of MTG Learning Media (P) Ltd.
Payments should be made directly to : MTG Learning Media (P) Ltd,
406, Taj Apartment, Near Safdarjung Hospital, Ring Road, New Delhi 29.
We have not appointed any subscription agent.

MATHS MUSING

Maths Musing was started in January 2003 issue of Mathematics Today with the suggestion of Shri Mahabir Singh. The aim of Maths Musing is to augment the chances of bright students seeking admission into IITs with additional study material.

During the last 10 years there have been several changes in JEE pattern. To suit these changes Maths Musing also adopted the new pattern by changing the style of problems. Some of the Maths Musing problems have been adapted in JEE benefitting thousand of our readers. It is heartening that we receive solutions of Maths Musing problems from all over India.

Maths Musing has been receiving tremendous response from candidates preparing for JEE and teachers coaching them. We do hope that students will continue to use Maths Musing to boost up their ranks in JEE Main and Advanced.

Prof. Dr. Ramanaiah Gundala, Former Dean of Science and Humanities, Anna University, Chennai

PROBLEM Set 135

JEE MAIN

1. In a triangle ABC, if $\tan A : \tan B : \tan C = 2 : 3 : 4$, then $\frac{\Delta}{R^2} =$

(a) $\frac{9\sqrt{6}}{35}$ (b) $\frac{18\sqrt{6}}{35}$ (c) $\frac{36\sqrt{2}}{35}$ (d) $\frac{18\sqrt{3}}{35}$

2. Let $\{x\} = x - [x]$. If $\{a^{-1}\} = \{a^2\}$ and $2 < a^2 < 3$, then $a^{12} - 144\, a^{-1} =$

(a) 279 (b) 299 (c) 231 (d) 233

3. Let n be the number of permutations of the letters of the word ALLAHABAD such that no two A's are adjacent. The sum of the digits of n is

(a) 9 (b) 11 (c) 13 (d) 7

4. In triangle ABC, the vertices A, B, C are given by the complex numbers $9i$, 12, 0 respectively. Equilateral triangles BCD and ACE are formed externally. If z_1 and z_2 are the centroids of the two equilateral triangles, then $|z_1 - z_2|^2$ is $p + q\sqrt{3}$, where $p + q =$

(a) 98 (b) 100 (c) 111 (d) 121

5. A tangent to $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ meets the ellipse $\frac{x^2}{a} + \frac{y^2}{b} = a + b$ at the points P and Q. The angle between the tangents at P and Q is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

JEE ADVANCED

6. In triangle ABC, h_1, h_2, h_3 are the lengths of altitudes from A, B, C, which are natural numbers forming an increasing H.P. If $h_2 = 24$, then $a : b : c =$

(a) 4 : 3 : 2 (b) 6 : 5 : 4

(c) 8 : 7 : 6 (d) 10 : 9 : 6

COMPREHENSION

The parametric equations of a parabola are $x = u\cos\alpha \cdot t$, $y = u\sin\alpha \cdot t - \frac{1}{2}gt^2$, t is a parameter.

7. Its latus rectum is

(a) $\frac{2u^2}{g}\sin^2\alpha$ (b) $\frac{2u^2}{g}\cos^2\alpha$

(c) $\frac{u^2}{2g}\sin^2\alpha$ (d) $\frac{u^2}{2g}\cos^2\alpha$

8. Its focus is

(a) $\left(\frac{u^2}{2g}\sin 2\alpha, \frac{u^2}{2g}\cos 2\alpha\right)$ (b) $\left(\frac{u^2}{2g}\sin 2\alpha, \frac{-u^2}{2g}\cos 2\alpha\right)$

(c) $\left(\frac{u^2}{2g}\sin^2\alpha, \frac{-u^2}{2g}\cos^2\alpha\right)$ (d) $\left(\frac{u^2}{2g}\cos^2\alpha, \frac{-u^2}{2g}\sin^2\alpha\right)$

INTEGER MATCH

9. A straight line with negative slope passes through the point (1, 2) and cuts x-axis at A and y-axis at B. O is the origin. The minimum value of the semi perimeter of triangle OAB is

MATCHING LIST

10. Match the following.

	List-I	List-II
P.	The least length of a normal chord of $xy = 2$ is	1. 2
Q.	$\tan\frac{\pi}{5}\tan\frac{2\pi}{5}\tan\frac{3\pi}{5}\tan\frac{4\pi}{5}$ is	2. 3
R.	When 23^{23} is divided by 53, the sum of the digits of the remainder is	3. 4
S.	If $\frac{dy}{dx} = \frac{x-y}{x+y}$, $y(1) = 1$, then $(y(0))^2$ is	4. 5

	P	Q	R	S
(a)	1	2	3	4
(b)	4	3	2	1
(c)	4	3	1	2
(d)	3	4	2	1

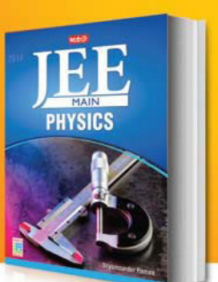
See Solution set of Maths Musing 134 on page no. 19

Prof. Ramanaiah is the author of MTG JEE(Main & Advanced) Mathematics series

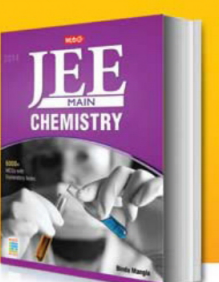
mtG

BEST TOOLS FOR SUCCESS IN

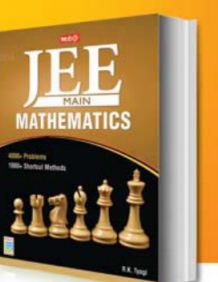
JEE Main



₹575



₹650



₹550

- Based on latest pattern of JEE Main
- Illustrations and MCQs to clear your concepts
- Covers complete syllabus
- Also contains previous years' MCQs of AIEEE, JEE Main and other Engineering Entrance Exams.

Success in JEE 2014 Now Made Easy !

Buy online at www.mtg.in

Available at leading book-shops throughout India.

mtG BOOKS

For more information or to place your order :
Call at : 0124 - 4951200 @ Mail at : info@mtg.in
Log on to : www.mtg.in

PRACTICE PAPER 2014

JEE Main

SECTION-I

Single Correct Answer Type

1. From a common point on the lines, $xy - 3y - 4x + 12 = 0$ mutually perpendicular lines L_1 and L_2 are drawn such that they intercept triangles of equal area with coordinate axes. If θ_1 and θ_2 are inclination of L_1 and L_2 with coordinate axes, then $|\theta_1 + \theta_2|$ is equal to

(a) $\tan^{-1}\left(\frac{48}{7}\right)$ (b) $\tan^{-1}\left(\frac{24}{7}\right)$

(c) $\tan^{-1}\left(\frac{1}{7}\right)$ (d) $\tan^{-1}\left(\frac{2}{7}\right)$

2. **Statement-1 :** Number of common tangents of $x^2 + y^2 - 2x - 4y - 95 = 0$ and $x^2 + y^2 - 6x - 8y + 16 = 0$ is zero.

Statement-2 : If $C_1C_2 < |r_1 - r_2|$, then there will be no common tangent (where C_1, C_2 are the centres and r_1, r_2 are radii of circles).

(a) Both Statement-1 and Statement-2 are true, Statement-2 is a correct explanation for Statement-1.

(b) Both Statement-1 and Statement-2 are true, but Statement-2 is not a correct explanation for Statement-1.

(c) Statement-1 is true and Statement-2 is false.

(d) Statement-1 is false and Statement-2 is true.

3. Let $a = \cos\alpha + \cos\beta - \cos(\alpha + \beta)$ and $b = 4\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\cos\frac{\alpha+\beta}{2}$. Then value of $a - b$ is

(a) 0 (b) 1

(c) –1 (d) none of these

4. Let $f(x)$ be a polynomial function of second degree. If $f(1) = f(-1)$ and a, b, c, are in A.P., then $f'(a)$, $f'(b)$ and $f'(c)$ are in

(a) A.P. (b) G.P.

(c) H.P. (d) A.G.P.

5. The area of the region bounded by the curves $y = 2^x$, $y = 2x - x^2$ and $x = 2$, is

(a) $\frac{3}{\log 2}$ sq. unit (b) $\frac{3}{\log 2} - \frac{4}{9}$ sq. unit

(c) $\frac{3}{2} - \frac{\log 2}{9}$ (d) none of these

6. The ratio in which the line segment joining the points (4, –6) and (3, 1) is divided by the parabola $y^2 = 4ax$ is

(a) $\frac{-20 \pm \sqrt{155}}{11} : 11$ (b) $\frac{-2 \pm \sqrt{155}}{11} : 1$

(c) $-20 \pm 2\sqrt{155} : 11$ (d) $-20 \pm \sqrt{155} : 11$

7. If the variance of 1, 2, 3, 4, 5, ..., 10 is $\frac{99}{12}$, then the standard deviation of 3, 6, 9, 12, ..., 30 is

(a) $\frac{297}{4}$ (b) $\frac{3}{2}\sqrt{33}$

(c) $\frac{3}{2}\sqrt{99}$ (d) $\sqrt{\frac{99}{12}}$

8. The value of $\lim_{n\to\infty}\left[\frac{2n}{2n^2-1}\cos\frac{n+1}{2n-1} - \frac{n}{1-2n}\cdot\frac{n(-1)^n}{n^2+1}\right]$ is

(a) 1 (b) –1

(c) 0 (d) none of these

9. If $A = \begin{bmatrix} a & b & c \\ x & y & z \\ p & q & r \end{bmatrix}$ and $B = \begin{bmatrix} q & -b & y \\ -p & a & -x \\ r & -c & z \end{bmatrix}$ then

(a) $|A| = |B|$ (b) $|A| = -|B|$

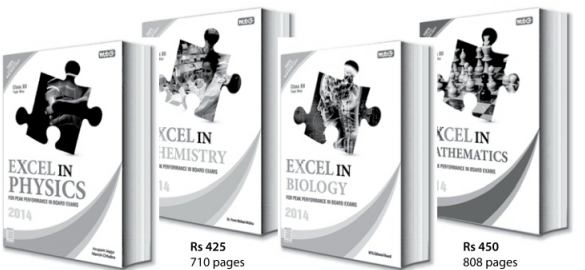
(c) $|A| = 2|B|$ (d) none of these

10. If $y(x-y)^2 = x$, then $\int \frac{1}{x-3y}\,dx$ is equal to

(a) $\frac{1}{3}\log\{(x-y)^2 + 1\}$ (b) $\frac{1}{4}\log\{(x-y)^2 - 1\}$

(c) $\frac{1}{2}\log\{(x-y)^2 + 1\}$ (d) $\frac{1}{6}\log\{(x^2-y^2) - 1\}$

Concerned about your performance in Class XII Boards?



Rs 425
720 pages

Rs 425
710 pages

Rs 350
496 pages

Rs 450
808 pages

Well, fear no more, help is at hand.....
To excel, studying in right direction is more important than studying hard. Which is why we created the Excel Series. These books – for Physics, Chemistry, Biology & Mathematics – have been put together totally keeping in mind the prescribed syllabus and the pattern of CBSE's Board examinations, so that students prepare and practice with just the right study material to excel in board exams.

Did you know nearly all questions in CBSE's 2013 Board Examination were a part of our Excel books? That too fully solved ?!

HIGHLIGHTS:

- Comprehensive theory strictly based on NCERT, complemented with illustrations, activities and solutions of NCERT questions
- Practice questions & Model Test Papers for Board Exams
- Latest CBSE Board Sample Papers (Solved)
- Previous years' CBSE Board Examination Papers (Solved)

*Scan now with your smartphone or tablet**

Visit www.mtg.in for latest offers and to buy online!

MtG Learning Media

Available at all leading book shops throughout the country.
For more information or for help in placing your order:
Call 0124-4951200 or email info@mtg.in
**Application to read QR codes required*

11. If the difference of two unit vectors is again a unit vector, then the angle between them is
(a) 30° (b) 40°
(c) 60° (d) 90°

12. f is a function defined on the interval [–1, 1] such that $f(\sin 2x) = \sin x + \cos x$.

Statement-1 : If $x \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$, then $f(\tan^2 x) = \sec x$.

Statement-2 : $f(x) = \sqrt{1+x}, \forall x \in [-1, 1]$.
(a) Both Statement-1 and Statement-2 are true, Statement-2 is a correct explanation for Statement-1.
(b) Both Statement-1 and Statement-2 are true, but Statement-2 is not a correct explanation for Statement-1.
(c) Statement-1 is true and Statement-2 is false.
(d) Statement-1 is false and Statement-2 is true.

13 Image point of (5, 4, 6) in the plane $x + y + 2z - 15 = 0$ is
(a) (3, 2, 2) (b) (2, 3, 2)
(c) (2, 2, 3) (d) (–5, – 4, – 6)

14. Let P be a variable point. From P, PQ and PR are tangents drawn to the parabola $y^2 = 4x$. If $\angle QPR$ is always 45°, then locus of P is
(a) $x^2 - y^2 + 6x + 1 = 0$ (b) $x^2 + y^2 + 6x + 1 = 0$
(c) $x^2 - y^2 - 6x + 1 = 0$ (d) none of these

15. The number of values of x where the fuction $f(x) = 2(\cos 3x + \cos \sqrt{3}x)$ attains its maximum, is
(a) 1 (b) 2
(c) 0 (d) infinite

16. If two adjacent sides of a cyclic quadrilateral are 2 and 5 and the angle between them is 60°. If the third side is 3, then the remaining fourth side is
(a) 2 (b) 3
(c) 4 (d) 5

17. If $\frac{1}{a} + \frac{1}{c} + \frac{1}{a-b} + \frac{1}{c-b} = 0$ and $b \neq a + c$, then a, b, c are in
(a) A.P. (b) G.P.
(c) H.P. (d) none of these

18. In a series of $2n$ observations, half of them equal a and remaining half equal $-a$. If the standard deviation of the observations is 2, then $|a|$ equals
(a) $\frac{1}{n}$ (b) $\sqrt{2}$
(c) 2 (d) $\frac{\sqrt{2}}{n}$

19. The maximum number of points into which 4 circles and 4 straight lines intersect is
(a) 26 (b) 50
(c) 56 (d) 72

20. Five coins whose faces are marked 2, 3 are tossed. The chance of obtaining a total of 12 is
(a) $\frac{1}{32}$ (b) $\frac{1}{16}$
(c) $\frac{3}{16}$ (d) $\frac{5}{16}$

21. Statement-1 : A point on the straight line $2x + 3y - 4z = 5$ and $3x - 2y + 4z = 7$ can be determined by taking $x = k$ and then solving the two equations for y and z, where k is any real number.

Statement-2 : If $c' \neq kc$, then the straight line $ax + by + cz + d = 0$, $kax + kby + c'z + d' = 0$, does not intersect the plane $z = \alpha$, where α is any real number.
(a) Both Statement-1 and Statement-2 are true, Statement-2 is a correct explanation for Statement-1.
(b) Both Statement-1 and Statement-2 are true, but Statement-2 is not a correct explanation for Statement-1.
(c) Statement-1 is true and Statement-2 is false.
(d) Statement-1 is false and Statement-2 is true.

22. If $\cos^{-1}\left(\frac{p}{a}\right) + \cos^{-1}\left(\frac{q}{b}\right) = \alpha$ and

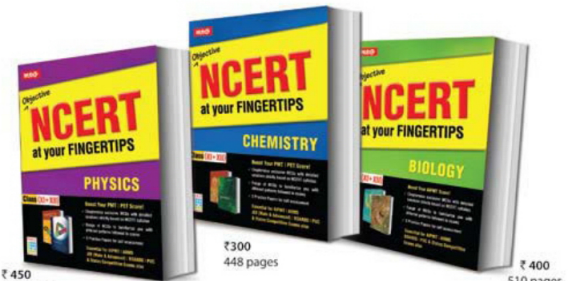
$\frac{p^2}{a^2} + k\cos\alpha + \frac{q^2}{b^2} = \sin^2\alpha$, then the value of k is
(a) $\frac{2pq}{ab}$ (b) $\frac{pq}{ab}$
(c) $2pq$ (d) $-\frac{2pq}{ab}$

23. If the coefficients of 5th, 6th and 7th terms in the expansion of $(1 + x)^n$ be in A.P., then the value of n is
(a) 7 only (b) 14 only
(c) 7 or 14 (d) none of these

24. The sum of the first n terms of the series

$1^2 + 2.\,2^2 + 3^2 + 2.4^2 + \ldots..$ is $\frac{n(n+1)^2}{2}$ when n is even. When n is odd, the sum is
(a) $\frac{3n(n+1)}{2}$ (b) $\frac{n^2(n+1)}{2}$
(c) $\frac{n(n+1)^2}{4}$ (d) $\left(\frac{n(n+1)}{2}\right)^2$

How to select the correct answer faster?



₹ 450
532 pages

₹300
448 pages

₹ 400
510 pages

The answer is practice....

Our team has seen that in NEET, AIPMT, AIIMS and other PMTs, Multiple Choice Questions (MCQs) are based on the NCERT syllabus. Largely !! With Objective NCERT at your FINGERTIPS, you can become a pro at handling MCQs. Practice to increase your accuracy and improve timing with a bank of over 10,000 questions, all taken from NCERT course books. Don't take our word for it. Hear it from the pros…

Vidit Panchal says
"This is an excellent book...actually the reason for my success in NEET and other PMTs like MGIMS Wardha and JIPMER, 2013. It is the only book currently available in the market which is exactly based on NCERT. AWESOME CREATION!! ☺☺☺"

Ashok Bhandari says
"The only Objective Biology Book you ever need!! Excellent MCQs (95% of them strictly according to NCERT!) It is about 15-20% at higher level than the actual NEET paper. But if you can do 90% of a chapter's MCQs of this book within an hour, NEET will look like a cakewalk! MTG has proved that they are still the best in publishing market even after 30 years.☺☺☺"

Swadeep Biswas says
"This book has helped me a lot to secure a rank in NEET!!! I recommend this book to all medical aspirants!! Thanks a lot!!! ☺☺☺"

*Scan now with your smartphone or tablet**

Visit **www.mtg.in** for latest offers and to buy online!

mtG Learning Media

Available at all leading book shops throughout the country.
For more information or for help in placing your order:
Call 0124-4951200 or email info@mtg.in

**Application to read QR codes required*

25. For any vector $\vec{r}$, the value of $\hat{i}\times(\vec{r}\times\hat{i})+\hat{j}\times(\vec{r}\times\hat{j})+\hat{k}\times(\vec{r}\times\hat{k})$ is

(a) $\vec{0}$ (b) $2\vec{r}$

(c) $-2\vec{r}$ (d) none of these

26. The value of 'a', for which one root of the quadratic equation $(a^2-5a+3)x^2+(3a-1)x+2=0$ is twice as large as the other, is

(a) $-\frac{1}{3}$ (b) $\frac{2}{3}$

(c) $-\frac{2}{3}$ (d) $\frac{1}{3}$

27. If $a_n = 2^{2^n}+1$, then for $n>1$, last digit of a_n is

(a) 3 (b) 5

(c) 7 (d) 8

28. The equation of the tangent to the curve $x=\frac{3at}{1+t^2}$, $y=\frac{3at^2}{1+t^2}$ at the point $t=2$ is

(a) $4x+3y+a=0$ (b) $3x+4y-a=0$

(c) $4x+3y-12a=0$ (d) none of these

29. A parabola is drawn with focus at (3, 4) and vertex at the focus of the parabola $y^2-12x-4y+4=0$. The equation of the parabola is

(a) $x^2-6x-8y+25=0$ (b) $y^2-8x-6y+25=0$

(c) $x^2-6x+8y-25=0$ (d) $x^2+6x-8y-25=0$

30. For any two complex numbers z_1, z_2 and any two real numbers a, b, the value of $|az_1-bz_2|^2+|bz_1+az_2|^2$ is equal to

(a) $(a+b)(|z_1|^2+|z_2|^2)$

(b) $(a^2+b^2)(|z_1|^2+|z_2|^2)$

(c) $(a^2+b^2)(|z_1|+|z_2|)$

(d) none of these

SOLUTIONS

1. (b): Common point is (3, 4).

Let L_1 and L_2 be

$y-4=m(x-3)$ and $y-4=-\frac{1}{m}(x-3)$

Now, $A_1=A_2$

$$\frac{1}{2}\frac{(4m+3)^2}{m}=\frac{(4-3m)^2}{m}\times\frac{1}{2} \Rightarrow 7m^2+48m-7=0$$

$$m_1+m_2=-\frac{48}{7},\ m_1m_2=-1$$

$$\Rightarrow |\theta_1+\theta_2|=\tan^{-1}\left(\frac{24}{7}\right)$$

2. (a): $C_1(1, 2)$, $r_1=10$

$C_2(3, 4)$, $r_2=3$

$\therefore\ C_1C_2=2\sqrt{2}<|r_1-r_2|=7$

$\therefore$ The Statements are true.

3. (b): $a=2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}-2\cos^2\frac{\alpha+\beta}{2}+1$

$$=2\cos\frac{\alpha+\beta}{2}\left(\cos\frac{\alpha-\beta}{2}-\cos\frac{\alpha+\beta}{2}\right)+1$$

$$=2\cos\frac{\alpha+\beta}{2}\cdot 2\sin\frac{\alpha}{2}\sin\frac{\beta}{2}+1=b+1$$

$\Rightarrow\ a-b=1.$

4. (a): Let $f(x)=ax^2+bx+c$

Then, $f(1)=a+b+c$

and $f(-1)=a-b+c$

Since, $f(1)=f(-1)$

$\Rightarrow\ a+b+c=a-b+c$

$\Rightarrow\ 2b=0$

$\Rightarrow\ b=0$

i.e., $f(x)=ax^2+c$

$\therefore\ f'(x)=2ax$

$f'(a)=2a^2, f'(b)=2ab, f'(c)=2ac$

Now, $2f'(b)=f'(a)+f'(c)$

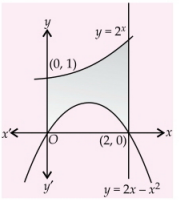
If $2\cdot 2ab=2a^2+2ac$

If $2b=a+c$

If a, b, c are in A.P., which is given.

$\therefore\ f'(a), f'(b), f'(c)$ are in A.P.

5. (d):



Required area

$$=\int_0^2[2^x-(2x-x^2)]dx$$

$$=\int_0^2(2^x-2x+x^2)dx$$

$$=\left[\frac{2^x}{\log 2}-x^2+\frac{x^3}{3}\right]_0^2$$

$$=\frac{4}{\log 2}-4+\frac{8}{3}-\frac{1}{\log 2}$$

$$=\frac{3}{\log 2}-\frac{4}{3}\text{ sq. unit}$$

6. (c) : Let required ratio be $k : 1$

then, $\left(\frac{3k+4}{k+1}, \frac{k-6}{k+1}\right)$ is lying on $y^2 = 4x$

$\therefore \quad (k-6)^2 = 4(3k+4)(k+1)$

$\Rightarrow \quad 11k^2 + 40k - 20 = 0$

$\Rightarrow \quad k = \frac{-40 \pm \sqrt{1600 + 4 \times 220}}{22}$

or $\quad k = \frac{-20 \pm \sqrt{620}}{11} = \frac{-20 \pm 2\sqrt{155}}{11}$

$\therefore \quad$ Required ratio $= -20 \pm 2\sqrt{155} : 11$

7. (b): Given, $\sigma_{10}^2 = \frac{99}{12} = \frac{33}{4}$

$\Rightarrow \quad \sigma_{10} = \frac{\sqrt{33}}{2}$

SD of required series $= 3\sigma_{10} = \frac{3\sqrt{33}}{2}$

8. (c) : $\lim_{n\to\infty}\left[\frac{2n}{2n^2-1}\cos\frac{n+1}{2n-1} - \frac{n}{1-2n}\cdot\frac{n(-1)^n}{n^2+1}\right]$

$= \lim_{n\to\infty}\left[\frac{2}{2-\frac{1}{n^2}}\cdot\frac{1}{n}\cos\left(\frac{1+\frac{1}{n}}{2-\frac{1}{n}}\right) - \frac{1}{\left(\frac{1}{n}-2\right)}\cdot\frac{(-1)^n}{\left(1+\frac{1}{n^2}\right)}\cdot\frac{1}{n}\right]$

$= \lim_{n\to\infty}\frac{1}{n}\left[\frac{2}{2-\frac{1}{n^2}}\cdot\cos\left(\frac{1+\frac{1}{n}}{2-\frac{1}{n}}\right) - \frac{1}{\left(\frac{1}{n}-2\right)}\cdot\frac{(-1)^n}{\left(1+\frac{1}{n^2}\right)}\right]$

$= 0 \times \left[\frac{2}{2}\times\cos\frac{1}{2} + \frac{1}{2}\times\frac{1}{1}\right] = 0$

9. (b) : $|B| = -\begin{vmatrix} q & -b & y \\ p & -a & x \\ r & -c & z \end{vmatrix} = \begin{vmatrix} q & b & y \\ p & a & x \\ r & c & z \end{vmatrix}$

$= -\begin{vmatrix} p & a & x \\ q & b & y \\ r & c & z \end{vmatrix} = \begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix} = \begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$

$= -\begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix} \Rightarrow |B| = -|A|$

10. (c) : We have, $y(x-y)^2 = x$

$\Rightarrow \quad d[y(x-y)^2] = dx$ [taking differential of both sides]

$\Rightarrow \quad (x-y)^2 dy + 2y(x-y)(dx-dy) = dx$

$\Rightarrow \quad (x-y)^2 dy - (x-y)^2 dx + 2y(x-y)(dx-dy) = [1-(x-y)^2]\,dx$

$\Rightarrow \quad [(x-y)^2 - 2y(x-y)](dy-dx) = [1-(x-y)^2]\,dx$

$\Rightarrow \quad (x-y)(x-3y)\,(dx-dy) = [(x-y)^2 - 1]\,dx$

$\Rightarrow \quad (x-y)\frac{(dx-dy)}{(x-y)^2-1} = \frac{dx}{x-3y}$

$\Rightarrow \quad \frac{dx}{x-3y} = \frac{(x-y)d(x-y)}{(x-y)^2-1}$

$\Rightarrow \quad \int\frac{1}{x-3y}dx = \frac{1}{2}\int\frac{2(x-y)d(x-y)}{(x-y)^2-1}$

$\Rightarrow \quad \int\frac{1}{x-3y}dx = \frac{1}{2}\log\{(x-y)^2-1\}$

11. (c) : Let $\vec{a} - \vec{b} = \vec{c}$ where $\vec{a}, \vec{b}, \vec{c}$ are all unit vectors.

$\therefore \quad (\vec{a}-\vec{b})^2 = \vec{c}^2 \Rightarrow |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}|\cdot|\vec{b}|\cos\theta = |\vec{c}|^2$

$\Rightarrow \quad 1 + 1 - 2(1)(1)\cos\theta = 1 \Rightarrow \cos\theta = \frac{1}{2}$

$\Rightarrow \quad \theta = 60°.$

12. (a) : $(f(\sin 2x))^2 = \sin^2 x + \cos^2 x + 2\sin x\cos x$

$= 1 + \sin 2x$

$\Rightarrow \quad f(x) = \sqrt{1+x}, \forall x \in [-1, 1]$

$\Rightarrow \quad$ If $x \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$, then

$f(\tan^2 x) = \sqrt{1+\tan^2 x} = \sec x$

13. (a) : Let Q be the image of the point $P(5, 4, 6)$ in the given plane, then PQ is normal to the plane. The direction ratios of PQ are 1, 1, 2. Since, PQ passes through (5, 4, 6) and has direction ratios (1, 1, 2). Therefore, equation of PQ is

$\frac{x-5}{1} = \frac{y-4}{1} = \frac{z-6}{2} = r \qquad$ (say)

$\therefore \quad x = r+5, y = r+4, z = 2r+6$

So, coordinates of Q be $(r+5, r+4, 2r+6)$

Let R be the mid point of PQ then coordinates of R are

$\left(\frac{r+5+5}{2}, \frac{r+4+4}{2}, \frac{2r+6+6}{2}\right)$ *i.e.*, $\left(\frac{r+10}{2}, \frac{r+8}{2}, \frac{2r+12}{2}\right)$

Since, R lies on the plane.

$\therefore \quad \frac{r+10}{2} + \frac{r+8}{2} + 2\left(\frac{2r+12}{2}\right) - 15 = 0$

$\Rightarrow \quad r + 10 + r + 8 + 4r + 24 - 30 = 0$

$\Rightarrow \quad 6r + 12 = 0 \Rightarrow r = -2$

So, coordinates of Q are (3, 2, 2).

14. (a) : Let coordinates of the point P be (h, k).

The equation of the tangents to $y^2 = 4x$ is

$SS_1 = T^2 \Rightarrow (k^2 - 4h)(y^2 - 4x) = (ky - 2x - 2h)^2$

$\Rightarrow \quad 4x^2 - 4kxy + 4hy^2 + 4(k^2 - 2h)x - 4khy + 4h^2 = 0 \quad ...(i)$

$\therefore \quad$ Angle between the tangents is 45°

$\therefore \quad \tan 45° = \frac{2\sqrt{4k^2 - 16h}}{4(h+1)}$

$\therefore$ Required locus is $(x+1)^2 = y^2 - 4x$

$\Rightarrow$ $x^2 - y^2 + 6x + 1 = 0$

15. (a) : We have,

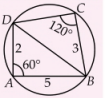
$f(x) = 2(\cos 3x + \cos\sqrt{3}x)$

$= 4\cos\left(\frac{3+\sqrt{3}}{2}\right)x \cdot \cos\left(\frac{3-\sqrt{3}}{2}\right)x \le 4$

and it is equal to 4 when both $\cos\left(\frac{3+\sqrt{3}}{2}\right)x$ and $\cos\left(\frac{3-\sqrt{3}}{2}\right)x$ are equal to 1 for a value of x.

This is possible only when $x = 0$

16. (a) : In ΔABD,



$\cos 60° = \frac{2^2 + 5^2 - BD^2}{2(3)(5)}$

$\Rightarrow$ $BD^2 = 19$

Now, in ΔBCD

$\cos 120° = \frac{CD^2 + 9 - 19}{2(3)(CD)}$

$\Rightarrow$ $CD^2 + 3CD - 10 = 0$

$\Rightarrow$ $CD = -5, 2$

$\Rightarrow$ $CD = 2$ $(\because\ CD \neq -5)$

17. (c) : We have $\frac{1}{a} + \frac{1}{c} + \frac{1}{a-b} + \frac{1}{c-b} = 0$

$\Rightarrow$ $\frac{1}{a} + \frac{1}{c-b} = -\left[\frac{1}{a-b} + \frac{1}{c}\right]$

$\Rightarrow$ $\frac{c-b+a}{a(c-b)} = -\frac{(c+a-b)}{(a-b)c}$

$\Rightarrow$ $\frac{1}{a(c-b)} = -\frac{1}{(a-b)c}$ $(\because\ b \neq a + c)$

$\Rightarrow$ $ac - bc = -ac + ab$

$\Rightarrow$ $2ac = b(a+c) \Rightarrow b = \frac{2ac}{a+c}$

$\therefore$ a, b, c are in H.P.

18. (c) : In the $2n$ observations, half of them equal to a and remaining half equal to $-a$. Then, the mean of total $2n$ observations is equal to zero.

$\therefore$ $SD = \sqrt{\frac{\Sigma(x-\bar{x})^2}{N}}$,

$\Rightarrow$ $2 = \sqrt{\frac{\Sigma x^2}{2n}}$

$\Rightarrow$ $4 = \frac{\Sigma x^2}{2n} \Rightarrow 4 = \frac{2na^2}{2n}$

$\Rightarrow$ $a^2 = 4$

$\Rightarrow$ $|a| = 2$

19. (b): Maximum points of intersection of 4 lines $= {}^4C_2 \times 1 = 6$

Maximum points of intersection of 4 circles $= {}^4C_2 \times 2 = 12$

Maximum point of intersection of 4 lines and 4 circles $= 4 \times 8 = 32$

$(\because$ A line can cut 4 circles at atmost 8 points)

$\therefore$ Total no. of points of intersection $= 6 + 12 + 32 = 50$

20. (d): The probability of getting a number either 2 or 3 in one toss is $\frac{1}{2}$

Condition for getting the sum 12 in five tossed is (2, 2, 2, 3, 3).

$\therefore$ Required probability $= {}^5C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2$

$= \frac{5\times 4}{2\times 1}\left(\frac{1}{2}\right)^5$

$= 10 \cdot \frac{1}{2^5} = \frac{5}{16}$

21. (b): Statement 1

$3y - 4z = 5 - 2k$

$-2y + 4z = 7 - 3k$

$\therefore$ $x = k,\ y = 12 - 5k,\ z = \frac{31 - 13k}{4}$ is a point on the line for all real values of k.

Statement 1 is true.

Statement 2 direction ratio of the straight line are $< bc' - kbc,\ kac - ac',\ 0 >$ direction ratios of normal to be plane $< 0, 0, 1 >$

Now, $0 \times (bc' - kbc) + 0 \times (kac - ac') + 1 \times 0 = 0$

$\therefore$ The straight line is parallel to the plane.

Statement 2 is true but does not explain statement 1.

22. (d): $\cos^{-1}\left[\frac{p}{a}\cdot\frac{q}{b} - \sqrt{\left(1-\frac{p^2}{a^2}\right)}\sqrt{\left(1-\frac{q^2}{b^2}\right)}\right] = \alpha$

$\Rightarrow$ $\frac{pq}{ab} - \sqrt{\left(1-\frac{p^2}{a^2}\right)}\sqrt{\left(1-\frac{q^2}{b^2}\right)} = \cos\alpha$

$\Rightarrow$ $\left(\frac{pq}{ab} - \cos\alpha\right)^2 = 1 - \frac{p^2}{a^2} - \frac{q^2}{b^2} + \frac{p^2q^2}{a^2b^2}$

$\Rightarrow$ $\frac{p^2q^2}{a^2b^2} + \cos^2\alpha - \frac{2pq}{ab}\cos\alpha = 1 - \frac{p^2}{a^2} - \frac{q^2}{b^2} + \frac{p^2q^2}{a^2b^2}$

$\Rightarrow$ $\frac{p^2}{a^2} - \frac{2pq}{ab}\cos\alpha + \frac{q^2}{b^2} = 1 - \cos^2\alpha = \sin^2\alpha$

Is there a fool-proof method to guarantee your success in JEE ADVANCED - 2014?

It is time to give you a clear perspective of the relationship between preparing for the exam and succeeding at it.

According to the findings of our panel of experts, apart from starting as early as 2 years in advance and putting in 3 to 5 hours everyday, successful students devote 2 years before the exams getting familiar with the problems which have appeared in the last JEEs. This is the secret of their success.

Our Key For Your Success

It is in the last area, that we can really help you. We have categorized chapterwise problems and solutions from 36 years papers of JEE engaging some of the finest minds in the country. The result of this laborious exercise has emerged as the **MTG 36 years Chapterwise Solutions.**



A Must for Every IIT Hopeful

Without doubt the **36 years Chapterwise Solutions** is the single most powerful tool available to a serious student.

It is a tool designed to fine tune for all the efforts that you have put in until now and our meticulously demonstrated solutions will clarify your doubt beyond doubt.

So go ahead. Lay your hands on the most comprehensive success tool available.

Get hold of the **MTG 36 years Chapterwise Solutions** just when it matters.

Buy online at www.**mtg.in**

Available at leading bookshops throughout India.

MTG BOOKS

For more information or to place your order :
Call at : 0124 - 4951200 @ Mail at : info@mtg.in
Log on to : www.mtg.in

But $\frac{p^2}{a^2}+k\cos\alpha+\frac{q^2}{b^2}=\sin^2\alpha$

$\therefore\quad k=-\frac{2pq}{ab}$

23. (c) : Coefficients of $T^5={}^nC_4$, $T^6={}^nC_5$ and $T^7={}^nC_6$
According to the given condition,

$2\,{}^nC_5={}^nC_4+{}^nC_6$

$\Rightarrow\quad 2\left[\frac{n!}{(n-5)!5!}\right]=\left[\frac{n!}{(n-4)!4!}+\frac{n!}{(n-6)!6!}\right]$

$\Rightarrow\quad 2\left[\frac{6}{(n-5)}\right]=\left[\frac{5.6}{(n-4)(n-5)}+1\right]$

$\Rightarrow\quad \frac{12}{(n-5)}=\frac{30+n^2-9n+20}{(n-4)(n-5)}$

$\Rightarrow\quad n^2-21n+98=0$

$\Rightarrow\quad (n-7)(n-14)=0$

$\Rightarrow\quad n=7$ or 14

24. (b): Let n be even.

$\therefore\quad S_n=1^2+2.2^2+3^2+2.4^2+.....+n\,.\,n^2=\frac{n(n+1)^2}{2}$

$\therefore\quad S_{n+1}=1^2+2\,.\,2^2+3^2+2\,.\,4^2+.....+n\,.\,n^2+(n+1)^2$

$=\frac{n(n+1)^2}{2}+(n+1)^2=\frac{(n+1)^2(n+2)}{2}$

Hence $n+1$ is odd.

Replacing $n+1$ by n, we get

$S_n=\frac{(n-1+1)^2(n-1+2)}{2}=\frac{n^2(n+1)}{2}$

25. (b): Let $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$. Then,

$\hat{i}\times(\vec{r}\times\hat{i})+\hat{j}\times(\vec{r}\times\hat{j})+\hat{k}\times(\vec{r}\times\hat{k})$

$=(\hat{i}\cdot\hat{i})\vec{r}-(\hat{i}\cdot\vec{r})\hat{i}+(\hat{j}\cdot\hat{j})\vec{r}-(\hat{j}\cdot\vec{r})\hat{j}+(\hat{k}\cdot\hat{k})\vec{r}-(\hat{k}\cdot\vec{r})\hat{k}$

$=\vec{r}-x\hat{i}+\vec{r}-y\hat{j}+\vec{r}-z\hat{k}$

$=3\vec{r}-(x\hat{i}+y\hat{j}+z\hat{k})=3\vec{r}-\vec{r}=2\vec{r}$

26. (b): Let the roots be α and 2α.

$\therefore\quad \alpha+2\alpha=-\frac{3a-1}{a^2-5a+3}$ and $\alpha.2\alpha=\frac{2}{a^2-5a+3}$

$\therefore\quad \alpha=\frac{1-3a}{3(a^2-5a+3)}$

and $\alpha^2=\frac{1}{a^2-5a+3}$

$\therefore\quad \left(\frac{1-3a}{3(a^2-5a+3)}\right)^2=\frac{1}{a^2-5a+3}$

$\Rightarrow\quad 1+9a^2-6a=9(a^2-5a+3)$

$\Rightarrow\quad 39a=26\Rightarrow a=2/3.$

27. (c) : For $n=2$, $a_n=2^{2^2}+1=16+1=17$

Assume that, $a_k=2^{2^k}+1=10\,m+7$, where $k>1$ and m is some positive integer.

Now, $a_{k+1}=2^{2^{k+1}}+1=(2^{2^k})^2+1$

$=(10\,m+6)^2+1$

$=10\,(10\,m^2+12\,m+3)+7$

Hence, last digit of a_n is 7, $\forall\ n>1$.

28. (c) : We have,

$\frac{dx}{dt}=\frac{3a(1+t^2)-3at\cdot 2t}{(1+t^2)^2}=\frac{3a(1-t^2)}{(1+t^2)^2}$ and $\frac{dy}{dt}=\frac{6at}{(1+t^2)^2}$

$\therefore\quad \frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{6at}{3a(1-t^2)}=\frac{2t}{1-t^2}$

$\therefore\quad \left[\frac{dy}{dx}\right]_{t=2}=-\frac{4}{3}$

Also, when $t=2$, then $x=\frac{6a}{5}$, $y=\frac{12a}{5}$

Hence, the equation of the tangent is

$y-\frac{12a}{5}=-\frac{4}{3}\left(x-\frac{6a}{5}\right)\Rightarrow 4x+3y-12a=0$

29. (a) : $\because\ y^2-12x-4y+4=0$

$\Rightarrow\quad (y-2)^2=12x$

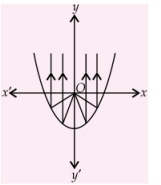
Its vertex is (0, 2) and $a=3$, its focus is (3, 2).

Hence, for the required parabola; focus is (3, 4), vertex is (3, 2) and $a=2$

Hence, the equation of the parabola is

$(x-3)^2=4(2)(y-2)$

or $x^2-6x-8y+25=0$



30. (b): $|az_1-bz_2|^2=(az_1-bz_2)(\overline{az_1-bz_2})$

$=(az_1-bz_2)(a\bar{z}_1-b\bar{z}_2)$

$=a^2z_1\bar{z}_1-abz_1\bar{z}_2-ab\bar{z}_1z_2+b^2z_2\bar{z}_2$

$=a^2|z_1|^2-ab(z_1\bar{z}_2+\overline{z_1\bar{z}_2})+b^2|z_2|^2$

$=a^2|z_1|^2-ab\,.\,2\,\mathrm{Re}(z_1\bar{z}_2)+b^2|z_2|^2$

Similarly,

$|bz_1+az_2|^2=b|z_1|^2+ab\,.\,2\ \mathrm{Re}\,(z_1\bar{z}_2)+a^2|z_2|^2$

$\therefore\quad |az_1-bz_2|^2+|bz_1+az_2|^2$

$=a^2|z_1|^2+b^2|z_2|^2+b^2|z_1|^2+a^2|z_2|^2$

$=(a^2+b^2)\,(|z_1|^2+|z_2|^2)$

20

Challenging Problems For Entrance Exams

SINGLE OPTION CORRECT

1. If $f(x)=\int_2^x t(t-3)^3(t-5)^5(t-7)^7(e^t-1)dt$ has maxima at $x=a$, then $\int_0^{a-3}(11x^{10}-5x^4-1)dx=$

(a) 2014 (b) 2015 (c) 2016 (d) 2017

2. If $f_r(K)=^{2K-1}P_r$ and $f_{n-1}(n+1):f_n(n)=5:12$, then $n=$

(a) 4 (b) 5 (c) 6 (d) 7

3. Let $f(x)=(x+1)^2+K(K+1+2x)+6=0$. If both roots of $f(x)=0$ be negative, then least prime value of K is

(a) 5 (b) 7 (c) 11 (c) 13

4. If $f(x)=\cot x$, then find the number of solutions of $f(x)+\sqrt{1+\{f(x)\}^2}=|f(x)|$ in $[0, 2\pi]$

(a) 0 (b) 1
(c) 2 (d) more than 2

5. If $y=f(x)$ be a real function so that

$3f\left(\frac{x+y}{3}\right)=f(x)+f(y)\ \forall\ x, y\in R$ and

$f(0)=2=f'(0)$, then the graph will be a

(a) straight line passing through origin
(b) parabola passing through (1, 1)
(c) straight line passing through (0, 2)
(d) none of these

6. If $f(x)=\begin{vmatrix}\cos(x+a) & \sin(x+a) & \sin(b-c)\\ \cos(x+b) & \sin(x+b) & \sin(c-a)\\ \cos(x+c) & \sin(x+c) & \sin(a-b)\end{vmatrix}$ such that $f(2014)=19$, then $\sum_{r=1}^{106}f(r)$ is

(a) >2014 (b) <2014
(c) $=2014$ (d) none of these

7. If $f(x)$ is an identity function, then

$\lim_{x\to\infty}\left\{\frac{f(x)\,f(x+1)+1}{f(x)\,f(x+4)+2}\right\}^{3f(x)-1}=$

(a) e^{-9} (b) e^{-6}
(c) -9 (d) none of these

8. If $f(x)=1+\sum_{r=1}^{2014}\frac{x^r}{r}$ then $f'(1):f(0)=K^3+2K^2-2$, then $K=$

(a) 1 (b) 14 (c) 12 (d) 16

9. If $f:[-5, 4]\to R$ then domain of $f([\,|2x|\,])$ is (where $[\cdot]$ denotes greatest integer function)

(a) $\left(-\frac{3}{2}, \frac{3}{2}\right)$ (b) $\left(-\frac{5}{2}, \frac{5}{2}\right)$

(c) $\left(-\frac{1}{2}, \frac{1}{2}\right)$ (d) none of these

10. If $f: R\to R$ defined as $f(x)=x^3+2x+3$ and $g(x)$ be its inverse, then $g''(3)=$

(a) 3 (b) 0
(c) 6 (d) does not exist

11. If $f\left(\frac{x}{y}\right)=f(x)-f(y)\ \forall x, y\in R-\{0\}$ and

$\lim_{t\to0}\frac{f(1+t)}{t}=1+f(1)$ then $xf''(x)=$

(a) $-f'(x)$ (b) $f'(x)$
(c) $f(x)$ (d) none of these

12. If $f(x)=a_0x^n+a_1x^{n-1}+\ldots\ldots\ldots+a_n$ and roots of $f(x)=0$ are all real, then

(a) $ff''>(f')^2$ (b) $ff''<(f')^2$
(c) $ff''=(f')^2$ (d) none of these

13. If $f(x)=\int_{1+a}^{x}\frac{dt}{f(t)}$ and $\int_{a+1}^{2}\frac{dx}{f(x)}=\sqrt{3}$, then domain of $f(x)$ is

(a) $(1, \infty)$ (b) $\left(-\infty, \frac{1}{2}\right)$

(c) $\left[\frac{1}{2}, \infty\right)$ (d) none of these

By : **Anil Kr. Gupta (AKG Classes), ASANSOL, Kolkata** (W.B). Mob.: 09832230099

14. If $f(x)=\dfrac{[|x|+x]}{[x]}$ then $f(x)$ is (where $[\cdot]$ is greatest integer function)

(a) continuous at $x=0$

(b) discontinuous at $x=0$ because $\lim\limits_{x\to 0^-} f(x)$ does not exist

(c) discontinuous at $x=0$ because $\lim\limits_{x\to 0^+} f(x)$ does not exist

(d) zero when x tends to 0

MORE THAN ONE OPTION CORRECT

15. If any function $f(x, y)$ be defined in such a way that for every non-negative integer x and y we have $f(x+1, y+1)=f(x, f(x+1, y))$; $f(x+1, 0)=f(x, 1)$ and $f(0, x)=1+x$, then

(a) $f(1,0)+f(2,0)=f(3,0)$

(b) $f(1,1)$, $f(2,1)$ and $f(2,2)$ are in A.P.

(c) $f(0,1)$, $f(0,2)$, $f(1,3)$, $f(1,5)$ all are prime numbers

(d) $f(1,1)+f(2,2)=f(1,3)+f(2,1)$

16. If $f(x)=x[x]$, then

(a) $\lim\limits_{x\to 0^-} f(x)\neq \lim\limits_{x\to 0^+} f(x)$

(b) $\lim\limits_{x\to 0^+} f(x)=f(0)$

(c) $\lim\limits_{x\to 0^-} f(x)=f(0)$

(d) $\lim\limits_{x\to 0^-} f(x)=\lim\limits_{x\to 0^+} f(x)\neq f(0)$

17. If $f(x)=|x+3|-2|x+2|+|x-1|$ but $f(x)=m^2+3m$ has no solution them $m\in$

(a) $(-\infty,-4)$ (b) $(-2,-1)$

(c) $(1,\infty)$ (d) $(-1,1)$

18. If $f(x)=(a^2+a+1)x^2+bx+c$ and $f(1-t)-f(1+t)=0$ $\forall\ t\in R$ then

(a) $f(1-\sqrt{3})<f(\sqrt{3})<f(2+\sqrt{3})<f(4)$

(b) $f(\sqrt{3})<f(1-\sqrt{3})<f(2+\sqrt{3})<f(4)$

(c) $f(2+\sqrt{3})>f(4)>f(\sqrt{3})>f(1-\sqrt{3})$

(d) $f(4)>f(2+\sqrt{3})>f(1-\sqrt{3})>f(\sqrt{3})$

19. If $f(x)=\left(\dfrac{K^2-4K}{K^2+K+2}\right)x^3-2x+e^{3\log K}$ and $f(x)$ is monotonically decreasing on R then $K\in$

(a) $(-1,1)$ (b) $(-1,4)$

(c) $(0,4)$ (d) $(-1,10)$

20. If $f(x)=\sum\limits_{r=1}^{10}(-1)^{r+1}x^{r-1}$ but $f(x)$ can also be expressed as $\sum\limits_{r=0}^{9} a_r(x+1)^r$, then

(a) $a_1=45$ (b) $a_1=-45$

(c) $a_2=240$ (d) $a_2=210$

SOLUTIONS

1. (a): $f'(x)=x(x-3)^3(x-5)^5(x-7)^7(e^x-1)$

Scheme for sign of $f'(x)$ is as follows

$$\overset{-}{\underset{0}{|}}\ \overset{-}{\underset{3}{|}}\ \overset{+}{\underset{5}{|}}\ \overset{-}{\underset{7}{|}}\ \overset{+}{|}$$

$\therefore$ $f'(x)$ changes sign from +ve to –ve at $x=5$

$\therefore$ $f(x)$ has maxima at $x=5 \Rightarrow a=5$

$\therefore$ $\int\limits_{0}^{a-3}(11x^{10}-5x^4-1)dx=[x^{11}-x^5-x]_0^2$

$=2048-32-2=2014$

2. (d): $\because$ $f_r(K)=2^{K-1}P_r$ and $f_{n-1}(n+1):f_n(n)=5:12$

$\therefore$ $2^{(n+1)-1}P_{n-1}:2^{n-1}P_n=5:12$

$\Rightarrow \dfrac{\lfloor 2n+1}{\lfloor n+2}\cdot\dfrac{\lfloor n-1}{\lfloor 2n-1}=\dfrac{5}{12} \Rightarrow \dfrac{(2n+1)2n}{(n+2)(n+1)n}=\dfrac{5}{12}$

$\Rightarrow 5(n^2+3n+2)=48n+24 \Rightarrow 5n^2-33n-14=0$

$\Rightarrow n=7, \dfrac{-2}{5}$ But, $n\neq -\dfrac{2}{5}$ $\therefore$ $n=7$

3. (b): Equation can be written as

$x^2+2(K+1)x+K^2+K+7=0$

Both roots are negative

$\Rightarrow -2(K+1)<0 \Rightarrow K>-1$ ...(1)

$D\geq 0 \Rightarrow 4(K+1)^2-4(K^2+K+7)\geq 0 \Rightarrow K\geq 6$ ...(2)

Also, $K^2+K+7>0$ [$\because$ product of roots will be +ve]

$\left(K+\dfrac{1}{2}\right)^2+\dfrac{27}{4}>0$, always true

$\therefore$ From (1) & (2), $K\geq 6$

$\therefore$ Least prime value of $K=7$

4. (c): We have, $\cot x+\sqrt{1+\cot^2 x}=|\cot x|$

$\Rightarrow \cot x+|\operatorname{cosec} x|=|\cot x|$

If x lies in 1st quadrant, $\cot x+\operatorname{cosec} x=\cot x$

$\Rightarrow \operatorname{cosec} x=0$ (absurd)

If x lies in 2nd quadrant, $\cot x+\operatorname{cosec} x=-\cot x$

$\Rightarrow \dfrac{2\cos x}{\sin x}+\dfrac{1}{\sin x}=0 \Rightarrow \cos x=-\dfrac{1}{2} \Rightarrow x=\dfrac{2\pi}{3}$

If x lies in 3rd quadrant, $\cot x-\operatorname{cosec} x=\cot x$

$\Rightarrow \operatorname{cosec} x=0$ (absurd)

If x lies in 4th quadrant, $\cot x-\operatorname{cosec} x=-\cot x$

$2\cos x-1=0 \Rightarrow x=\dfrac{5\pi}{3}$

$\therefore$ 2 solutions exist in $[0, 2\pi]$

5. (c): $\because$ $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

$=\lim\limits_{h\to 0}\dfrac{3f\left(\dfrac{3x+3h}{3}\right)-3f\left(\dfrac{3x+3(0)}{3}\right)}{3h}$

$=\lim\limits_{h\to 0}\dfrac{f(3x)+f(3h)-\{f(3x)+f(0)\}}{3h}$

$=\lim\limits_{h\to 0}\dfrac{f(0+3h)-f(0)}{3h}=f'(0)=2$

$\therefore$ On integrating, $f(x)=2x+C$

Putting $x = 0 \Rightarrow f(0) = 0 + C \ \therefore C = 2 \quad [\because f(0) = 2]$
$\Rightarrow f(x) = 2x + 2 \Rightarrow y = f(x) = 2x + 2$
This is a straight line passing through (0, 2)

6. (c) : $\because f'(x) = 0$
$\therefore$ On integrating, we get $f(x)$ = constant
However, $f(2014) = 19 \ \therefore f(x) = 19 \ \forall \ x \in R$

$\therefore \sum_{r=1}^{106} f(r) = f(1) + f(2) + f(3) + ... + f(106)$
$= 19 \times 106 = 2014$

7. (a) : $\because f(x) = x$ is an identity function

$\therefore \lim_{x\to\infty} \left\{\frac{f(x)\,f(x+1)+1}{f(x)\,f(x+4)+2}\right\}^{3f(x)-1}$

$= \lim_{x\to\infty} \left\{\frac{x^2+x+1}{x^2+4x+2}\right\}^{3x-1} \quad [1^\infty \text{ form}]$

$= e^{\lim_{x\to\infty}\left(\frac{x^2+x+1}{x^2+4x+2}-1\right)(3x-1)}$

$= e^{\lim_{x\to\infty}\frac{(-3x-1)(3x-1)}{x^2+4x+2}} = e^{\lim_{x\to\infty}\frac{(-9x^2+1)}{x^2+4x+2}} = e^{-9}$

8. (c) : $f(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{3} + ... + \frac{x^{2014}}{2014}$

$f(0) = 1$ & $f'(x) = 1 + x + x^2 + ... + x^{2013} \Rightarrow f'(1) = 2014$
$\therefore f'(1) : f(0) = K^3 + 2K^2 - 2$ gives $2014 = K^3 + 2K^2 - 2$
$\Rightarrow K^2(K+2) = 2016$ is satisfied by $K = 12$

9. (d): We have $-5 \le [\,|2x|\,] \le 4 \ \Rightarrow \ -5 \le |2x| < 5$
$\Rightarrow \ 0 \le |2x| < 5 \quad [\because \ |2x| \ge 0]$

$\Rightarrow \ |x| < \frac{5}{2} \ \Rightarrow \ -\frac{5}{2} < x < \frac{5}{2} \Rightarrow x \in \left(-\frac{5}{2}, \frac{5}{2}\right)$

10. (b): $\because \ f(0) = 3 \ \therefore \ g(3) = 0 \qquad (\because \ g = f^{-1})$
$\because \ f^{-1}(x) = g(x) \ \Rightarrow \ f\{g(x)\} = x \ \Rightarrow \ f'\{g(x)\}\cdot g'(x) = 1$

$\Rightarrow \quad g'(x) = \frac{1}{f'\{g(x)\}} \qquad ...(1)$

$\Rightarrow \quad g''(x) = \frac{-1}{[f'\{g(x)\}]^2}\cdot f''\{g(x)\}\cdot g'(x) = \frac{-f''\{g(x)\}}{[f'\{g(x)\}]^3}$

[from (1)]

$g''(3) = -\frac{f''\{g(3)\}}{[f'\{g(3)\}]^3} = \frac{-f''(0)}{\{f'(0)\}^3} \qquad ... (2)$

$\because \ f'(x) = 3x^2 + 2 \Rightarrow f''(x) = 6x \ \Rightarrow f'(0) = 2, f''(0) = 0$
Hence, $g''(3) = 0$

11. (a) : $x = y = 1 \ \Rightarrow \ f(1) = f(1) - f(1) \Rightarrow f(1) = 0 \quad ...(1)$

$\left[\because \ f\left(\frac{x}{y}\right) = f(x) - f(y)\right] \qquad ...(2)$

$\because \ f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{f\left(\frac{x+h}{x}\right)}{\frac{h}{x}\cdot x}$

[from (2)]

$= \lim_{h\to 0} \frac{f\left(1+\frac{h}{x}\right)}{\frac{h}{x}}\cdot\frac{1}{x} = \lim_{t\to 0}\frac{f(1+t)\frac{1}{x}}{t}\left[\frac{h}{x} = t\right]$

$= \{1 + f(1)\}\cdot\frac{1}{x} = \frac{1}{x} \ [\because f(1) = 0]$

$\Rightarrow \ xf'(x) = 1 \ \Rightarrow \ xf''(x) + 1\cdot f'(x) = 0$
$\Rightarrow \ xf''(x) = -f'(x)$

12. (b): Let α_i (i=1, 2, 3, ..., n) be n real roots of $f(x) = 0$
$\therefore \ f(x) = a(x-\alpha_1)(x-\alpha_2).....(x-\alpha_n)$
$\Rightarrow \ \log f(x) = \log a + \log(x-\alpha_1) + \log(x-\alpha_2) + ...$
$....+ \log(x-\alpha_n)$

$\Rightarrow \quad \frac{1}{f(x)}\cdot f'(x) = 0 + \frac{1}{x-\alpha_1} + \frac{1}{x-\alpha_2} + ... + \frac{1}{x-\alpha_n}$

$\Rightarrow \quad \frac{f''(x)\cdot f(x) - f'(x)\cdot f'(x)}{\{f(x)\}^2}$

$= -\frac{1}{(x-\alpha_1)^2} - \frac{1}{(x-\alpha_2)^2} - ..... - \frac{1}{(x-\alpha_n)^2}$

$\Rightarrow \quad \frac{f''f - (f')^2}{f^2} < 0 \ \Rightarrow \ ff'' < (f')^2$

13. (c) : $\because \ f(x) = \int_{1+a}^{x} \frac{dt}{f(t)} \ \therefore \ f'(x) = \frac{1}{f(x)} \qquad ...(1)$

[on differentiating w.r.t. x]

$\therefore \ f(x) = \int_{1+a}^{x} \frac{dx}{f(x)}$ [using property]

$= \int_{1+a}^{x} f'(x)dx = f(x) - f(1+a) \ \therefore \ f(1+a) = 0$

Also, $\int_{a+1}^{2} \frac{dx}{f(x)} = \sqrt{3} \ \Rightarrow \ \sqrt{3} = \int_{a+1}^{2} f'(x)dx$

$= f(2) - f(a+1) = f(2) \ [\because \ f(1+a) = 0]$
From (1), $2f(x)f'(x) = 2 \ \Rightarrow \ \{f(x)\}^2 = 2x + C$
[on integrating]

Putting $x = 2 \Rightarrow (\sqrt{3})^2 = 2\cdot 2 + C \ \Rightarrow \ C = -1$
$\therefore \ \{f(x)\}^2 = 2x - 1 \ \Rightarrow \ f(x) = \pm\sqrt{2x-1} \ [\because \ f(2) = \sqrt{3}]$

$\therefore$ Domain of $f(x)$ is $\left[\frac{1}{2}, \infty\right)$

14. (c) : L.H.L. $= \lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{[\,|x|+x\,]}{[x]}$

$= \lim_{h\to 0} \frac{[\,|0-h|+(0-h)\,]}{[0-h]} = \lim_{h\to 0} \frac{[h-h]}{[-h]} = \lim_{h\to 0} \frac{0}{(-1)} = 0$

R.H.L. $= \lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{[\,|x|+x\,]}{[x]}$

$= \underset{h\to 0}{\text{Lt}} \dfrac{[|0+h|+(0+h)]}{[0+h]}$

$= \underset{h\to 0}{\text{Lt}} \dfrac{[2h]}{[h]}$, which does not exist.

$\therefore$ $f(x)$ is discontinuous as R.H.L. does not exist.
Note : Here, $f(0)$ also does not exist.

15. (a, b, c, d) : $\because$ $f(x+1, y+1) = f(x, f(x+1, y))$ ...(1)
$f(x+1, 0) = f(x, 1)$ ...(2)
and $f(0, x) = 1 + x$ ...(3)
$\therefore$ $f(1, 0) = f(0, 1)$ [using (2)]
$= 1 + 1$ [using (3)] $= 2$
$\therefore$ $f(1, 0) = f(0, 1) = 2$ $\therefore$ $f(2, 0) = f(1, 1)$ [using (2)]
$= f(0, f(1, 0))$ [using (1)]
$= 1 + f(1, 0)$ [using (3)] $= 1 + 2 = 3$
Also, $f(0, 2) = 1 + 2 = 3$ [using (3)]
$\therefore$ $f(2, 0) = f(1, 1) = f(0, 2) = 3$
Again, $f(3, 0) = f(2, 1)$ [using (2)]
$= f(1, f(2, 0))$ [using (1)] $= f(1, 3)$ [as above]
$= f(0, f(1, 2))$ [using (1)]
$= 1 + f(1, 2)$ [using (3)]
$= 1 + f(0, f(1, 1))$ [using (1)]
$= 1 + 1 + f(1, 1)$ [using (3)]
$= 2 + 3$ [as above] $= 5$
$\therefore$ $f(3, 0) = f(2, 1) = f(1, 3) = 5$
Further $f(2, 2) = f(1, f(2, 1)) = f(1, 5)$ [as above]
$= f(0, f(1, 4))$ [using (1)]
$= 1 + f(1, 4)$ [using (3)]
$= 1 + f(0, f(1, 3))$ [using (1)]
$= 1 + 1 + f(1, 3)$ [using (3)]
$= 2 + 5$ [as above]
$= 7$ $\therefore$ $f(2, 2) = f(1, 5) = 7$
Hence, all given options are correct.

16. (b, c) : $\underset{x\to 0^-}{\text{Lt}} f(x) = \underset{x\to 0^-}{\text{Lt}} x[x] = \underset{h\to 0}{\text{Lt}} (0-h)[0-h]$

$= \underset{h\to 0}{\text{Lt}} (-h)(-1) = \underset{h\to 0}{\text{Lt}} h = 0$

$\underset{x\to 0^+}{\text{Lt}} f(x) = \underset{x\to 0^+}{\text{Lt}} x[x] = \underset{h\to 0}{\text{Lt}} (0+h)[0+h] = \underset{h\to 0}{\text{Lt}} h(0) = 0$

And, $f(0) = 0$

17. (a, b, c) :

Here, $f(x) = \begin{cases} -(x+3) + 2(x+2) - (x-1), & x < -3 \\ (x+3) + 2(x+2) - (x-1), & -3 \le x < -2 \\ (x+3) - 2(x+2) - (x-1), & -2 \le x < 1 \\ (x+3) - 2(x+2) + (x-1), & x \ge 1 \end{cases}$

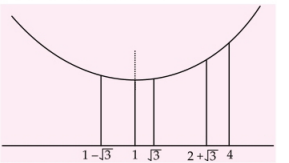
i.e., $f(x) = \begin{cases} 2, & x < -3 \\ 2x + 8, & -3 \le x < -2 \\ -2x, & -2 \le x < 1 \\ -2, & x \ge 1 \end{cases}$

$\therefore$ $f_{max} = 4$ (at $x = -2$) and $f_{min} = -2$ (at $x = 1$)
$\therefore$ $f(x) = m^2 + 3m$ will have no solution if
$m^2 + 3m > 4$ or < -2
i.e. if $m^2 + 3m - 4 > 0$ or $m^2 + 3m + 2 < 0$
i.e. if $(m + 4)(m - 1) > 0$ or $(m + 2)(m + 1) < 0$
i.e. if $\{m < -4$ or $m > 1\}$ or $\{-2 < m < -1\}$

18. (b, d) : $\because$ Coefficient of $x^2 = a^2 + a + 1$

$= \left(a + \dfrac{1}{2}\right)^2 + \dfrac{3}{4} > 0$

$\therefore$ Graph of $f(x)$ will be concave upward and a parabola.
$\because$ $f(1-t) = f(1+t) \Rightarrow f(x)$ is symmetric about $x = 1$



$\therefore$ From the figure, it is clear that
$f(\sqrt{3}) < f(1 - \sqrt{3}) < f(2 + \sqrt{3}) < f(4)$

19. (b, c, d) : If $f'(x) = \dfrac{K^2 - 4K}{K^2 + K + 2} \cdot 3x^2 - 2 + 0$

$\Rightarrow$ $f'(x) = \dfrac{K^2 - 4K}{K^2 + K + 2} \cdot 3x^2 - 2$

Being the graph as monotonically decreasing,
$f'(x) < 0$

$\Rightarrow$ $K^2 - 4K < 0$ as $\dfrac{3x^2}{K^2 + K + 2} \ge 0$

$\Rightarrow$ $0 < K < 4$

20. (b, c) : $\because$ $f(x) = \sum_{r=0}^{9} a_r (x+1)^r$

$= a_0 + a_1(x+1) + a_2(x+1)^2 + ... + a_9(x+1)^9$
$\therefore$ $f'(x) = a_1 + a_2 \cdot 2(x+1) + ... + a_9 \cdot 9(x+1)^8$
$f''(x) = 2a_2 + 6a_3(x+1) + ...$
$\therefore$ $f'(-1) = a_1$ and $f''(-1) = 2a_2$

Also, $f(x) = \sum_{r=1}^{10} (-1)^{r+1} x^{r-1} = 1 - x + x^2 - .....- x^9$

$f'(x) = -1 + 2x - 3x^2 + ...$ ...(1)
$f''(x) = 2 - 6x + 12x^2 - .....$ ...(2)

From (1), $f'(-1) = -1 - 2 - 3 - ..... - 9 = -\dfrac{(9)(9+1)}{2} = -45$
$\therefore$ $a_1 = -45$
From (2), $f''(-1) = 2 - 6(-1) + 12(-1)^2 - .....$
$= 2 + 6 + 12 + ...$
$= 1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + ... + 8 \cdot 9$
$= 1(1+1) + 2(2+1) + 3(3+1) + ... + 8(8+1)$
$= \dfrac{8(8+1)(16+1)}{6} + \dfrac{8(8+1)}{2}$
$= 204 + 36 = 240$

MATHS MUSING

SOLUTION SET-134

1. (d) : $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1, \vec{a}\cdot\vec{b} = \frac{1}{2}, |\vec{a}\times\vec{b}| = \frac{\sqrt{3}}{2}$

$\vec{a}\cdot\vec{c} = \frac{1}{2} = \alpha\left(1+\frac{1}{2}\right) \Rightarrow \alpha = \frac{1}{3}$

$|\vec{c}| = 1 \Rightarrow 1 = 3\alpha^2 + \frac{3}{4}\beta^2 \Rightarrow \beta = \frac{2\sqrt{2}}{3}, \frac{\beta}{\alpha} = 2\sqrt{2}$

2. (d) : $P(HHH) + P(HHTT) + P(HTTTT) + P(TTTTTT)$

$= \frac{1}{2^3} + \binom{4}{2}\frac{1}{2^4} + \binom{5}{1}\frac{1}{2^5} + \frac{1}{2^6} = \frac{43}{64}.$

3. (a) : Integration by parts,

$5 = -\cos x\, f(x)\big|_0^\pi + \int_0^\pi \cos x.f'(x)dx + \int_0^\pi f''(x)\sin x dx$

$= f(\pi) + f(0) = 2 + f(0)$

$\therefore \quad f(0) = 5 - 2 = 3$

4. (c) : Let $x + \frac{1}{x} = t$

$\therefore$ The given equation becomes,

$\Rightarrow t^2 - 3t - 4 = 0$

$t = 4, -1 \Rightarrow t = 4 \Rightarrow x = 2 \pm \sqrt{3}$

$t = -1 \Rightarrow x = \text{cis}\left(\pm\frac{2\pi}{3}\right)$

Sum of the moduli $2 + 2 + 1 + 1 = 6$.

5. (b) : Let $CD = x$, $DA = y$

$AC^2 = 19 = x^2 + y^2 + xy$

$4\sqrt{3} = \frac{1}{2}\cdot 2\cdot 5\frac{\sqrt{3}}{2} + \frac{1}{2}xy\cdot\frac{\sqrt{3}}{2} \Rightarrow xy = 6$

$x^2 + y^2 = 13,\ xy = 6 \Rightarrow x = 2,\ y = 3$

Perimeter $= 2 + 5 + 2 + 3 = 12$

6. (b, c) : Circles through (3, 2) and (5, 4) are

$(x-3)(x-5) + (y-2)(y-4) + \lambda(x-y-1) = 0$

(2, 0) lies on it, $\lambda = -11$

$y = 0 \Rightarrow t^2 - 8t + 15 + 8 - 11t + 11 = 0$

$t^2 - 19t + 34 = 0, \Rightarrow t = 2, 17$

7. (b) : $\frac{2h_1h_3}{h_1+h_3} = 16 \Rightarrow (h_1 - 8)(h_3 - 8) = 64.$

$\frac{1}{h_3} + \frac{1}{16} > \frac{1}{h_1} \Rightarrow \frac{1}{16} > \frac{1}{h_1} - \frac{1}{h_3} = \frac{h_3 - h_1}{8(h_3 + h_1)}$

$\Rightarrow \quad 3h_1 > h_3 \Rightarrow h_1 = 12,\ h_3 = 24$

$\therefore \quad a = \frac{\Delta}{6}, b = \frac{\Delta}{8}, c = \frac{\Delta}{12}$

$\Rightarrow \Delta = 256\sqrt{\frac{3}{5}}, s = 48\sqrt{\frac{3}{5}}, r = \frac{\Delta}{s} = \frac{16}{3}$

8. (d) : $R = \frac{abc}{4\Delta} = \frac{256}{15}$

9. (8) : AAA I NRSV

$3 + 1 + 1 \Rightarrow \binom{5}{2}\cdot\frac{5!}{3!} = 200$ words

$2 + 1 + 1 + 1 \Rightarrow \binom{5}{3}\cdot\frac{5!}{2!} = 600$ words

$1 + 1 + 1 + 1 + 1 \Rightarrow \binom{6}{5}\cdot 5! = 720$ words

$\therefore \quad n = 1520$. Sum of the digits is 8.

10. (c) :

P. $S = 1^2 + 3^2 + 5^2 + ... + 21^2 = \binom{23}{3} = 1771$

Q. $S = 1^3 + 3^3 + 5^3 + ... + 19^3 = 10^2(2\times 10^2 - 1) = 19900$

R. $S = -1^2 + 3^2 - 5^2 + ... - 33^2 + 35^2$

$= 2(1 + 3 + 5 + ... + 35) = 2 \times 18^2$

S. $S = 1^3 - 3^3 + 5^3 - 7^3 + ... + 29^3$

$= 15(4 \times 15^2 - 3) = 13455$

Solution Sender of Maths Musing

SET-134

1. Khwaja Sami Baig : Azamgarh (UP)
2. Ayaz Ahmed : Jharkhand
3. Gouri Sankar Adhikary : Medinipur (WB)

PRACTICE PAPER 2014

JEE Main & Advanced

SECTION-I

SINGLE CORRECT ANSWER TYPE

This section 9 single correct answer type questions. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

1. If $b-c$, $2b-x$ and $b-a$ are in H.P., then $a-\left(\frac{x}{2}\right)$, $b-\left(\frac{x}{2}\right)$ and $c-\left(\frac{x}{2}\right)$ are in

(a) A.P. (b) G.P.
(c) H.P. (d) none of these

2. A function $f: Q^+ \to Q^+$ is defined such that $f(x)+f(y)+2xyf(xy)=\frac{f(xy)}{f(x+y)}$, then $f(1)$ is equal to

(a) 1 (b) 0
(c) 2 (d) none of these

3. The number of seven digit integers, with sum of the digits equal to 10 and formed by using the digits 1, 2 and 3 only, is

(a) 55 (b) 66 (c) 77 (d) 88

4. In a complex plane the points A and B are at $z_1 = 5 - 2i$ and $z_2 = 1 + i$. If $P(z)$ moves such that $|z - z_1| = 2\,|z - z_2|$, then the maximum area of ΔPAB is

(a) 5 (b) 1/3 (c) 25/3 (d) 50/3

5. If in a ΔABC, $\cos A + 2\cos B + \cos C = 2$, then a, b, c are in

(a) A.P. (b) H.P. (c) G.P. (d) A.G.P.

6. A function $g(\theta)=\int_0^{\sin^2\theta} f(x)dx+\int_0^{\cos^2\theta} f(x)dx$ is defined in the interval $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ where $f(x)$ is an increasing function, $g(\theta)$ is increasing in the interval

(a) $\left(-\frac{\pi}{2},0\right)$ (b) $\left(-\frac{\pi}{2},-\frac{\pi}{4}\right)$
(c) $\left(0,\frac{\pi}{2}\right)$ (d) $\left(-\frac{\pi}{4},0\right)$

7. $\int_0^{\infty}\frac{e^{x/a}-e^{b/x}}{x}dx,\ (ab\neq 0)$ is equal to

(a) a/b (b) $x/2$
(c) ab (d) none of these

8. If $y=\frac{x}{\ln|cx|}$ (where c is an arbitrary constant) is the general solution of the differential equation $\frac{dy}{dx}=\frac{y}{x}+\phi\left(\frac{x}{y}\right)$, then the function $\phi\left(\frac{x}{y}\right)$ is

(a) $\frac{x^2}{y^2}$ (b) $-\frac{x^2}{y^2}$ (c) $\frac{y^2}{x^2}$ (d) $-\frac{y^2}{x^2}$

9. If A is a skew symmetric matrix, then $B=(I-A)(I+A)^{-1}$ is (where I is an identity matrix of same order as of A)

(a) idempotent matrix
(b) symmetric matrix
(c) orthogonal matrix
(d) none of these

SECTION-II

MULTIPLE CORRECT ANSWER TYPE

This section contains 4 multiple correct answer(s) type questions. Each question has 4 choices (a), (b), (c) and (d), out of which ONE OR MORE is/are correct.

10. If a, b, c, d are in A.P. and $\int_0^2 f(x)dx=-4$, where $f(x)=\begin{vmatrix} x+a & x+b & x+a-c \\ x+b & x+c & x-1 \\ x+c & x+d & x-b+d \end{vmatrix}$, then the common difference of the A.P. may be

(a) 1 (b) –1 (c) 2 (d) 4

11. Let z_1 and z_2 be two complex numbers such that $z_1^2 - 4z_2 = 16 + 20i$. If α and β are roots of $x^2+z_1x+z_2+M=0$ (where M is complex number) and $|(\alpha-\beta)^2| = 28$, then

(a) maximum value of $|M|$ is $7+\sqrt{41}$
(b) maximum value of $|M|$ is $5+\sqrt{41}$

(c) minimum value of $|M|$ is $7-\sqrt{41}$

(d) minimum value of $|M|$ is $5-\sqrt{41}$

12. The system of equations $ax+by+cz=q-r$, $bx+cy+az=r-p$ and $cx+ay+bz=p-q$ is

(a) consistent if $p=q=r$

(b) inconsistent if $a=b=c$ and p, q, r are distinct

(c) consistent if a, b, c are distinct and $a+b+c\neq 0$

(d) all of above

13. If the tangents of the angles A and B of a triangle ABC satisfy the equation $abx^2-c^2x+ab=0$, then

(a) $\tan A=\frac{a}{b}$

(b) $\tan B=\frac{b}{a}$

(c) $\cos C=0$

(d) $\sin^2A+\sin^2B+\sin^2C=2$

SECTION-III

REASONING TYPE

This section contains 3 reasoning type questions. Each question contains Statement-1 and Statement-2. Each question has 4 choices (a), (b), (c) and (d) out of which ONLY ONE is correct.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(b) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.

(c) Statement-1 is true, Statement-2 is false.

(d) Statement-1 is false, Statement-2 is true.

14. **Statement-1:** $f(x)=\sin^{-1}[x]$ and $g(x)=\cos^{-1}[x]$ are identical functions (where [.] denotes greatest integer function)

Statement-2 : Identical functions have the same domain.

15. **Statement-1 :** Let A, B, C are three independent events and $P(A)=\frac{1}{2}, P(B)=\frac{1}{3}, P(C)=\frac{1}{6}$, then $P(\bar{A}\bar{B}C)=\frac{1}{18}$.

Statement-2 : If A, B, C are 3 independent events then $\bar{A}, \bar{B}, C$ are also independent.

16. **Statement-1 :** There exist no triangle ABC which has the property that $\cos A=\frac{3}{5}$ and $\tan B-\tan C=\frac{1}{3}$.

Statement-2 : In a triangle ABC;
$\tan A+\tan B+\tan C=\tan A\tan B\tan C$.

SECTION IV

MATRIX-MATCH TYPE

This section contains 2 questions.

Each question contains statements given in two columns which have to be matched. The statements in Column I are labelled as A, B, C and D while the statements in Column II are labelled as p, q, r, s and t.

	p	q	r	s	t
A	(p)	(q)	(r)	(s)	(t)
B	(p)	(q)	(r)	(s)	(t)
C	(p)	(q)	(r)	(s)	(t)
D	(p)	(q)	(r)	(s)	(t)

The answers to these questions have to be appropriately bubbled as illustrated in the following example. If the correct matches are A – p, s, t ; B – q, r ; C – p, q ; D – s, t, then the correctly bubbled matrix will look like the above.

17. Match the following:

Column-I		Column-II	
(A)	The solution set of $\log_{100}\lvert x+y\rvert=\frac{1}{2}$, $\log_{10}y-\log_{10}\lvert x\rvert=\log_{100}4$ is	(p)	$\{\sqrt{2}, 2\}$
(B)	The solution set of $4(\log_2 x)^2+1=2\log_2 y$ and $\log_2 x^2\geq\log_2 y$ is	(q)	$\{1, 1\}$
(C)	The solution set of $\log_4 x-\log_2 y=0$ and $x^2-5y^2+4=0$ is	(r)	$\{-10, 20\}$
(D)	The solution set of the equation $2x^2-16x+3y^2-12y+44=0$ is	(s)	$\{4, 2\}$
		(t)	$\left\{\frac{10}{3}, \frac{20}{3}\right\}$

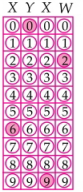
18. If $\vec{x}, \vec{y}, \vec{z}$ are unit vectors such that $\vec{x}+\vec{y}+\vec{z}=\vec{a}$, $\vec{x}\times(\vec{y}\times\vec{z})=\vec{b}$, $(\vec{x}\times\vec{y})\times\vec{z}=\vec{c}$, $\vec{a}\cdot\vec{x}=\frac{3}{2}$, $\vec{a}\cdot\vec{y}=\frac{7}{4}$, $\vec{a}=2\hat{a}$, then match column–I with column–II.

Column-I		Column-II	
(A)	$\vec{x}=$	(p)	$-4\vec{c}$
(B)	$\vec{y}=$	(q)	$\frac{4}{3}(\vec{c}-\vec{b})$
(C)	$\vec{z}=$	(r)	$\frac{1}{3}(3\vec{a}+4\vec{b}+8\vec{c})$
(D)	$3\vec{x}+2\vec{y}+3\vec{z}=$	(s)	$(3\vec{a}+4\vec{c})$
		(t)	$\frac{2}{3}(\vec{c}-\vec{b})$

SECTION-V

INTEGER TYPE

This section contains 4 questions. The answer to each of the questions is a single digit integer, ranging from 0 to 9. The appropriate bubbles below the respective question numbers in the ORS have to be darkened. For example, if the correct answers to question numbers X, Y, Z and W (say) are 6, 0, 9 and 2 respectively, then the correct darkening of bubbles will look like the following:



19. Find the number of solution(s) of trigonometric equation $\sin(\cos x) = \cos(\sin x)$, where $x \in [0, 2\pi]$

20. If $a(x^2 + y^2 + 2y + 1) = (x - 2y + 3)^2$ is an ellipse and $a \in (b, \infty)$, then find 'b'.

21. Let $f(x) = (x + 1)(x + 2)(x + 3)(x + 4) + 5$ where $x \in [-6, 6]$. If the range of the function is $[a, b]$ where $a, b, \in N$, then find the value of a.

22. If two of the lines represented by $x^4 + x^3y + cx^2y^2 - xy^3 + y^4 = 0$ bisect the angle between the other two, then find the value of $c + 10$.

SOLUTIONS

1. (b): If, x, y, z are in H.P.

then, $x - \frac{y}{2}, \frac{y}{2}, z - \frac{y}{2}$ are in G.P.

2. (a): Let $f(1) = a$

Then, $f(x) + a + 2xf(x) = \dfrac{f(x)}{f(x+1)}$

$$f(x+1) = \frac{f(x)}{(1+2x)f(x) + a}$$

Hence $f(2) = \dfrac{1}{4}; f(3) = \dfrac{1}{5+4a}; f(4) = \dfrac{1}{7+5a+4a^2}$

Put $x = y = 2$, then $2f(2) + 8f(4) = \dfrac{f(y)}{f(y)} = 1$

So, $\dfrac{1}{2} + \dfrac{8}{7+5a+4a^2} = 1 \Rightarrow a = 1$

3. (c): Possible ways are

(i) 2 three times, 1 four times

Number of ways = $\dfrac{7!}{3!\,4!}$

(ii) 2 once, 3 once and 1 five times.

Number of ways = $\dfrac{7!}{5!}$

Required number = 77

4. (c): z lies on the circle with centre $\left(\dfrac{-1}{3}, 2\right)$ and radius = $\dfrac{10}{3}$, then

maximum area of $\Delta PAB = \dfrac{1}{2} \times \dfrac{10}{3} \times 5 = \dfrac{25}{3}$

5. (a): $\cos A + \cos C = 2(1 - \cos B)$ and $\sin\dfrac{B}{2} \neq 0$

So, $2\sin\left(\dfrac{A+C}{2}\right)\cos\left(\dfrac{A-C}{2}\right) = 2\sin B$

$\Rightarrow \sin A + \sin C = 2\sin B$

6. (d)

7. (d)

8. (d): $y = \dfrac{x}{\ln|cx|}$

$$\phi\left(\frac{x}{y}\right) = \frac{dy}{dx} - \frac{y}{x} = \frac{\ln|cx|.1 - x.\frac{1}{cx}.c}{(\ln|cx|)^2} - \frac{1}{\ln|cx|}$$

$$= \frac{1}{\ln|cx|} - \frac{1}{(\ln|cx|)^2} - \frac{1}{\ln|cx|} = -\frac{y^2}{x^2}$$

9. (c): $B = (I - A)(I + A)^{-1}$; $B^T = (I + A^T)^{-1}(I - A^T)$

$= (I - A)^{-1}(I + A)$

$BB^T = (I - A)(I + A)^{-1}(I - A)^{-1}(I + A)$

$= (I - A)(I - A)^{-1}(I + A)^{-1}(I + A) = I$

(As $(I - A)(I + A) = (I + A)(I - A)$)

10. (a, b): $b - a = c - b = d - c = \lambda$ (say)

$$f(x) = \begin{vmatrix} x+a & x+b & x+a-c \\ x+b & x+c & x-1 \\ x+c & x+d & x-b+d \end{vmatrix} = \begin{vmatrix} x+a & x+b & x+a-c \\ \lambda & \lambda & -1+2\lambda \\ \lambda & \lambda & 1+2\lambda \end{vmatrix}$$

$[R_2 \to R_2 - R_1, R_3 \to R_3 - R_2]$

$$= \begin{vmatrix} x+a & \lambda & x+a-c \\ \lambda & 0 & -1+2\lambda \\ \lambda & 0 & 1+2\lambda \end{vmatrix} \qquad [C_2 \to C_2 - C_1]$$

$f(x) = -\lambda(\lambda + 2\lambda^2 + \lambda - 2\lambda^2) = -2\lambda^2$

$\int_0^2 -2\lambda^2 dx = -4 \Rightarrow 2\lambda^2[x]_0^2 = 4$

$\Rightarrow 2\lambda^2 \times 2 = 4 \Rightarrow \lambda^2 = 1 \Rightarrow \lambda = \pm 1$

11. (a, c): $(\alpha - \beta)^2 = (16 + 20i - 4M)$

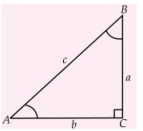
$\Rightarrow |16 + 20i - 4M| = 28$

12. (a, c)

13. (a, b, c, d): $abx^2 - c^2x + ab = 0$

$\tan A + \tan B = \dfrac{c^2}{ab}$ ... (1)

$\tan A \tan B = 1$ ... (2)



From (1), $\dfrac{\sin(A+B)}{\cos A \cos B} = \dfrac{c^2}{ab}$

$\Rightarrow \quad \dfrac{\sin C}{\cos A \cos B} = \dfrac{\sin^2 C}{\sin A \sin B}$

$\Rightarrow \quad \tan A \tan B = \sin C = 1$

$\Rightarrow \quad C = \dfrac{\pi}{2}; \quad \therefore \ \cos C = 0$

$\therefore \quad a^2 + b^2 = c^2$

$\sin^2 A + \sin^2 B = \sin^2 C = 1 \ \Rightarrow \ \sin^2 A + \sin^2 B + \sin^2 C = 2$

Also, $\tan A = \dfrac{a}{b}$ and $\tan B = \dfrac{b}{a}$

14. (d)

15. (a): $P(\bar{A}\bar{B}C) = P(\bar{A}) \cdot P(\bar{B}) \cdot P(C) = \dfrac{1}{2} \times \dfrac{2}{3} \times \dfrac{1}{6} = \dfrac{1}{18}$

16. (d): $\tan A = \dfrac{4}{3}$

$\tan A + \tan B + \tan C = \tan A \tan B \tan C$

$\dfrac{4}{3} + \tan B + \tan C = \dfrac{4}{3} \times \tan B \tan C$

$(\tan B - \tan C)^2 = (\tan B + \tan C)^2 - 4 \tan B \tan C$

$= \dfrac{16}{9}(\tan B \tan C - 1)^2 - 4 \tan B \tan C$

Let $\tan B \tan C = x$

$\dfrac{1}{9} = \dfrac{16}{9}(x-1)^2 - 4x \ \Rightarrow \ 1 = 16(x-1)^2 - 36x$

$\Rightarrow \quad 16x^2 - 68x + 15 = 0$

So, we get real value of x.

17. (A) - r, t ; (B) - p ; (C) - q, s ; (D) - s

(A) Since, $\log_{100} |x+y| = \dfrac{1}{2} \Rightarrow |x+y| = (100)^{1/2} = 10$

$\therefore \quad |x+y| = 10$ ... (i)

and $\log_{10} y - \log_{10} |x| = \log_{100} 4 \qquad (\because \ y > 0)$

$\Rightarrow \quad \log_{10}\left(\dfrac{y}{|x|}\right) = \log_{10^2} 2^2 = \dfrac{2}{2}\log_{10} 2$

$\Rightarrow \quad \log_{10}\left(\dfrac{y}{|x|}\right) = \log_{10} 2 \ \Rightarrow \ \dfrac{y}{|x|} = 2$

$\Rightarrow \quad y = 2|x|$ ... (ii)

From equations (i) and (ii), $|x + 2|x|| = 10$

Case I : $x > 0$, $|3x| = 10 \ \Rightarrow \ 3|x| = 10 \ \Rightarrow \ 3x = 10$

$\therefore \quad x = \dfrac{10}{3}$

From equation (ii), $y = \dfrac{20}{3}; \ \therefore \ \left\{\dfrac{10}{3}, \dfrac{20}{3}\right\}$

Case II : $x < 0$, $|x - 2x| = 10 \ \Rightarrow \ |x| = 10 \ \Rightarrow \ -x = 10$

$\therefore \quad x = -10$

From equation (ii), $y = 2|-10| = 20; \ \therefore \ \{-10, 20\}$

(B) Let $\log_2 x = A$ and $\log_2 y = B$

$\therefore \quad 4A^2 + 1 = 2B$ ... (i)

and $2A \geq B$ or $4A \geq 2B$

$\Rightarrow \quad 4A \geq 4A^2 + 1$ [from equation (i)]

$\Rightarrow \quad (2A-1)^2 \leq 0 \ \Rightarrow \ 2A - 1 = 0$ or $A = 1/2$

From equation (i), $4\left(\dfrac{1}{2}\right)^2 + 1 = 2B$

$\therefore \quad B = 1$

$\Rightarrow \quad \log_2 x = \dfrac{1}{2}$ and $\log_2 y = 1 \ \Rightarrow \ x = 2^{1/2}$ and $y = 2^1$

or $\quad x = \sqrt{2}, \ y = 2. \ \therefore \ \{\sqrt{2}, 2\}$

(C) $\because \quad \log_4 x = \log_2 y \ \Rightarrow \ \log_{2^2} x = \log_2 y$

$\Rightarrow \quad \dfrac{1}{2}\log_2 x = \log_2 y$

$\Rightarrow \quad \log_2 x = 2\log_2 y = \log_2 y^2 \ \Rightarrow \ x = y^2$ ... (i)

and $x^2 - 5y^2 + 4 = 0$

$\Rightarrow \quad x^2 - 5x + 4 = 0 \ \Rightarrow \ (x-1)(x-4) = 0$

$\therefore \quad x = 1, \ y = 4$

Then, $y = 1$, $y = \pm 2$ (but $y > 0$)

$\therefore$ Solution set are {1, 1}, {4, 2}

(D) $2(x^2 - 8x + 16) + 3(y^2 - 4y + 4) = 0$

$\Rightarrow \quad 2(x-4)^2 + 3(y-2)^2 = 0 \ \Rightarrow \ x = 4, \ y = 2$

18. (A) - r, (B) - p, (C) - q, (D) - s

(A) $\vec{x} + \vec{y} + \vec{z} = \vec{a}$

$\left|\vec{x} + \vec{y} + \vec{z}\right|^2 = 4 \ \Rightarrow \ 3 + 2(\vec{x}\cdot\vec{y} + \vec{x}\cdot\vec{z} + \vec{y}\cdot\vec{z}) = 4$

$\Rightarrow \quad \vec{x}\cdot\vec{y} + \vec{y}\cdot\vec{z} + \vec{z}\cdot\vec{x} = \dfrac{1}{2}$ ... (i)

But $\vec{a}\cdot\vec{x} = \dfrac{3}{2} \ \Rightarrow \ (\vec{x} + \vec{y} + \vec{z})\cdot\vec{x} = \dfrac{3}{2}$

$\Rightarrow \quad 1 + \vec{x}\cdot\vec{y} + \vec{x}\cdot\vec{z} = \dfrac{3}{2}$

$\vec{x}\cdot\vec{y} + \vec{x}\cdot\vec{z} = \dfrac{1}{2}$ ... (ii)

and $(\vec{x} + \vec{y} + \vec{z})\cdot\vec{y} = \dfrac{7}{4} \ \Rightarrow \ \vec{x}\cdot\vec{y} + \vec{z}\cdot\vec{y} = \dfrac{3}{4}$ ... (iii)

From (i), (ii) and (iii) $\Rightarrow \ \vec{x}\cdot\vec{y} = \dfrac{3}{4}; \ \vec{y}\cdot\vec{z} = 0; \ \vec{z}\cdot\vec{x} = -\dfrac{1}{4}$


BUY ONLINE
Now you can buy
MTG Books & Magazines
Log on to : www.mtg.in

Now, $\vec{x}\times(\vec{y}\times\vec{z})=\vec{b} \Rightarrow -\frac{1}{4}\vec{y}-\frac{3}{4}\vec{z}=\vec{b}$

and $(\vec{x}\times\vec{y})\times\vec{z}=\vec{c} \Rightarrow -\frac{1}{4}\vec{y}=\vec{c}$

$\therefore \vec{y}=-4\vec{c}$

So, $\vec{z}=\frac{4}{3}(\vec{c}-\vec{b})$ and

$\vec{x}=\vec{a}-\vec{y}-\vec{z}=\vec{a}+4\vec{c}+\frac{4}{3}(\vec{b}-\vec{c})=\frac{1}{3}[3\vec{a}+4\vec{b}+8\vec{c}]$

So, $3\vec{x}+2\vec{y}+3\vec{z}=3\vec{a}+4\vec{b}+8\vec{c}-8\vec{c}+4\vec{c}-4\vec{b}=3\vec{a}+4\vec{c}$

19. (0) : Because $\frac{-\pi}{2}<-1\le\sin x\le 1<\frac{\pi}{2}$, $\cos(\sin x)>0$

Hence, $\sin(\cos x)>0$ and so $\cos x>0$, so the only possible solutions can lie in the interval $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$

If x is a solution then $-x$ is also a solution.

$\sin(\cos x)=\sin\left(\frac{\pi}{2}-\sin x\right)$; $\cos x=\frac{\pi}{2}-\sin x$

or $\sin x+\cos x=\frac{\pi}{2}>\sqrt{2}$

20. (5) : $PS=ePM \Rightarrow a\in(5,\infty)$, then $b=5$

21. (4) : $f(x)=(x^2+5x+4)(x^2+5x+6)+5$

Let $x^2+5x+5=t$. Then $f(t)=(t^2-1)+5$

$-\frac{5}{4}\le t\le 71$, then range of $f(x)$ will be $[4, 5045]$

$a=4$

22. (4) : $\because m_1m_2m_3m_4=1$, so product of slopes of each pair is -1

Let a pair of straight line $ax^2+2hxy-ay^2=0$

Equation of bisectors $\frac{x^2-y^2}{2a}=\frac{xy}{h}$

$\Rightarrow hx^2-2axy-hy^2=0$

So, $(ax^2+2hxy-ay^2)(hx^2-2axy-hy^2)=0$

$\Rightarrow ah(x^4+y^4)+2(h^2-a^2)(x^3y-xy^3)-6ahx^2y^2=0$

$\Rightarrow ah=1, c=-6ah \Rightarrow c=-6 \Rightarrow c+10=4$

ATTENTION COACHING INSTITUTES:
a great offer from MTG

MTG offers **"Classroom Study Material"** for JEE (Main & Advanced), AIPMT and FOUNDATION MATERIAL for Class 8, 9, 10, 11 & 12 **with YOUR BRAND NAME & COVER DESIGN**.

This study material will save you lots of money spent on teachers, typing, proof-reading and printing. Also, you will save enormous time. Normally, a good study material takes 2 years to develop. But you can have the material printed with your logo delivered at your doorstep.

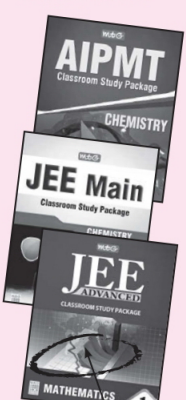
Profit from associating with MTG Brand – the most popular name in educational publishing for JEE (Main & Advanced)/AIPMT/PMT ....

Order sample chapters on Phone/Fax/e-mail.

Phone : 0124-4951200, 09312680856
e-mail : sales@mtg.in | www.mtg.in

★ EXCELLENT QUALITY ★
★ CONTENT ★ PAPER ★ PRINTING

CLASSROOM STUDY MATERIAL



Your logo here

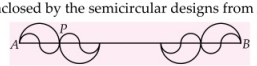
YOU ASKED WE ANSWERED

Do you have a question that you just can't get answered?

Use the vast expertise of our mtg team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough.

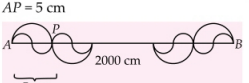
The best questions and their solutions will be printed in this column each month.

Q.1 AB is a line segment 2000 cm long. The following design of semicircles is drawn on AB, with AP = 5 cm and repeating the designs. Find the area enclosed by the semicircular designs from A to B.



– Anand, WB

Ans. $AP = 5$ cm



$\therefore$ The number of designs repeated $= \dfrac{2000}{5} = 400$

The area for one design is the area of the semi circle on AP, as the two smaller semi circular regions are equal and one of the semi circular regions is included in the bigger semi circular region the other is omitted.

$\therefore$ The area of all the 400 semi circular regions

$= \dfrac{1}{2}\pi \times \dfrac{25}{4} \times 400$ sq cm. $= 1250\pi$ sq. cm

Q.2 Find the number of real values of x such that $\{(x+1)^3\} = x^3$ where $\{x\} = x - [x]$ is

– Rajesh Kr., Rajasthan

Ans. Clearly, $0 \le x < 1$ as x^3 is the fractional part of $\{(x+1)^3\}$

(1) $(x+1)^3 = x^3 \Rightarrow x^3 + 3x^2 + 3x + 1 = x^3$

$\Rightarrow 3x^2 + 3x + 1 = 0$ has imaginary roots.

No real solution.

(2) $(x+1)^3 = 1 + x^3 \Rightarrow 3x^2 + 3x = 0 \Rightarrow x = 0$ or -1.

$x = 0$ alone is acceptable.

(3) $(x+1)^3 = 2 + x^3 \Rightarrow 3x^2 + 3x - 1 = 0$

$\Rightarrow \quad x = \dfrac{-3 \pm \sqrt{21}}{6}$.

We can accept only the positive value for x.

$\Rightarrow \quad x = \dfrac{-3 + \sqrt{21}}{6}$. Second Solution.

(4) $(x+1)^3 = 3 + x^3 \Rightarrow 3x^2 + 3x - 2 = 0$

i.e., $x = \dfrac{-3 \pm \sqrt{33}}{6}$

$x = \dfrac{-3 + \sqrt{33}}{6}$ alone is acceptable.

(5) $(x+1)^3 = 4 + x^3 \Rightarrow 3x^2 + 3x - 3 = 0$ *i.e.*,

$x = \dfrac{-3 \pm \sqrt{45}}{6} = \dfrac{-1 \pm \sqrt{5}}{2}$

Again $x = \dfrac{-1 + \sqrt{5}}{2}$ is only acceptable.

(6) $(x+1)^3 = 5 + x^3 \Rightarrow \quad x = \dfrac{-3 \pm \sqrt{57}}{6}$

$\Rightarrow \quad x = \dfrac{-3 + \sqrt{57}}{6}$ is acceptable.

(7) $(x+1)^3 = 6 + x^3 \Rightarrow \quad x = \dfrac{-3 \pm \sqrt{69}}{6}$

$x = \dfrac{-3 + \sqrt{69}}{6}$.

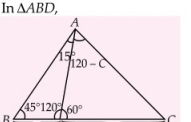
For $(x+1)^3 = 7 + x^3$ onwards positive x value is greater than or equal to 1.

So, there are six solutions.

Q.3 In triangle ABC, $\angle ABC = 45°$, Point D is on BC so that $2BD = CD$, and $\angle DAB = 15°$. Find the value of $\angle ACB$.

– Suraj Singh, Navi Mumbai

Ans. In ΔABD,



$\dfrac{AD}{\sin 45°} = \dfrac{BD}{\sin 15°} = \dfrac{1}{\sin 15°}$

$\Rightarrow \quad AD = \dfrac{\sin 45°}{\sin 15°} = \dfrac{1}{\sqrt{2}\sin 15°}$

In ΔADC, $\dfrac{AD}{\sin C} = \dfrac{DC}{\sin(120 - C)} = \dfrac{2}{\sin(120 - C)}$

$\Rightarrow \quad \dfrac{1}{\sqrt{2}\sin 15} = \dfrac{2\sin C}{\dfrac{\sqrt{3}}{2}\cos C + \dfrac{1}{2}\sin C}$

$= \dfrac{4}{\sqrt{3}\cot C + 1}$

$$\Rightarrow \quad \sqrt{3}\cot C + 1 = 4\sqrt{2}\sin 15 = 4\sqrt{2}\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)$$

$$= 2(\sqrt{3}-1)$$

$$\Rightarrow \quad \sqrt{3}\cot C = 2\sqrt{3} - 2 - 1 = 2\sqrt{3} - 3 = \sqrt{3}(2-\sqrt{3})$$

$$\Rightarrow \quad \cot C = 2 - \sqrt{3}$$

$$\Rightarrow \quad \tan C = 2 + \sqrt{3} = \tan 75°$$

$$\therefore \quad C = 75°$$

Q.4 Let $b \neq 0$ and for $j = 0, 1, 2, \ldots, n$. Let S_j be the area of the region bounded by y-axis and the curve $x.e^{ay} = \sin by$, $\frac{j\pi}{b} \le y \le \frac{(j+1)\pi}{b}$. Show that S_0, S_1, S_2,...., S_n are in Geometric Progression. Also find their sum for $a = -1$ and $b = \pi$.

– Aarushi, Patna

Ans. Here, $S_j = \left|\int_{j\pi/b}^{(j+1)\pi/b} x\,dy\right| = \left|\int_{j\pi/b}^{(j+1)\pi/b} e^{-ay}\cdot\sin by\,dy\right|$

$$= \left|\left\{\frac{e^{-ay}}{a^2+b^2}(-a\sin by - b\cos by)\right\}_{j\pi/b}^{(j+1)\pi/b}\right|$$

$$= \left|\frac{e^{-a(j+1)\pi/b}}{a^2+b^2}\times(-b)(-1)^{j+1} - \frac{e^{-aj\pi/b}}{a^2+b^2}(-b)(-1)^j\right|$$

$$= |b|\left\{\frac{e^{-a(j+1)\pi/b}}{a^2+b^2} + \frac{e^{-aj\pi/b}}{a^2+b^2}\right\}$$

$$= |b|\frac{e^{-aj\pi/b}}{a^2+b^2}\{e^{-a\pi/b}+1\}; \qquad \{j = 0,1,2,\ldots..,n\}$$

Now,

$$\frac{S_{j+1}}{S_j} = |b|\frac{e^{-a(j+1)\pi/b}}{a^2+b^2}\{e^{-a\pi/b}+1\} \Bigg/ |b|\frac{e^{-aj\pi/b}}{a^2+b^2}\{e^{-a\pi/b}+1\}$$

$$= e^{-a\pi/b} \text{ for all } j = 0, 1, 2, \ldots, n$$

Hence, S_0, S_1, S_2, ..., S_n are in G.P. with common ratio $e^{-a\pi/b}$.

For $a = -1$ and $b = \pi$, we have

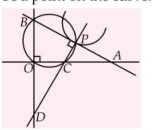
$$S_j = \frac{e^j\pi(e+1)}{\pi^2+1};\ j = 0, 1, 2, \ldots..., n$$

$$\therefore \quad \sum_{j=0}^{n} S_j = \sum_{j=0}^{n}\frac{e^{j\pi}(e+1)}{\pi^2+1} = \frac{(e+1)\pi}{\pi+1}\left\{\frac{e^{n+1}-1}{e-1}\right\}$$

Q.5 The tangent and a normal to a curve at any point P meet the x and y axis at A, B, C and D respectively. Find the equation of the curve passing through (1, 0) if the centre of circle through O, C, P and B lies on the line $y = x$ (where O is origin).

– Abhijit Kr., Bhubaneswar

Ans. Let $P(x, y)$ be a point on the curve.



$$\Rightarrow \quad C \equiv \left(x + y\frac{dy}{dx}, 0\right)$$

$$B \equiv \left(0, y - x\frac{dy}{dx}\right)$$

Circle passing through O, C, P and B has its centre at mid-point of BC.

Let the centre of the circle be (α, β)

$$\Rightarrow \quad 2\alpha = x + y\frac{dy}{dx} \text{ and } 2\beta = y - x\frac{dy}{dx}$$

and since $\beta = \alpha, y - x\frac{dy}{dx} = x + y\frac{dy}{dx}$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y-x}{y+x}$$

Let $y = vx$

$$\Rightarrow \quad x\frac{dv}{dx} = -\frac{(1+v^2)}{1+v} \Rightarrow \quad \frac{1+v}{v^2+1}dv = -\frac{dx}{x}$$

Integrating both sides we, get

$$\int\frac{1+v}{v^2+1}dv = -\int\frac{dx}{x}$$

$$\Rightarrow \quad \frac{1}{2}\int\frac{2v}{v^2+1}dv + \int\frac{dv}{v^2+1} = -\int\frac{dx}{x}$$

$$\Rightarrow \quad \frac{1}{2}\log|v^2+1| + \tan^{-1}|v| = -\log x + c$$

$$\Rightarrow \quad \log\{(\sqrt{v^2+1})x\} + \tan^{-1}v = c$$

$$\Rightarrow \quad \log\sqrt{x^2+y^2} + \tan^{-1}\frac{y}{x} = c$$

Put $x = 1$ and $y = 0$,

$\Rightarrow$ $\log 1 + \tan^{-1}0 = c \Rightarrow c = 0$

$\therefore$ Required curve is

$$(\log\sqrt{x^2+y^2}) + \tan^{-1}\left(\frac{y}{x}\right) = 0$$

■■

MOCK TEST PAPER 2014

JEE Main

Exam on 6th April 2014

* ALOK KUMAR, B.Tech, IIT Kanpur

1. M is the mid point of side AB of an equilateral triangle ABC. P is a point on BC such that $AP + PM$ is minimum. If $AB = 20$ then $AP + PM$ is
(a) $10\sqrt{7}$ (b) $10\sqrt{3}$ (c) $10\sqrt{5}$ (d) 10

2. If in an equilateral triangle, inradius is a rational number then which of the following is not true?
(a) circum-radius is always rational
(b) area is always irrational
(c) ex-radii are always rational
(d) perimeter is always rational

3. For $x^2 - (\alpha + 2)|x| + 9 = 0$ to have real solutions, the range of 'α' is
(a) $(-\infty, 4]$ (b) $[4, \infty)$
(c) $(-\infty, 7] \cup [11, \infty)$ (d) $[-4, \infty)$

4. The number of solutions of $\sin^2 x \cos^2 x = 1 + \cos^2 x \sin^4 x$ in the interval $[0, 2\pi]$ is
(a) 0 (b) 1 (c) 2 (d) 3

5. If A is an orthogonal matrix of order n, then the value of $|\text{adj}(\text{adj } A)|$ is
(a) 0 (b) ± 1 (c) n (d) $n - 2$

6. If $\log_{0.5} \sin x = 1 - \log_{0.5} \cos x$, then number of values of $x \in [-2\pi, 2\pi]$ is
(a) 1 (b) 2 (c) 3 (d) 4

7. If $f(x) = \dfrac{\sin 3x}{\sin x}$, $x \neq n\pi$ then the range of value of $f(x)$ for real values of x is
(a) $[-1, 3]$ (b) $(-\infty, -1)$
(c) $(3, \infty)$ (d) $[-1, 3)$

8. The value of $\sum_{r=0}^{10} \cos^3 \dfrac{\pi r}{3}$ is equal to
(a) $-9/2$ (b) $-7/2$ (c) $-9/8$ (d) $-1/8$

9. If $x > 1$, $y > 1$, $z > 1$ and x, y, z are in G.P. then $(\ln x^2)^{-1}$, $(\ln xy)^{-1}$, $(\ln xz)^{-1}$ are in
(a) A.P. (b) G.P.
(c) H.P. (d) none of these

10. $A = \begin{bmatrix} t & t+1 \\ t-1 & t \end{bmatrix}$ is a matrix such that $AA' = I_2$ then trace A is
(a) 0 (b) 1 (c) -1 (d) 2

11. If $a, b, c, d > 0$; $x \in R$ and $(a^2 + b^2 + c^2)x^2 - 2(ab + bc + cd)x + b^2 + c^2 + d^2 \leq 0$, then $\begin{vmatrix} 33 & 14 & \log a \\ 65 & 27 & \log b \\ 97 & 40 & \log c \end{vmatrix} =$
(a) 1 (b) -1
(c) 0 (d) none of these

12. Let $f: R \to R$ be a positive decreasing function with $\lim_{x \to \infty} \dfrac{f\left(\dfrac{x - x^3}{6}\right)}{f(x)} = 1$, then $\lim_{x \to \infty} \dfrac{f(\sin x)}{f(x)} =$
(a) 1 (b) $\pi/2$ (c) 1/6 (d) $\pi/36$

13. Let $p(x)$ be a function on R such that $p'(x) = p'(5 - x)$ $\forall\, x \in [0, 5]$, $p(0) = 1$ and $p(5) = 7$ then $\int_1^4 p(x)\,dx$ equals
(a) 8 (b) 12 (c) 6 (d) 4

14. Let $f: R \to R$ and $\phi(x) = \dfrac{f(x)}{3^x - 1} + \dfrac{f(x)}{2} + 1$ be an even function then f is
(a) odd (b) even
(c) neither even nor odd
(d) $f(x)$ has to be a constant

15. If $x \in (-\infty, -2)$ and and $y^3 - 3y + x = 0$, then
(a) y is not a function of x
(b) y is not a monotonic function
(c) y is an increasing function of x
(d) y is a decreasing function of x

16. If $f(x) = \cot^{-1} x + \dfrac{1}{2} \ln x$, $x \in \left[\dfrac{1}{\sqrt{3}}, \sqrt{3}\right]$, then greatest value of $f(x)$ is
(a) $\dfrac{\pi}{6} - \dfrac{1}{4}\ln 3$ (b) $\dfrac{\pi}{6} + \dfrac{1}{2}\ln 3$
(c) $\dfrac{1}{4}\ln 3 - \dfrac{\pi}{6}$ (d) $\dfrac{\pi}{6} + \dfrac{\ln 3}{4}$

* Alok Kumar is a winner of INDIAN NATIONAL MATHEMATICS OLYMPIAD (INMO-91).
He trains IIT and Olympiad aspirants.

17. The range of $f(x)=\tan^{-1}\sqrt{[x]+[-x]}+\sqrt{3-|x|}+\frac{1}{x^2}$ is (where [.] denotes G.I.F)

(a) $\left[\frac{1}{4},\infty\right)$ (b) $\left\{\frac{1}{4}\right\}\cup[2,\infty)$

(c) $\left\{\frac{1}{9},\frac{5}{4},1+\sqrt{2}\right\}$ (d) $\left\{\frac{5}{4},1+\sqrt{2}\right\}$

18. The solution of differential equation $x^2dy-y^2dx+xy^2(x-y)dy=0$ is

(a) $\ln\left|\frac{xy}{x-y}\right|+\frac{y^2}{2}=C$ (b) $\ln\left|\frac{x+y}{x-y}\right|+\frac{y^2}{2}=C$

(c) $\ln\left|\frac{x-y}{xy}\right|+\frac{y^2}{2}=C$ (d) $\ln\left|\frac{x-y}{x+y}\right|+\frac{y^2}{2}=C$

19. The area of the region bounded by $y=x\ln x$, $y=2x-2x^2$ is

(a) 7/12 (b) 5/2 (c) 1/12 (d) 4

20. $\int\sin 51x(\sin x)^{49}dx$ equals

(a) $\frac{\sin 50x(\sin x)^{50}}{50}+C$

(b) $\frac{\cos 50x(\sin x)^{50}}{50}+C$

(c) $\frac{\cos 50x(\cos x)^{50}}{50}+C$

(d) $\frac{\sin 50x(\sin x)^{51}}{51}+C$

21. If f be a polynomial function satisfying $f(x^2+x+3)+2f(x^2-3x+5)=6x^2-10x+17\ \forall\ x\in R$ then

(a) f is a decreasing function

(b) $f(x)=0$ has a root in (0, 2)

(c) $f(x)$ is an odd function

(d) no such polynomial exist

22. If $|f''(x)|\le 1\ \forall\ x\in R$ and $f(0)=0=f'(0)$ then which of the following cannot be true?

(a) $f\left(\frac{1}{3}\right)=\frac{1}{5}$ (b) $f\left(-\frac{1}{3}\right)=\frac{1}{4}$

(c) $f(3)=4$ (d) $f(-3)=\frac{1}{3}$

23. Let $f(x)=\left[\frac{1}{\sin\{x\}}\right]$ where [.] and {.} denote the greatest integer and fractional part function respectively. The range of f is

(a) the set of integers = I

(b) the set of natural numbers = N

(c) the set of whole numbers = W

(d) {2, 3, 4, ...}

24. The chord of contact of tangents from any point of circle $x^2+y^2=a^2$ with respect to the circle $x^2+y^2=b^2$ touches the circle $x^2+y^2=c^2$ where $(a, b, c>0)$ then

(a) $b<\frac{a+c}{2}$

(b) $\frac{1}{1+\log a},\frac{1}{1+\log b},\frac{1}{1+\log c}$ are in A.P.

(c) a, b, c are in A.P.

(d) $b>\sqrt{ac}$

25. The locus of mid-point of the chord of the circle with diameter as minor axis of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\ (a>b)$ which subtend right angle at centre of ellipse is

(a) $x^2+y^2=2b^2$ (b) $x^2+y^2=\frac{a^2+b^2}{2}$

(c) $2(x^2+y^2)=b^2$ (d) $x^2+y^2=\frac{b^2}{4}$

26. Let α, β, γ are the roots of the equation $x^3+3x^2-6x-8=0$. If $\left(\frac{1}{\alpha},\alpha\right),\left(\frac{1}{\beta},\beta\right)$ and $\left(\frac{1}{\gamma},\gamma\right)$ are the vertices of the triangle, then

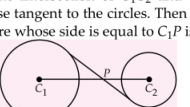
(a) centroid of the triangle is $\left(-\frac{3}{4},-1\right)$

(b) orthocentre of the triangle is $\left(-\frac{1}{8},-8\right)$

(c) circumcentre of the triangle is $\left(\frac{29}{8},-\frac{23}{16}\right)$

(d) centroid of the triangle is $\left(\frac{1}{4},-1\right)$

27. In the figure, two circles with centres C_1 and C_2 are 35 units apart, *i.e.* $C_1C_2=35$. The radii of the circles with centres C_1 and C_2 are 12 and 9 respectively. If P is the intersection of C_1C_2 and a common transverse tangent to the circles. Then the area of the square whose side is equal to C_1P is



(a) 324 (b) 400 (c) 20 (d) 225

28. The equation of the conjugate hyperbola of the hyperbola $3x^2-5xy-2y^2+5x+11y-8=0$ is

(a) $3x^2-5xy-2y^2+5x+11y+8=0$

(b) $3x^2-5xy-2y^2+5x+11y+16=0$

(c) $3x^2-5xy-2y^2+5x+11y-16=0$

(d) $3x^2-5xy-2y^2+5x+11y-12=0$

29. On shifting the origin to a point P, the axes remaining parallel to the original axes, the equation $ax + by + c = 0$ is transformed to $ax + by + c + k = 0$. Then the locus of P is

(a) $ax - by + c = 0$ (b) $ax + by + k = 0$

(c) $ax - by - k = 0$ (d) $ax + by - k = 0$

30. If the normals at P and Q on the parabola $y^2 = 4ax$ meet on it at R. Then the directrix of the locus of mid point of PQ is

(a) $2x + 3a = 0$ (b) $2x + 5a = 0$

(c) $2x + a = 0$ (d) $2x - 5a = 0$

31. Angle subtended by common tangents of two ellipses $9(x-2)^2 + 4y^2 = 36$ and $9(x+1)^2 + y^2 = 9$ at the origin is

(a) $\frac{\pi}{2}$ (b) $\tan^{-1}\left(\frac{7}{9}\right)$

(c) $\tan^{-1}\left(\frac{9}{7}\right)$ (d) $\frac{\pi}{4}$

32. $C_1 : x^2 + y^2 = r^2$ and $C_2 : \frac{x^2}{16} + \frac{y^2}{9} = 1$ intersect at four distinct points A, B, C and D. Their common tangents form a square $A'B'C'D'$. Then the ratio of area of the circle C_1 to the area of circumcircle $\Delta\, A'B'C'$ is

(a) 9/16 (b) 3/4 (c) 1 (d) 1/2

33. The ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is such that it has the least area but contains the circle $(x-1)^2 + y^2 = 1$. Then the eccentricity of the ellipse is

(a) $\sqrt{\frac{2}{3}}$ (b) $\sqrt{\frac{1}{3}}$ (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

34. The ellipse $4x^2 + 9y^2 = 36$ and the hyperbola $a^2x^2 - y^2 = 4$ intersect at right angles then the equation of the circle through the point of intersection of two conic is

(a) $x^2 + y^2 = 5$

(b) $\sqrt{5}(x^2 + y^2) = 3x + 4y$

(c) $\sqrt{5}(x^2 + y^2) - 3x + 4y = 0$

(d) $x^2 + y^2 = 25$

35. If S_1 and S_2 are the foci of the hyperbola whose transverse axis length is 4 and conjugate axis length is 6, S_3 and S_4 are the foci of the conjugate hyperbola, then the area of the quadrilateral $S_1S_2S_3S_4$ is

(a) 24 (b) 26 (c) 22 (d) 28

36. Let I be the incentre of the triangle ABC, where $\frac{\overrightarrow{BC}}{|\overrightarrow{BC}|} + \frac{\overrightarrow{BA}}{|\overrightarrow{BA}|} = \frac{\overrightarrow{BI}}{k}$ then the diameter of the circumcircle of the triangle is

(a) $k\left(\cos\frac{A}{2} + \cos\frac{C}{2}\right)$ (b) $k\left(\sin\frac{A}{2} + \sin\frac{C}{2}\right)$

(c) $k\left(\cot\frac{A}{2} + \cot\frac{C}{2}\right)$ (d) $k\left(\tan\frac{A}{2} + \tan\frac{C}{2}\right)$

37. The reciprocal of the distance between two points, one on each of the lines $\frac{x-2}{3} = \frac{y-4}{2} = \frac{z-5}{5}$ and $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$

(a) cannot be less than 9

(b) having minimum value $5\sqrt{3}$

(c) cannot be greater than 78

(d) cannot be $2\sqrt{19}$

38. Equation of the plane containing the straight line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$ is

(a) $x + 2y - 2z = 0$ (b) $3x + 2y - 2z = 0$

(c) $x - 2y + z = 0$ (d) $5x + 2y - 4z = 0$

39. The distance between the plane $x - 2y + z - 6 = 0$ and the plane containing the sets of points $(1 + 2\lambda, 2 + 3\lambda, 3 + 4\lambda)$ and $(2 + 3\mu, 3 + 4\mu, 4 + 5\mu)$, where λ, μ are parameters, is

(a) $\sqrt{3}/2$ (b) $\sqrt{6}$ (c) $\sqrt{12}$ (d) $2\sqrt{6}$

40. Three numbers a, b, c are choosen randomly from the set of natural numbers. The probability that '$a^2 + b^2 + c^2$' is divisible by 7 is

(a) 1/3 (b) 1/4 (c) 1/5 (d) 1/7

41. If the equation $z^2 + z + \alpha = 0$ has a purely imaginary root and α lies on the circle $|z| = 1$ then the imaginary part of that root, is (are)

(a) $\pm\sqrt{2}$ (b) 0

(c) $\pm\sqrt{2-\sqrt{2}}$ (d) $\pm\sqrt{\frac{\sqrt{5}-1}{2}}$

42. Let $z(\alpha, \beta) = \cos\alpha + e^{i\beta}\sin\alpha$ ($\alpha, \beta \in R$, $i = \sqrt{-1}$) then the exaustive set of values of modulus of $z(\theta, 2\theta)$ as θ varies is

(a) $[0, 1]$ (b) $[0, \sqrt{2}]$

(c) $[1, 2]$ (d) $[\sqrt{2}, 2]$

43. If $|z| = 1$ and $z \neq \pm 1$ then one of the possible values of $\arg(z) - \arg(z+1) - \arg(z-1)$ is
(a) $-\pi/6$ (b) $\pi/3$ (c) $-\pi/2$ (d) $\pi/4$

44. Let X be a set containing n elements. The no. of all the ordered triplets (A, B, C) such that C is a subset of B and B is a proper subset of A where $A \subseteq X$, is
(a) 4^n (b) 3^n
(c) $4^n - 3^n$ (d) $3^n - 2^n$

45. If two events A and B are such that $P(\bar{A}) = 0.3$, $P(B) = 0.4$, $P(A \cap \bar{B}) = 0.5$, then the value of $P(B / A \cup \bar{B}))$ is
(a) 1/2 (b) 1/4 (c) 3/4 (d) 4/5

46. If the third power of a natural no. ends with a prime digit then the probability of its fourth power ending not with a prime digit, is
(a) 3/10 (b) 9/10 (c) 4/9 (d) 3/4

47. A fair coin is tossed 10 times and the outcomes are listed. Let H_i be the event that the i^{th} outcome is a head and A_m be the event that the list contains exactly m heads, then
(a) H_3 and A_4 are independent
(b) A_1 and A_9 are independent
(c) H_2 and A_5 are independent
(d) H_4 and H_8 are not independent

SOLUTIONS

1. (a): Take the reflection of ΔABC in BC.
$PM = PM'$
$PA + PM = PA + PM'$ it is minimum when $M'PA$ lies in a line
Now apply cosine rule in triangle ABM'
We will get $AM' = 10\sqrt{7}$

2. (d): Clearly, $r = \dfrac{R}{2} \Rightarrow R \in Q$

Now, $r_1 = 4R\sin\dfrac{A}{2}\cos\dfrac{B}{2}\cos\dfrac{C}{2} = 4R\left(\dfrac{1}{2}\right)\left(\dfrac{\sqrt{3}}{2}\right)^2 \in Q$

Similarly, $r_2, r_3 \in Q$.

Now, $\Delta = \dfrac{abc}{4R} = 2R^2 \sin A \sin B \sin C = 2R^2\left(\dfrac{\sqrt{3}}{2}\right)^3 \notin Q$

Also, $s = a + b + c = 2R(\sin A + \sin B + \sin C) = 3\sqrt{3}R \notin Q$

3. (b): $\alpha = \dfrac{x^2+9}{|x|} - 2 = |x| + \dfrac{9}{|x|} - 2$

$\Rightarrow \alpha \geq 4$

4. (a): $\sin^2 x \cos^2 x\,(1 - \sin^2 x) = 1$
$\sin^2 x \cos^4 x = 1$
No values of x for L.H.S. = R.H.S

5. (b): $AA' = I \Rightarrow |A| = \pm 1$
$\therefore \quad |\text{adj}\,(\text{adj}\cdot A)| = |A|^{(n-1)^2} = \pm 1$

6. (d): $\log_{0.5}\sin x = 1 - \log_{0.5}\cos x$
$\therefore \quad x \in [-2\pi, 2\pi]$, $\sin x > 0$ and $\cos x > 0$
$\sin x \cos x = 1/2$
$\sin 2x = 1$; $2x \in [-4\pi, 4\pi] \Rightarrow$ 4 solutions

7. (d): $3 - 4\sin^2 x = y$

$\therefore \quad \sin^2 x = \dfrac{3-y}{4}$.

But $0 < \sin^2 x \leq 1$ $(\because \sin x = 0 \Rightarrow x = n\pi]$

$\therefore \quad 0 < \dfrac{3-y}{4} \leq 1$ or $0 < 3 - y \leq 4$

or $-3 \leq y < 3$

8. (d): $I = \sum_{r=0}^{10} \dfrac{1}{4}\left(\cos\dfrac{3\pi r}{3} + 3\cos\dfrac{\pi r}{3}\right)$

$= \sum_{r=0}^{10} \dfrac{1}{4}\left(\cos\pi r + 3\cos\dfrac{\pi r}{3}\right) = \dfrac{1}{4}(I_1 + I_2)$

$\therefore \quad I_1 = \sum_{r=0}^{10} \cos\pi r = 1 - 1 + 1 - 1 + \ldots\ldots - 1 + 1 = 1$

$$I_2 = 3\sum_{r=0}^{10} \cos\frac{\pi r}{3} = \frac{3\cos\left(\frac{10}{2}\frac{\pi}{3}\right)\sin\frac{11\pi}{3}}{\sin\frac{\pi}{6}} = -\frac{1\times 3}{2} = -\frac{3}{2}$$

$$\Rightarrow \quad I = \frac{1}{4}\left(1 - \frac{3}{2}\right) = -\frac{1}{8}$$

9. (c): $x > 1$, $y > 1$, $z > 1$
$x, y, z \rightarrow$ G.P. $\Rightarrow$ $\ln x, \ln y, \ln z$ are in A.P.
$2\ln x, \ln xy, \ln xz$ are in A.P.
$(\ln x^2)^{-1}, (\ln xy)^{-1}, (\ln xz)^{-1}$ are in H.P.

10. (c): $AA' = I_2 \Rightarrow t = 0$

11. (c): We have
$(a^2 + b^2 + c^2)x^2 - 2(ab + bc + cd)x + b^2 + c^2 + d^2 \leq 0$
$\Rightarrow \quad (ax - b)^2 + (bx - c)^2 + (cx - d)^2 \leq 0$
$\Rightarrow \quad (ax - b)^2 + (bx - c)^2 + (cx - d)^2 = 0$
$\Rightarrow \quad \dfrac{b}{a} = \dfrac{c}{b} = \dfrac{d}{c} = x$
$\Rightarrow \quad b^2 = ac$ or $2\log b = \log a + \log c$

Now, $\begin{vmatrix} 33 & 14 & \log a \\ 65 & 27 & \log b \\ 97 & 40 & \log c \end{vmatrix} = \begin{vmatrix} 130 & 54 & \log a + \log c \\ 65 & 27 & \log b \\ 97 & 40 & \log c \end{vmatrix}$

(Apply $R_1 \rightarrow R_1 + R_3$]

$\begin{vmatrix} 0 & 0 & 0 \\ 65 & 27 & \log b \\ 97 & 40 & \log c \end{vmatrix} = 0$ [Apply $R_1 \rightarrow R_1 - 2R_2$]

12. (a): $\dfrac{x-x^3}{6} < \sin x < x \;\forall\; x > 0$

$\Rightarrow \quad f\left(\dfrac{x-x^3}{6}\right) > f(\sin x) > f(x)$

$\Rightarrow \quad \dfrac{f\left(\dfrac{x-x^3}{6}\right)}{f(x)} > \dfrac{f(\sin x)}{f(x)} > 1$

According to Sandwich theorem, $\lim\limits_{x\to\infty} \dfrac{f(\sin x)}{f(x)} = 1$

13. (b): $I = \int_1^4 p(x)\,dx$

$I = \int_1^4 p(5-x)\,dx \;;\quad 2I = \int_1^4 (p(x)+p(5-x))\,dx$

Now, $p'(x) = p'(5-x) \;\Rightarrow\; p(x) + p(5-x) = C$

$C = 1 + 7 = 8 \quad \Rightarrow \quad 2I = \int_1^4 8\,dx = 24 \;\Rightarrow\; I = 12$

14. (a): $\phi(x) = \dfrac{f(x)}{2}\left(\dfrac{3^x+1}{3^x-1}\right) + 1 \;\Rightarrow\; f$ is odd

15. (d): $y^3 - 3y + x = 0$

$x = -y^3 + 3y < -2 \;\Rightarrow\; y^3 - 3y - 2 > 0$

$\Rightarrow \quad (y+1)^2 (y-2) > 0 \;\Rightarrow\; y > 2$

$\dfrac{dy}{dx} = \dfrac{1}{3(1-y^2)} < 0$

16. (d): $f(x) = \cot^{-1} x + \dfrac{1}{2}\ln x$

$f'(x) = -\dfrac{1}{1+x^2} + \dfrac{1}{2x} = \dfrac{-2x+1+x^2}{2x(1+x^2)} = \dfrac{(1-x)^2}{2x(1+x^2)}$

f is increasing $\Rightarrow\; f_{\max} = f(\sqrt{3}) = \dfrac{\pi}{6} + \dfrac{1}{4}\ln 3$

17. (c): $D_f = \{-3, -2, -1, 1, 2, 3\}$

Range $= \left\{\dfrac{1}{9}, \dfrac{5}{4}, 1+\sqrt{2}\right\}$

18. (a): D.E. is $\dfrac{dy}{y^2} - \dfrac{dx}{x^2} + y\left(\dfrac{1}{y} - \dfrac{1}{x}\right)dy = 0$

$\Rightarrow \quad \dfrac{\dfrac{dy}{y^2} - \dfrac{dx}{x^2}}{\dfrac{1}{y} - \dfrac{1}{x}} + y\,dy = 0 \quad \Rightarrow \quad -\ln\left|\dfrac{1}{y} - \dfrac{1}{x}\right| + \dfrac{y^2}{2} = C$

$\Rightarrow \quad \ln\left|\dfrac{xy}{x-y}\right| + \dfrac{y^2}{2} = C$

19. (a): $\int_0^1 (-x\ln x + 2x - 2x^2)\,dx$

$= \dfrac{1}{4} + 1 - \dfrac{2}{3} = \dfrac{3+12-8}{12} = \dfrac{7}{12}$

20. (a): $\sin 51x\,(\sin x)^{49} = (\sin x \cdot \cos 50x)\,(\sin x)^{49} + \cos x \sin 50x\,(\sin x)^{49}$

$= \dfrac{d}{dx}((\sin x)^{50} \sin 50x)$

50

21. (b): It is obvious from the given functional relation that degree of polynomial is 1. Putting $f(x) = ax + b$ we get $a = 2$, $b = -3$ *i.e.* $f(x) = 2x - 3$

22. (a): According to L.M.V.T,

$\left|\dfrac{f'(x) - f'(0)}{x}\right| = |f''(C_1)| \le 1 \;\Rightarrow\; |f'(x)| \le 1$

Again, $\left|\dfrac{f(x) - f(0)}{x}\right| = |f'(C_2)| \le 1 \;\Rightarrow\; |f(x)| \le |x|$

23. (b): $0 \le \{x\} < 1 \;;\quad 0 \le \sin\{x\} < \sin 1$

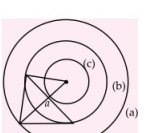
$\dfrac{1}{\sin 1} < \dfrac{1}{\sin\{x\}} < \infty$

$\left[\dfrac{1}{\sin\{x\}}\right] \in \{1, 2, 3, ...\}$

24. (a): $\dfrac{1}{2}a\sqrt{b^2 - c^2}$

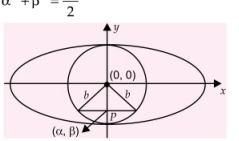
$= \dfrac{1}{2}b\sqrt{a^2 - b^2}$

$\Rightarrow \quad b = \sqrt{ac}$



$\because \quad AM \ge GM$

25. (c): $\alpha^2 + \beta^2 = \dfrac{b^2}{2}$



26. (c): $\Sigma\alpha = -3$, $\Sigma\alpha\beta = -6$, $\alpha\beta\gamma = 8$

Since $\left(\dfrac{1}{\alpha}, \alpha\right)\left(\dfrac{1}{\beta}, \beta\right)$ and $\left(\dfrac{1}{\gamma}, \gamma\right)$ lie on the hyperbola $xy = 1$. So therefore orthocentre will be $\left(-\alpha\beta\gamma, -\dfrac{1}{\alpha\beta\gamma}\right)$

27. (b): $C_1P = \dfrac{35 \times 12}{21} = 20$

Area $= (20)^2 = 400$

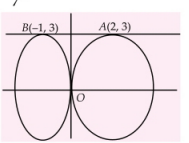
28. (c) : Equation of asymptotes of hyperbola is given by $3x^2 - 5xy - 2y^2 + 5x + 11y + k = 0$
This gives two lines if $k = -12$
Hence the equation of conjugate hyperbola is given by 2(equation of asymptotes) – equation of hyperbola = 0

29. (d): $x = X + \alpha,\ y = Y + \beta$
$aX + bY + a\alpha + b\beta + c = 0$
$\Rightarrow\ a\alpha + b\beta + c = c + k$
$a\alpha + b\beta - k = 0$
So, locus of (α, β) is $ax + by - k = 0$

30. (b): Normal at $P(t_1)$ and $Q(t_2)$ meet the curve again at R. So, $t_1t_2 = 2$.
Hence the locus of midpoint of PQ is $y^2 = 2ax + 4a^2$.
Hence the directrix is $2x + 5a = 0$.

31. (c) : $m_{OA} = \frac{3}{2},\ m_{OB} = -3$

Hence $\tan\theta = \frac{9}{7}$, where $\theta = \angle AOB$



32. (d): If $A'B'C'D'$ is a square, then tangents $y = \pm x \pm 5$ for which diagonal length $A'C' = 10$
Area of circumcircle of $\Delta A'B'C' = 25\pi$

Area of circle $C_1 = \frac{25\pi}{2}$

Hence the ratio is 1 : 2.

33. (a) : Area be least if both the curves touch each other hence solving both the equation together will gives us $a = \frac{1}{e\sqrt{1-e^2}}$

Area of ellipse $= \pi ab = \frac{\pi}{e^2\sqrt{1-e^2}} = \frac{\pi}{\sqrt{e^4 - e^6}}$

Area will be minimum when $e^4 - e^6$ is maximum

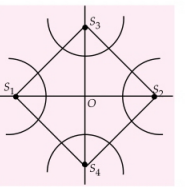
hence $e^2 = \frac{2}{3} \ \Rightarrow\ e = \sqrt{\frac{2}{3}}$

34. (a) : Since ellipse and hyperbola intersect orthogonally, they are confocal hence $a = 2$
Let $P(x_1, y_1)$ be the point of intersection of both the curves therefore,
$4x_1^2 + 9y_1^2 = 36$ and $4x_1^2 - y_1^2 = 4$
$\Rightarrow\ x_1^2 + y_1^2 = 5$
Hence the equation of circle is $x^2 + y^2 = 5$

35. (b): Area = 4 × area ΔS_2OS_4

$= 4 \times \frac{1}{2} ae \times be_1 = 2abee_1 = 12ee_1 = 12 \times \frac{\sqrt{13}}{2} \times \frac{\sqrt{13}}{3}$

= 26 sq. units



36. (c) : Taking modulus both sides

$$2\cos\frac{B}{2} = \frac{1}{k}|\overrightarrow{BI}| = \frac{1}{k}\frac{r}{\sin(B/2)} = \frac{kR\sin(A/2)\sin(C/2)}{k}$$

$$\Rightarrow\ 2R = \frac{k\sin\left(\frac{A+C}{2}\right)}{\sin(A/2)\sin(C/2)} = k(\cot(A/2) + \cot(C/2))$$

37. (c) : The shortest distance (SD)

$$= \frac{\begin{vmatrix} 2-1 & 4-2 & 5-3 \\ 2 & 3 & 4 \\ 3 & 2 & 5 \end{vmatrix}}{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 2 & 5 \end{vmatrix}} = \frac{1}{\sqrt{78}}$$

So, $\frac{1}{SD} = \sqrt{78}$

38. (c) : Vector along the required plane is

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 2 \\ 4 & 2 & 3 \end{vmatrix} = 8\hat{i} - \hat{j} - 10\hat{k}$$

So, normal vector $(\vec{n})$ to the plane is

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 8 & -1 & -10 \\ 2 & 3 & 4 \end{vmatrix} = 26\hat{i} - 52\hat{j} + 26\hat{k}$$

So, equation of the plane is

$\vec{r} \cdot \vec{n} = 0 \ \Rightarrow\ x - 2y + z = 0$

39. (b): Normal vector : $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = -\hat{i} + 2\hat{j} - \hat{k}$

Equation of plane : $-1(x-1)+2(y-2)-1(z-3)=0$

$\Rightarrow \quad x-2y+z=0$

So, required distance $=\dfrac{|6|}{\sqrt{1+4+1}}=\sqrt{6}$

40. (d): Numbers are of the form: $7k$, $7k+1$, $7k+2$, $7k+3$, $7k+4$, $7k+5$, $7k+6$ their squares: $7k$, $7k+1$, $7k+4$, $7k+2$, $7k+2$, $7k+1$, $7k+1$.

So, for $a^2+b^2+c^2$ to be a multiple of 7, either all the three squares should be of the form $7k$ or they belong to the categories $7k+1$, $7k+2$, $7k+4$ separately.

So, required probability $=\left(\frac{1}{7}\right)^3+3!\left(\frac{2}{7}\right)\left(\frac{2}{7}\right)\left(\frac{2}{7}\right)=\frac{1}{7}$

41. (d): Let $z=i\beta$ ($\beta \in R$) be a root, then

$-\beta^2+i\beta+\alpha=0 \quad \Rightarrow \quad \alpha=\beta^2-i\beta$

Now as $|\alpha|=1$

$\Rightarrow \beta^4+\beta^2=1 \Rightarrow \beta^2=\left(\dfrac{-1\pm\sqrt{5}}{2}\right) \Rightarrow \beta=\pm\sqrt{\dfrac{-1\pm\sqrt{5}}{2}}$

42. (b): $|z|(\theta,2\theta)=|\cos\theta+e^{i2\theta}\sin\theta|$

$=|\cos\theta+\sin\theta\cos2\theta+i\sin\theta\sin2\theta|$

$=\sqrt{(\cos\theta+\sin\theta\cos2\theta)^2+\sin^2\theta\sin^2 2\theta}$

$=\sqrt{1+\sin4\theta} \in [0,\sqrt{2}]$

43. (c): $\arg(z)-\arg(z+1)-\arg(z-1)$

$=\arg\left(\dfrac{z}{z^2-1}\right)=\arg\left(\dfrac{z}{z^2-z\bar{z}}\right)=\arg\left(\dfrac{1}{z-\bar{z}}\right)$

= arg(purely imaginary number)

44. (c): $C \subseteq B \subset A \subseteq X$

The number of ways $=\sum_{i=0}^{n} {}^nC_i\left[\sum_{j=0}^{i-1} {}^iC_j\left[\sum_{k=0}^{j} {}^jC_k\right]\right]$

$=\sum_{i=0}^{n} {}^nC_i\sum_{j=0}^{i-1} {}^iC_j\,2^j=\sum_{i=0}^{n} {}^nC_i(3^i-2^i)=4^n-3^n$

45. (b): $P(B/(A\cup\bar{B}))=\dfrac{P(B\cap(A\cup\bar{B}))}{P(A\cup\bar{B})}$

$=\dfrac{P(A)-P(A\cap\bar{B})}{P(A)+P(\bar{B})-P(A\cap\bar{B})}=\dfrac{0.7-0.5}{0.7+0.6-0.5}=\dfrac{0.2}{0.8}=\dfrac{1}{4}$

46. (d): Total cases are when numbers ending with 3, 5, 7, or 8

Favourable cases are when numbers are ending with 3, 7, or 8

So, the required probability = 3/4

47. (c): $P(H_i)=\dfrac{1}{2}, \quad P(A_m)=\dfrac{{}^{10}C_m}{2^{10}}$

$P(H_i\cap A_m)=\dfrac{{}^9C_{m-1}}{2^{10}}$

For H_i and A_m to be independent,

$\dfrac{{}^9C_{m-1}}{2^{10}}=\dfrac{1}{2}\times\dfrac{{}^{10}C_m}{2^{10}} \quad \Rightarrow \quad 1=\dfrac{1}{2}\times\dfrac{10}{m} \quad \Rightarrow \quad m=5$

Do you want yourself to be updated about

- NEW arrivals at MTG Book store
- Special offers on MTG Books and Magazines
- Exams alerts, instant reply to your queries
- Important questions for forthcoming examinations
- Study tips, quizzes, flowcharts, learning strategies, news from science world, and much more…



Like us on **Facebook**

https://www.facebook.com/pcmbtoday

MOCK TEST PAPER 2014

JEE Advanced

Exam on 25 May 2014

* ALOK KUMAR, B.Tech, IIT Kanpur

SECTION-I

Single Correct Answer Type

This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

1. If $Z_n = \text{cis}\dfrac{\pi}{n(n+1)(n+2)}$ for n = 1, 2, 3, ..., then principal argument of $Z = \lim\limits_{n\to\infty}(Z_1 \cdot Z_2 \cdot Z_3 \ldots\ldots Z_n)$ is

(a) $\pi/2$ (b) $\pi/3$ (c) $2\pi/3$ (d) $\pi/4$

2. Let 'x' be in radians with $0 < x < \pi/2$. If $\sin(2\sin x) = \cos(2\cos x)$, then $\tan x + \cot x$ can be written as $\dfrac{p}{\pi^q - r}$ where $p, q, r \in N$, then $\dfrac{p+q+r}{25} =$

(a) 1 (b) 6 (c) 2 (d) 3

3. $\sin\dfrac{\pi}{3} + \dfrac{1}{2}\sin\dfrac{2\pi}{3} + \dfrac{1}{3}\sin\dfrac{3\pi}{3} + \ldots\ldots\infty =$

(a) 0 (b) $\pi/2$ (c) $\pi/4$ (d) $\pi/3$

4. The bisectors of ΔABC meets its circumcircle in the points D, E, F respectively, then $\dfrac{\text{Area of } \Delta DEF}{\text{Area of } \Delta ABC} =$

(a) $\dfrac{R}{2r}$ (b) $\dfrac{r}{2R}$ (c) $\dfrac{2r}{R}$ (d) $\dfrac{R}{r}$

5. Locus of point of intersection of $x\sin\theta - y\cos\theta = \sqrt{x^2+y^2}$ and $\dfrac{\sin^2\theta}{a^2} + \dfrac{\cos^2\theta}{b^2} = \dfrac{1}{x^2+y^2}$ is

(a) $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ (b) $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

(c) $x^2 + y^2 = a^2 + b^2$ (d) $x^2 + y^2 = 2a^2$

6. The complete set of values of 'a' for which the equation $\cos^{-1}x + (\sin^{-1}y)^2 = \dfrac{a\pi^2}{4}$ holds good is

(a) $\left(0, \dfrac{4}{\pi} + 1\right)$ (b) $\left[0, \dfrac{4}{\pi} + 1\right]$

(c) R (d) ϕ

7. If $\log_6 30 = a$, $\log_{15} 24 = b$, then $\log_{12} 60 =$

(a) $\dfrac{2ba}{ab+b+1}$ (b) $\dfrac{ab+b-1}{2ab+1}$

(c) $\dfrac{2a(b+1)}{ab+b}$ (d) $\dfrac{2ab+2a-1}{ab+b+1}$

8. In an acute angled triangle ABC the circle on altitude AD as diameter cuts AB at P and AC at Q then $PQ =$

[Here 'R' is circumradius, 'r' is inradius, Δ is area, 's' is semi perimeter]

(a) R (b) r (c) Δ/R (d) s

9. The set of values of 'a' for which the equation $\sqrt{a}\cos x - 2\sin x = \sqrt{2} + \sqrt{2-a}$ possesses a solution is

(a) $-1 - \sqrt{5} \le a \le -1 + \sqrt{5}$ (b) $\sqrt{5} - 1 \le a \le 2$

(c) $a \in R$ (d) $a \in (-\infty, -1 - \sqrt{5})$

10. Let 'a' be a complex number such that $|a| < 1$ and Z_1, Z_2, Z_3 be the vertices of polygon such that $Z_k = 1 + a + a^2 + \ldots + a^{k-1}$ for all k = 1, 2, 3, ……, then the vertices of the polygon lie within a circle

(a) $|Z - a| = |a|$ (b) $\left|Z - \dfrac{1}{1-a}\right| = |a - 1|$

(c) $\left|Z - \dfrac{1}{1-a}\right| = \dfrac{1}{|1-a|}$ (d) $|Z - 1 + a| = |1 - a|$

11. $\sqrt{\log_2(\cos(\sin x - 1))} = \cos x$, then solution set of '$x$' is

(a) $\left\{x : x = (4n+1)\dfrac{\pi}{2}, n \in I\right\}$

(b) $\left\{x : x = (2n+1)\dfrac{\pi}{2}, n \in I\right\}$

(c) $\{x : x = n\pi, n \in I\}$

(d) $\left\{x : x = n\pi + \dfrac{\pi}{4}\right\}$

* Alok Kumar is a winner of INDIAN NATIONAL MATHEMATICS OLYMPIAD (INMO-91). He trains IIT and Olympiad aspirants.

12. If $\tan^{-1}(x^2 - x + a) + \cot^{-1}(2x^2 - ax + 1) = \frac{\pi}{2}$, for at least one real 'x' then parameter 'a' can't take integral values

(a) $\{-3, -2, -1\}$ (b) $\{-2, -1, 0, 1\}$
(c) $\{-2, -1, 0\}$ (d) $\{-3, -2, -1, 0, 1\}$

13. $\tan\frac{3\pi}{11} + 4\sin\frac{2\pi}{11} =$

(a) $\frac{-1}{2}$ (b) $\sqrt{7}$ (c) 1 (d) $\sqrt{11}$

14. Let a, b, c be the sides of right angled triangle and θ be the smallest angle of the triangle. If $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are also the sides of right angled triangle, then $\sin\theta =$

(a) $\frac{\sqrt{5}-1}{4}$ (b) $\frac{\sqrt{5}+1}{4}$
(c) $\frac{\sqrt{5}-1}{2}$ (d) $\frac{3-\sqrt{5}}{2}$

15. Let $A(z_a)$, $B(z_b)$, $C(z_c)$ be the vertices of ΔABC whose circumcircle is $|z| = r$. If the internal angular bisector of angle A meets the circumcircle again at $D(z_D)$, then

(a) $z_D = z_a z_c$ (b) $z_D^2 = z_b z_c$
(c) $z_D = \frac{z_b z_c}{z_a}$ (d) $z_D = \frac{-z_b z_c}{z_a}$

16. An ellipse with major and minor axes of length $10\sqrt{3}$ and 10 respectively slides along the co-ordinate axes and always remains confined in the first quadrant. The length of the arc of the locus of the centre of the ellipse is

(a) 10π (b) 5π (c) $5\pi/4$ (d) $5\pi/3$

17. If $b + c$, $c + a$, $a + b$ are in H.P., then a^2, b^2, c^2 are in

(a) A.P. (b) G.P. (c) H.P. (d) A.G.P.

18. The value of 'a' for which one root of the quadratic equation $(a^2 - 5a + 3)x^2 + (3a - 1)x + 2 = 0$ is twice as large as the other, is

(a) 2/3 (b) –2/3 (c) 1/3 (d) –1/3

19. Sum of first n terms of a sequence is given by $3S_n = T_n^2 + 3T_n - 2$, $(T_n > 0)$ where T_n is the n^{th} term of sequence, then the value of T_2^2 is

(a) $2 - \sqrt{2}$ (b) $2 + \sqrt{2}$
(c) $2 + 3\sqrt{2}$ (d) $3 + 2\sqrt{2}$

20. There is a point P on the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ such that its distance to the right directrix is the average of its distance to the two foci. Let the x-coordinate of P be m/n with m and n being integers, $(n > 0)$ having no common factor except 1. Then $n - m$ equals

(a) 59 (b) 69 (c) –59 (d) –69

21. The curves $C_1 : y = x^2 - 3$; $C_2 : y = kx^2$, $k < 1$ intersect each other at two different points. The tangent drawn to C_2, at one of the points of intersection $A = (a, y_1)(a > 0)$ meets C_1 again at $B(1, y_2)$, $(y_1 \neq y_2)$. Then value of $a =$

(a) 4 (b) 3 (c) 2 (d) 1

22. Let $A(1, 2)$, $B(3, 4)$ be two points and $C(x, y)$ be a point such that area of ΔABC is 3 sq. units and $(x - 1)(x - 3) + (y - 2)(y - 4) = 0$. Then maximum number of positions of C, in the xy plane is

(a) 2 (b) 4
(c) 8 (d) no such C exist

23. The reflection of the hyperbola $xy = 1$ in the line $y = 2x$ is the curve $12x^2 + r\,xy + s\,y^2 + t = 0$, then the value of '$r$' is

(a) –7 (b) 25 (c) –175 (d) 90

24. If a, b, c are three distinct numbers such that a, b, c are in A.P. and $b - a, c - b, a$ are in G.P. then $a : b : c$ is

(a) 2 : 3 : 4 (b) 3 : 4 : 5
(c) 1 : 3 : 5 (d) 1 : 2 : 3

25. A triangle is formed by a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the coordinate axes. The area of the triangle cannot be less than

(a) $\frac{a^2 + b^2}{2}$ sq. units
(b) $\frac{a^2 + ab + b^2}{3}$ sq. units
(c) $\frac{a^2 + 2ab + b^2}{2}$ sq. units
(d) ab sq. units

26. Chords of the parabola $y^2 = 4x$ touch the hyperbola $x^2 - y^2 = 1$. The locus of the point of intersection of the tangents drawn to the parabola at the extremities of such chords is

(a) a circle
(b) a parabola
(c) an ellipse
(d) a rectangular hyperbola

27. A chord of the hyperbola $x^2 - 2y^2 = 1$ is bisected at the point (–1, 1). Then the area of the triangle formed by the chord and the coordinate axes is

(a) 1 (b) 2 (c) 1/2 (d) 1/4

SECTION-II

Multiple Correct Answer Type

This section contains multiple correct answer(s) type questions. Each question has 4 choices (a), (b), (c) and (d), out of which ONE OR MORE is/are correct.

28. If α, β are two positive angles $< 2\pi$ such that $\cos\alpha + \cos(\alpha + \beta) + \cos(\alpha + \beta + \gamma) = 0$ and $\sin\alpha + \sin(\alpha + \beta) + \sin(\alpha + \beta + \gamma) = 0$, then

(a) $\beta = \frac{2\pi}{3}$ (b) $\beta = \frac{\pi}{3}$

(c) $\gamma = \frac{2\pi}{3}$ (d) $\gamma = \frac{\pi}{3}$

29. If $\cos x + \cos y = a$, $\cos 2x + \cos 2y = b$, $\cos 3x + \cos 3y = c$, then

(a) $\cos^2 x + \cos^2 y = 1 + \frac{b}{2}$

(b) $2a^3 + c = 3a(1 + b)$

(c) $\cos x \cos y = \frac{a^2}{2} - \left(\frac{b+2}{4}\right)$

(d) $a + b + c = 3abc$

30. If from a point $P(z_1)$ on the curve $|z| = 2$, pair of tangents are drawn to the curve $|z| = 1$ meeting $Q(z_2)$, $R(z_3)$, then

(a) complex number $\frac{z_1 + z_2 + z_3}{3}$ will lie on the curve $|z| = 1$

(b) $\left(\frac{4}{\bar{z}} + \frac{1}{\bar{z}_2} + \frac{1}{\bar{z}_3}\right)\left(\frac{4}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right) = 9$

(c) orthocentre and circumcentre of ΔPQR will coincide

(d) $\arg\left(\frac{z_2}{z_3}\right) = \frac{2\pi}{3}$

31. If the triplets $\log a$, $\log b$, $\log c$ and $\log a - \log 2b$, $\log 2b - \log 3c$, $\log 3c - \log a$ are in A.P., then

(a) $18(a + b + c)^2 = 18(a^2 + b^2 + c^2) + ab$

(b) a, b, c are in G.P.

(c) a, $2b$, $3c$ are in G.P.

(d) a, b, c can be lengths of sides of a triangle

32. If Q, S are two points on the circle $x^2 + y^2 = 4$ such that the tangents QP, SR are parallel. If PS, QR intersect at T, then $\left(\frac{QT}{PQ}\right)^2 + \left(\frac{ST}{RS}\right)^2 + PQ \cdot RS \neq$

(a) 5 (b) 10 (c) 16 (d) 17

33. The points on axis of parabola $3x^2 + 4x - 6y + 8 = 0$ from which three distinct normals can be drawn to it are

(a) $\left(\frac{-2}{3}, 2\right)$ (b) $\left(\frac{-2}{3}, 3\right)$

(c) $\left(\frac{-2}{3}, 4\right)$ (d) $\left(\frac{-2}{3}, 1\right)$

34. C_1, C_2 are two circles of radii a, b $(a < b)$ touching both the coordinate axes and have their centres in the first quadrant. Then the true statement(s) among the following is/are

(a) if C_1, C_2 touch each other, then $\frac{b}{a} = 3 + 2\sqrt{2}$

(b) if C_1,C_2 are orthogonal, then $\frac{b}{a} = 2 + \sqrt{3}$

(c) if C_1, C_2 intersect in such a way that their common chord has maximum length, then $\frac{b}{a} = 3$

(d) if C_2 passes through centre of C_1, then $\frac{b}{a} = 2 + \sqrt{2}$

35. $x^2 + y^2 = 1$ intersects the ellipse $x^2 + 2y^2 = 4$ at points P and Q, then the locus of the point of intersection of tangents to the ellipse at P and Q is a conic whose

(a) eccentricity is $\frac{\sqrt{3}}{2}$

(b) eccentricity is $\frac{\sqrt{5}}{2}$

(c) latus-rectum is of length 2 units

(d) foci are $(\pm 2\sqrt{5}, 0)$

36. If the parabolas $y^2 = 4kx$ $(k > 0)$ and $y^2 = 4(x - 1)$ do not have a common normal other than the axis of parabola, then $k \in$

(a) (0, 1) (b) (2, ∞) (c) (3, ∞) (d) (0, ∞)

37. An ellipse passes through the point (4, –1) and it's axes are along the axes of coordinates. If the line $x + 4y - 10 = 0$ is a tangent to it, then it's equation can be

(a) $\frac{x^2}{5} + \frac{y^2}{20} = 1$ (b) $\frac{x^2}{20} + \frac{y^2}{5} = 1$

(c) $\frac{x^2}{80} + \frac{y^2}{5/4} = 1$ (d) $\frac{x^2}{5/4} + \frac{y^2}{80} = 1$

38. Three distinct points A, B and C are given in the two-dimensional coordinate plane such that the ratio of the distance of any one of them from (2, –1) to its distance from (–1, 5) is 1 : 2. Then the centre of the circle passing through A, B and C is

(a) (1, 1) (b) (5, –7)
(c) (3, –3) (d) (4, – 8)

39. If two tangents can be drawn to the different branches of hyperbola $\frac{x^2}{1} - \frac{y^2}{4} = 1$ from the point (α, α^2), then
(a) $\alpha \in (-2, 0)$ (b) $\alpha \in (0, 2)$
(c) $\alpha \in (-\infty, -2)$ (d) $\alpha \in (2, \infty)$

SECTION-III

Comprehension Type

This section contains paragraphs. Based upon each paragraph, multiple choice questions have to be answered. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

Paragraph for Q. Nos. 40 to 42

Suppose $f(x) = 3x^3 - 13x^2 + 14x - 2$ and α, β, γ are roots of $f(x) = 0$ such that $\alpha < \beta < \gamma$.

40. $\tan^{-1}[\alpha] + \tan^{-1}[\beta + 1] + \tan^{-1}[\gamma + 1] =$ (where [.] denotes G.I.F.)
(a) $\frac{3\pi}{4}$ (b) π
(c) $\frac{\pi}{2}$ (d) can't be decided

41. $\lim_{n\to\infty} \alpha^{\lfloor n \rfloor} \beta^{1/n} = k\ (n \in N)$, then $\tan^{-1}(k) =$
(a) 1 (b) e (c) 0 (d) 2

42. $\tan^{-1}\alpha + \tan^{-1}\beta + \tan^{-1}\gamma =$
(a) $\pi/2$ (b) $3\pi/2$ (c) $\pi/4$ (d) $3\pi/4$

Paragraph for Q. Nos. 43 to 45

On the sides AB and BC of ΔABC, squares are drawn with centres D and E such that points C and D lie on the same side of line AB and the points A and E lie on either side of line BC. A, B, C are represented by the complex numbers 1, ω, ω^2 respectively. $\left(\text{Here } \omega = \frac{-1 + i\sqrt{3}}{2}\right)$

43. Angle between AC and DE is equal to
(a) $\pi/4$ (b) $\pi/3$ (c) $\pi/6$ (d) $\pi/2$

44. The length of DE is
(a) $\sqrt{\frac{3}{2}}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\sqrt{3}$ (d) $\sqrt{6}$

45. The length of AE is
(a) $\frac{3+\sqrt{3}}{4}$ (b) $\frac{3-\sqrt{3}}{2}$
(c) $\frac{3-\sqrt{3}}{4}$ (d) $\frac{3+\sqrt{3}}{2}$

Paragraph for Q. Nos. 46 to 48

Let ABC be triangle with incentre I, excentres I_1, I_2, I_3 and circumradius R. Then,

46. $I\,I_1 =$
(a) $a\ \text{cosec}(A/2)$ (b) $a\ \sec(A/2)$
(c) $a\ \sec(B/2)\ \sec(C/2)$
(d) $a\ \text{cosec}(B/2)\ \text{cosec}(C/2)$

47. $I_1I_2 =$
(a) $c\ \text{cosec}(C/2)$ (b) $a\ \text{cosec}(B/2)$
(c) $b\ \text{cosec}(A/2)$ (d) $c\ \sec(C/2)$

48. Area of $\Delta I_1I_2I_3 =$
(a) $\frac{abc}{R}$ (b) $\frac{abc}{r}$ (c) $\frac{abc}{2r}$ (d) $\frac{\Delta}{R}$

Paragraph for Q. Nos. 49 to 51

At times the methods of coordinates becomes effective in solving problems of properties of triangles. We may choose one vertex of the triangle as origin and one side passing through this vertex as x-axis. Thus without loss of generality, we can assume that every triangle ABC has a vertex situated at (0, 0), another at $(x, 0)$ and third one at (h, k).

49. If in ΔABC, $AC = 3$, $BC = 4$, medians AD and BE are perpendicular, then area of ΔABC ____ sq. units.
(a) $\sqrt{7}$ (b) $\sqrt{11}$ (c) $2\sqrt{2}$ (d) $2\sqrt{11}$

50. Suppose the bisector AD of the interior angle A of ΔABC divides side BC into segments $BD = 4$, $DC = 2$, then
(a) $b > c$ and $c < 4$
(b) $2 < b < 6$ and $c < 1$
(c) $2 < b < 6$ and $4 < c < 12$
(d) $b < c$ and $c > 4$

51. If in the above question, altitude $AE > \sqrt{10}$ suppose lengths of AB and AC are integers, then b will be
(a) 3 (b) 6 (c) 4 or 5 (d) 3 or 6

SECTION-IV

Matrix-Match Type

This section contains 6 questions.

Each question contains statements given in two columns, which have to be matched. Statements in Column I are labelled as A, B, C and D whereas statements in Column II are labelled as p, q, r and s. The answers to these questions have to be appropriately bubbled as illustrated in the following example. If the correct matches are A – p, s ; B – q, r ; C – p, q ; D – s, then the correct darkening of bubbles will look like the above:

	p	q	r	s
A	(p)	(q)	(r)	(s)
B	(p)	(q)	(r)	(s)
C	(p)	(q)	(r)	(s)
D	(p)	(q)	(r)	(s)

52. Let $t_1 = (\sin^{-1}x)^{\sin^{-1}x}$, $t_2 = (\sin^{-1}x)^{\cos^{-1}x}$, $t_3 = (\cos^{-1}x)^{\sin^{-1}x}$, $t_4 = (\cos^{-1}x)^{\cos^{-1}x}$, then

Column-I		Column-II	
(A)	$x \in (0, \cos 1)$	(p)	$t_1 > t_2 > t_4 > t_3$
(B)	$x \in \left(\cos 1, \frac{1}{\sqrt{2}}\right)$	(q)	$t_4 > t_3 > t_1 > t_2$
(C)	$x \in \left(\frac{1}{\sqrt{2}}, \sin 1\right)$	(r)	$t_2 > t_1 > t_4 > t_3$
(D)	$x \in (\sin 1, 1)$	(s)	$t_3 > t_4 > t_1 > t_2$

53. Match the following:

Column-I		Column-II	
(A)	$\angle C = 30°$ in ΔABC, $a = x^2 + x + 1$, $b = x^2 - 1$ and $c = 2x + 1$, then $x =$	(p)	3
(B)	The number of all possible values of θ where $0 < \theta < \pi$ for which system of equations $(y + z)\cos 3\theta = xyz(\sin 3\theta)$, $x\sin 3\theta = \frac{2\cos 3\theta}{y} + \frac{2\sin 3\theta}{z}$, $(xyz)\sin 3\theta = (y + 2z)\cos 3\theta + y\sin 3\theta$, have a solution (x_0, y_0, z_0) with $y_0 z_0 \neq 0$ is	(q)	0
(C)	The number of values of θ in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ such that $\theta \neq \frac{n\pi}{5}$ for $n = 0, \pm 1, \pm 2$ and $\tan\theta = \cot 5\theta$ as well as $\sin 2\theta = \cos 4\theta$ is	(r)	1
(D)	Let ω be a complex cube root of unity, then number of distinct complex numbers 'z' satisfying $\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0$ is	(s)	$\sqrt{3}+1$

54. Match the following:

Column-I		Column-II	
(A)	Maximum value of $\frac{a\bar{b} + \bar{a}b}{\lvert ab \rvert}$, where $a, b \in Z$ is	(p)	1
(B)	$\tan\left(\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{2}{9} + \tan^{-1}\frac{4}{33} + \ldots\ldots\infty\right) =$	(q)	0
(C)	If ω is a 2010^{th} root of unity, then $\log_2 \lvert 1 + \omega + \omega^2 + \ldots + \omega^{2008} - \omega^{-1} \rvert =$	(r)	2
(D)	If $\frac{\pi}{4} < A < \frac{\pi}{2}$, then $\tan^{-1}\left(\frac{1}{2}\tan 2A\right) + \tan^{-1}(\cot A) + \tan^{-1}(\cot^3 A) =$	(s)	3

55. If $y = m_i x + \frac{1}{m_i}$, $(i = 1, 2, 3)$ represent three straight lines whose slopes are roots of the equation $2m^3 - 3m^2 - 3m + 2 = 0$, then

Column-I		Column-II	
(A)	Algebraic sum of the intercepts made by the lines on x-axis is	(p)	$\frac{4\sqrt{2} + 9\sqrt{5}}{4}$
(B)	Algebraic sum of the intercepts made by the lines on y-axis is	(q)	$\frac{3}{2}$
(C)	Sum of the distances of the lines from origin is	(r)	$\frac{-21}{4}$
(D)	Sum of the lengths of the lines intercepted between the coordinate axes is	(s)	$\frac{5\sqrt{2} + 9\sqrt{5}}{10}$

56. A triangle ABC is inscribed in the parabola $y^2 = 4x$ with A as vertex and the orthocentre of the triangle as the focus of the parabola.

Column-I		Column-II	
(A)	The distance of the centroid of the triangle from the vertex A is not more than	(p)	1
(B)	The distance of the circumcentre of the triangle from the vertex A is more than	(q)	2
(C)	The distance of the incentre of the triangle from the vertex A is not less than	(r)	3
(D)	Distance between the incentre and the circumcentre of the triangle is less than	(s)	4
		(t)	5

57. Match the following:

Column-I		Column-II	
(A)	Locus of centre of circles touching $x^2 + y^2 - 4x - 4y = 0$ internally and $x^2 + y^2 - 6x - 6y + 17 = 0$ externally is	(p)	straight line
(B)	The locus of the point $(3h - 2, 3k)$ where (h, k) lies on the circle $x^2 + y^2 - 2x - 4y - 4 = 0$ is	(q)	Circle

(C)	Locus of centres of the circles touching the two circles $x^2 + y^2 + 2x = 0$ and $x^2 + y^2 - 6x + 5 = 0$ externally is	(r)	Ellipse
(D)	Extremities of a diagonal of a rectangle are (0, 0) and (4, 4). The locus of the extremities of the diagonal is	(s)	Part of Hyperbola

SECTION-V

Integer Answer Type

This section contains 7 questions. The answer to each of the questions is a single digit integer, ranging from 0 to 9. The appropriate bubbles below the respective question numbers in the ORS have to be darkened. For example, if the correct answers to question numbers X, Y, Z and W (say) are 6, 0, 9 and 2, respectively then the correct darkening of bubbles will look like the following:

X	Y	Z	W
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

58. If the angle between the asymptotes of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $\frac{\pi}{3}$. Then the eccentricity of conjugate hyperbola is

59. The equation $2^{2x} + (a-1)2^{x+1} + a = 0$ has roots of opposite signs, then $[a]$ is (where [.] denotes greatest integer function)

60. Tangent and normal at the ends A and C of focal chord AC of parabola $y^2 = 4x$ intersect at B and D, then minimum area of $ABCD$ is

61. The minimum distance of $4x^2 + y^2 + 4x - 4y + 5 = 0$ from the line $-4x + 3y = 3$ is

62. $A = (-3, 0)$ and $B = (3, 0)$ are the extremities of the base AB of triangle PAB. If the vertex P varies such that the internal bisector of angle APB of the triangle divides the opposite side AB into two segments AD and BD such that $AD : BD = 2 : 1$, then the maximum value of the length of the altitude of the triangle drawn through the vertex P is

63. The tangents drawn from the origin to the circle $x^2 + y^2 - 2rx - 2hy + h^2 = 0$ are perpendicular, then the sum of all possible values of h/r is

64. All terms of an A.P. are natural numbers. The sum of its first nine terms lies between 200 and 220. If the second term is 12, then first term is

SOLUTIONS

1. (d): $Z = \lim_{n\to\infty} (Z_1 \cdot Z_2 \cdot Z_3 \ldots Z_n)$

$$= \lim_{n\to\infty} \text{cis}\left(\frac{\pi}{1\cdot 2\cdot 3} + \frac{\pi}{2\cdot 3\cdot 4} + \ldots + \frac{\pi}{n(n+1)(n+2)}\right)$$

$$= \lim_{n\to\infty} \text{cis}\left(\frac{\pi}{2}\left(\frac{1}{2} - \frac{1}{(n+1)(n+2)}\right)\right) = \text{cis}\,\frac{\pi}{4}$$

$\text{Arg}(Z) = \frac{\pi}{4}$

For $\frac{1}{a_1a_2a_3} + \frac{1}{a_2a_3a_4} + \ldots\ldots + \frac{1}{a_na_{n+1}a_{n+2}}$

$$= \frac{1}{2(a_2 - a_1)}\left[\frac{1}{a_1a_2} - \frac{1}{a_{n+1}a_{n+2}}\right]$$

2. (c): $\sin(2\sin x) = \cos(2\cos x)$

$\sin(2\sin x) = \sin\left(\frac{\pi}{2} - 2\cos x\right)$

$2\sin x = \frac{\pi}{2} - 2\cos x \Rightarrow \sin x + \cos x = \frac{\pi}{4}$

$\Rightarrow 1 + \sin 2x = \frac{\pi^2}{16} \Rightarrow \sin 2x = \frac{\pi^2 - 16}{16}$

$\tan x + \cot x = 2\,\text{cosec}\,2x = \frac{2(16)}{\pi^2 - 16} = \frac{32}{\pi^2 - 16} = \frac{p}{\pi^q - r}$

$\Rightarrow p = 32, q = 2, r = 16 \Rightarrow \frac{p+q+r}{25} = 2$

3. (d): Let $s = \sin\frac{\pi}{3} + \frac{1}{2}\sin\frac{2\pi}{3} + \frac{1}{3}\sin\frac{3\pi}{3} + \ldots\ldots\infty$

$c = \cos\frac{\pi}{3} + \frac{1}{2}\cos\frac{2\pi}{3} + \frac{1}{3}\cos\frac{3\pi}{3} + \ldots\ldots\infty$

$c + is = \left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) + \frac{1}{2}\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

$$+ \frac{1}{3}\left(\cos\frac{3\pi}{3} + i\sin\frac{3\pi}{3}\right) + \ldots\ldots\infty$$

$$= e^{\frac{i\pi}{3}} + \frac{1}{2}\left(e^{\frac{i\pi}{3}}\right)^2 + \frac{1}{3}\left(e^{\frac{i\pi}{3}}\right)^3 + \ldots\ldots\infty$$

$\because \quad (\cos n\theta + i\sin n\theta) = (\cos\theta + i\sin\theta)^n = e^{in\theta}$

$$= -\log\left(1 - e^{\frac{i\pi}{3}}\right)\left\{\because\ x + \frac{x^2}{2} + \frac{x^3}{3} + \ldots\ldots\infty = -\log(1-x)\right\}$$

$$= -\log\left(1 - \left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)\right) = -\log\left(\left(1 - \frac{1}{2}\right) - i\frac{\sqrt{3}}{2}\right)$$

$$= -\log\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) = -\left(\log\sqrt{\frac{1}{4} + \frac{3}{4}} + i\,\text{Arg}\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)\right)$$

$(\because\ \log z = \log|z| + i\,\text{Arg}\,z)$

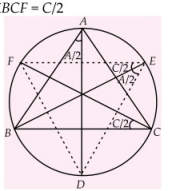
$= -\left(\log 1 + i\tan^{-1}(-\sqrt{3})\right) = i\tan^{-1}\sqrt{3}$

$\Rightarrow \quad s = \frac{\pi}{3}$

4. (a) : Since angles of same segment are equal

$\therefore \quad \angle BED = \angle BAD = A/2$

$\angle BEF = \angle BCF = C/2$



Now, $\angle DEF = \angle BEF + \angle BED = \dfrac{A+C}{2} = 90° - \dfrac{B}{2}$

Similarly, $\angle DFE = 90° - \dfrac{C}{2}$, $\angle EDF = 90° - \dfrac{A}{2}$

Area of $\Delta DEF = \dfrac{1}{2}(DE)(DF)\sin\left(90° - \dfrac{A}{2}\right)$

$= \dfrac{1}{2}(DE)(DF)\cos\dfrac{A}{2}$

Circumradius of ΔABC and ΔDEF are equal.

$\dfrac{DE}{\sin(90° - C/2)} = \dfrac{DF}{\sin(90° - B/2)} = \dfrac{EF}{\sin(90° - A/2)} = 2R$

$DE = 2R\cos(C/2)$, $DF = 2R\cos(B/2)$, $EF = 2R\cos(A/2)$

Area of $\Delta DEF = \dfrac{1}{2}[2R\cos(C/2)][2R\cos(B/2)]\cos(A/2)$

$= 2R^2\cos\dfrac{A}{2}\cos\dfrac{B}{2}\cos\dfrac{C}{2}$

$= 2R^2\sqrt{\dfrac{s(s-a)}{bc}\cdot\dfrac{s(s-b)}{ca}\cdot\dfrac{s(s-c)}{ab}}$

$\left(\because \cos\dfrac{A}{2} = \sqrt{\dfrac{s(s-a)}{bc}}$, similarly other's$\right)$

$= 2R^2\dfrac{s\Delta}{abc}$

$\dfrac{\text{Area of } \Delta DEF}{\text{Area of } \Delta ABC} = \dfrac{\dfrac{2R^2 s\Delta}{abc}}{\Delta} = \dfrac{2R^2 s}{4R\Delta} = \dfrac{Rs}{2\Delta} = \dfrac{R}{2r}$

$\left(\because R = \dfrac{abc}{4\Delta} \;\&\; rs = \Delta\right)$

5. (b): $x\sin\theta - y\cos\theta = \sqrt{x^2+y^2}$ ... (1)

$\dfrac{\sin^2\theta}{a^2} + \dfrac{\cos^2\theta}{b^2} = \dfrac{1}{x^2+y^2}$ ... (2)

From (1), $x^2\sin^2\theta + y^2\cos^2\theta - 2xy\sin\theta\cos\theta = x^2 + y^2$

Clearly, $x^2\cos^2\theta + y^2\sin^2\theta + 2xy\sin\theta\cos\theta = 0$

$\Rightarrow \quad x\cos\theta + y\sin\theta = 0 \;\Rightarrow\; \dfrac{x}{\sin\theta} = \dfrac{-y}{\cos\theta} = \mu$ (Assumed)

$\therefore \quad x^2 = \mu^2\sin^2\theta$, $y^2 = \mu^2\cos^2\theta \;\Rightarrow\; x^2 + y^2 = \mu^2$

$\therefore \quad \dfrac{x^2}{\sin^2\theta} = \dfrac{y^2}{\cos^2\theta} = x^2 + y^2$

$\Rightarrow \quad \sin^2\theta = \dfrac{x^2}{x^2+y^2}$, $\cos^2\theta = \dfrac{y^2}{x^2+y^2}$

From (2), $\dfrac{\sin^2\theta}{a^2} + \dfrac{\cos^2\theta}{b^2} = \dfrac{1}{x^2+y^2}$

$\dfrac{\frac{x^2}{x^2+y^2}}{a^2} + \dfrac{\frac{y^2}{x^2+y^2}}{b^2} = \dfrac{1}{x^2+y^2} \quad \Rightarrow \quad \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

6. (b): $0 \le \cos^{-1}x \le \pi$; $-\dfrac{\pi}{2} \le \sin^{-1}y \le \dfrac{\pi}{2}$

$\Rightarrow \quad 0 \le (\sin^{-1}y)^2 \le \dfrac{\pi^2}{4}$;

$0 \le \dfrac{a\pi^2}{4} \le \dfrac{\pi^2}{4} + \pi$

$\Rightarrow \quad 0 \le a\pi \le \pi + 4 \;\Rightarrow\; 0 \le a \le \dfrac{4}{\pi} + 1$

7. (d): $\log_{12}60 = \dfrac{\log_2 60}{\log_2 12} = \dfrac{2\log_2 2 + \log_2 3 + \log_2 5}{2\log_2 2 + \log_2 3}$

$= \dfrac{2 + \log_2 3 + \log_2 5}{2 + \log_2 3}$

$\log_6 30 = \dfrac{\log_2 30}{\log_2 6} = \dfrac{\log_2 2 + \log_2 3 + \log_2 5}{\log_2 2 + \log_2 3}$

$= \dfrac{1 + \log_2 3 + \log_2 5}{1 + \log_2 3}$ .....(i)

$\log_{15}24 = \dfrac{\log_2 24}{\log_2 15} = \dfrac{3\log_2 2 + \log_2 3}{\log_2 3 + \log_2 5} = \dfrac{3 + \log_2 3}{\log_2 3 + \log_2 5}$ ...(ii)

Put $\log_2 3 = x$, $\log_2 5 = y$ in (i) and (ii)

$\Rightarrow \quad a = \dfrac{1 + x + y}{1 + x}$ and $b = \dfrac{3 + x}{x + y}$

$a + ax = 1 + x + y$ and $bx + by = 3 + x$

$(a-1)x - y + a - 1 = 0$ ... (iii)

$(b-1)x + by - 3 = 0$ ... (iv)

Solving (iii) and (iv),

$x = \dfrac{b + 3 - ab}{ab - 1}$, $y = \dfrac{2a - b - 2 + ab}{ab - 1}$

$\Rightarrow \quad \log_{12}60 = \dfrac{2ab + 2a - 1}{ab + b + 1}$

8. (c) : In ΔAPQ, $\dfrac{PQ}{\sin A} = \dfrac{AP}{\sin\angle AQP} = \dfrac{AP}{\sin\angle ADP}$

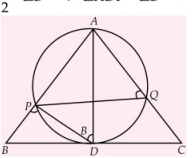
$\left(\text{Using } \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}\right)$



($\because \angle AQP, \angle ADP$ are the angles in same segment)

$\angle APD = \frac{\pi}{2}$ (angle in a semicircle)

$\therefore \quad \angle PDB = \frac{\pi}{2} - \angle B \quad \Rightarrow \quad \angle ADP = \angle B$



From ΔAPD, $\sin B = \frac{AP}{AD}$

$\frac{PQ}{\sin A} = \frac{AP}{\sin B} \Rightarrow \frac{PQ}{\sin A} = \frac{(AD)\sin B}{\sin B} \Rightarrow PQ = (AD)\sin A$

From ΔADC, $\sin C = \frac{AD}{AC} \quad \Rightarrow AD = AC\sin C = b\sin C$

$\Rightarrow \quad PQ = b\sin C \sin A = 2R\sin A \sin B \sin C = \Delta/R$

$\left(\text{Using } \frac{a}{2\sin A} = \frac{b}{2\sin B} = \frac{c}{2\sin C} = R\right)$

9. (b): $a \geq 0,\ 2 - a \geq 0 \ \Rightarrow \ a \leq 2 \ \geq \ 0 \leq a \leq 2$ ....(i)

$\frac{\sqrt{a}}{\sqrt{a+4}}\cos x - \frac{2}{\sqrt{a+4}}\sin x = \frac{\sqrt{2+\sqrt{2-a}}}{\sqrt{a+4}}$

$\left|\frac{\sqrt{2+\sqrt{2-a}}}{\sqrt{a+4}}\right| \leq 1 \ \Rightarrow \ \sqrt{2+\sqrt{2-a}} \leq \sqrt{a+4}$

$\Rightarrow \ 2+2-a+2\sqrt{2\sqrt{2-a}} \leq a+4 \ \Rightarrow \sqrt{2\sqrt{2-a}} \leq a$

$\Rightarrow \quad 2(2-a) \leq a^2 \Rightarrow \ a^2+2a-4 \geq 0$ or $(a+1)^2 - (\sqrt{5})^2 \geq 0$

$\Rightarrow \ a \leq -1-\sqrt{5}\ , a \geq -1+\sqrt{5}$

using $(x-a)\,(x-b) \geq 0 \ \forall\ a < b \Rightarrow \quad x \leq a,\ x \geq b$

$= \sqrt{5} - 1 \leq a \leq 2$ using (i)

10. (c): $Z_k = 1 + a + a^2 + \ldots\ldots + a^{k-1} = \frac{1-a^k}{1-a}$

$\Rightarrow \ Z_k - \frac{1}{1-a} = \frac{-a^k}{1-a}$

$\left|Z_k - \frac{1}{1-a}\right| = \left|\frac{-a^k}{1-a}\right| = \frac{|a|^k}{|1-a|} < \frac{1}{|1-a|} \qquad (\because |a| < 1)$

$\Rightarrow \ Z_k$ lies within the circle $\left|Z - \frac{1}{1-a}\right| = \frac{1}{|1-a|}$

11. (a): $\log_2(\cos(\sin x - 1)) \geq 0 \ \Rightarrow \ \cos(\sin x - 1) \geq 1$

$\Rightarrow \ \cos(\sin x - 1) = 1 \quad \Rightarrow \quad \sin x - 1 = 0$

$\Rightarrow \quad \sin x = 1 \ \Rightarrow \ x = (4n+1)\frac{\pi}{2}, n \in I$

RHS = 0 if $x = (2n+1)\frac{\pi}{2}, \forall\ n \in I$

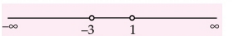
12. (c): $x^2 - x + a = 2x^2 - ax + 1$

$x^2 + (1-a)x + 1 - a = 0$

Since $x \in R$, $\Delta \geq 0$

$\Rightarrow \ (1-a)^2 - 4(1)(1-a) \geq 0$

$\Rightarrow \ (1-a)(1-a-4) \geq 0 \ \Rightarrow \ (1-a)(-a-3) \geq 0$



$\Rightarrow \quad (a-1)(a+3) \geq 0$

$\Rightarrow \quad a \in (-\infty, -3] \cup [1, \infty)$

$\Rightarrow \quad a \notin [-2, 0]$ i.e. $a \notin \{-2, -1, 0\}$

13. (d): Let $y = \tan\frac{3\pi}{11} + 4\sin\frac{2\pi}{11}$

$y\cos\frac{3\pi}{11} = \sin\frac{3\pi}{11} + 4\sin\frac{2\pi}{11}\cos\frac{3\pi}{11}$

$= \sin\frac{3\pi}{11} + 2\left[\sin\frac{5\pi}{11} - \sin\frac{\pi}{11}\right]$

Squaring both sides, we get

$y^2\cos^2\frac{3\pi}{11} = \sin^2\frac{3\pi}{11} + 4\sin^2\frac{5\pi}{11} + 4\sin^2\frac{\pi}{11}$

$+ 4\sin\frac{3\pi}{11}\sin\frac{5\pi}{11} - 4\sin\frac{3\pi}{11}\sin\frac{\pi}{11} - 8\sin\frac{5\pi}{11}\sin\frac{\pi}{11}$

$= \frac{1}{2}\left[1 - \cos\frac{6\pi}{11}\right] + 2\left[1 - \cos\frac{10\pi}{11}\right] + 2\left[1 - \cos\frac{2\pi}{11}\right]$

$+ 2\left[\cos\frac{2\pi}{11} - \cos\frac{8\pi}{11}\right] - 2\left[\cos\frac{2\pi}{11} - \cos\frac{4\pi}{11}\right]$

$- 4\left[\cos\frac{4\pi}{11} - \cos\frac{6\pi}{11}\right]$

$= \frac{9}{2} - 2\left[\cos\frac{2\pi}{11} + \cos\frac{4\pi}{11} + \cos\frac{6\pi}{11} + \cos\frac{8\pi}{11} + \cos\frac{10\pi}{11}\right]$

$+ \frac{11}{2}\cos\frac{6\pi}{11}$

$\Rightarrow \ 2y^2\cos^2\frac{3\pi}{11} = 9 + 11\cos\frac{6\pi}{11} - 4\left[\cos\frac{2\pi}{11} + \cos\frac{4\pi}{11}\right.$

$\left. + \cos\frac{6\pi}{11} + \cos\frac{8\pi}{11} + \cos\frac{10\pi}{11}\right]$

$$= 9 + 11\cos\frac{6\pi}{11} - \frac{2}{\sin\frac{2\pi}{11}}\left[2\sin\frac{2\pi}{11}\cos\frac{2\pi}{11} + 2\sin\frac{2\pi}{11}\cos\frac{4\pi}{11}\right.$$

$$\left. + 2\sin\frac{2\pi}{11}\cos\frac{6\pi}{11} + 2\sin\frac{2\pi}{11}\cos\frac{8\pi}{11} + 2\sin\frac{2\pi}{11}\cos\frac{10\pi}{11}\right]$$

$$= 9 + 11\cos\frac{6\pi}{11} - \frac{2}{\sin\frac{2\pi}{11}}\left[\sin\frac{4\pi}{11} + \sin\frac{6\pi}{11} - \sin\frac{2\pi}{11} + \sin\frac{8\pi}{11}\right.$$

$$\left. - \sin\frac{4\pi}{11} + \sin\frac{10\pi}{11} - \sin\frac{6\pi}{11} + \sin\frac{12\pi}{11} - \sin\frac{8\pi}{11}\right]$$

$$= 9 + 11\cos\frac{6\pi}{11} - \frac{2}{\sin\frac{2\pi}{11}}\left[-\sin\frac{2\pi}{11} + \sin\frac{10\pi}{10} + \sin\frac{12\pi}{11}\right]$$

$$= 9 + 11\cos\frac{6\pi}{11} - \frac{2}{\sin\frac{2\pi}{11}}\left[-\sin\frac{2\pi}{11} + 0\right] = 11\left(1 + \cos\frac{6\pi}{11}\right)$$

$$2y^2\cos^2\frac{3\pi}{11} = 22\cos^2\frac{3\pi}{11} \Rightarrow y^2 = 11 \Rightarrow y = \sqrt{11}$$

14. (c): $a^2 + b^2 = c^2$ and $\frac{1}{a^2} = \frac{1}{b^2} + \frac{1}{c^2}$

Let 'θ' be the smallest angle at the vertex A of ΔABC

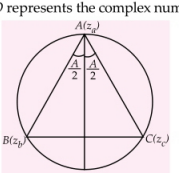
then $\sin\theta = \frac{a}{c}$

$$\Rightarrow \left(\frac{a}{c}\right)^2 = 1 - \left(\frac{a}{b}\right)^2 \Rightarrow \sin^2\theta + \tan^2\theta = 1$$

i.e., $\sin^4\theta - 3\sin^2\theta + 1 = 0$

$$\Rightarrow \sin\theta = \frac{\sqrt{5}-1}{2}\left(\text{as } \sin^2\theta = \frac{3-\sqrt{5}}{2} = \frac{6-2\sqrt{5}}{4} = \left(\frac{\sqrt{5}-1}{2}\right)^2\right)$$

15. (b): Let D represents the complex number $z = (z_D)$



$\angle BAD = \angle CAD = A/2$

$$\frac{z - z_a}{|z - z_a|} = \frac{z_b - z_a}{|z_b - z_a|}e^{iA/2}$$

$$\frac{(z - z_a)^2}{|z - z_a|^2} = \frac{(z_b - z_a)^2}{|z_b - z_a|^2}e^{iA} \quad \text{... (i)}$$

Similarly, $\frac{(z_c - z_a)^2}{|z_c - z_a|^2} = \frac{z - z_a}{|z - z_a|^2}e^{iA}$ ... (ii)

From (i) and (ii),

$$\frac{z - z_a}{\overline{z} - \overline{z}_a} = \frac{z_b - z_a}{\overline{z}_b - \overline{z}_a}e^{iA}, \quad \frac{z_c - z_a}{\overline{z}_c - \overline{z}_a} = \frac{z - z_a}{\overline{z} - \overline{z}_a}e^{iA}$$

$$\Rightarrow \frac{z - z_a}{\overline{z} - \overline{z}_a} \times \frac{\overline{z}_c - \overline{z}_a}{z_c - z_a} = \frac{z_b - z_a}{\overline{z}_b - \overline{z}_a} \times \frac{\overline{z} - \overline{z}_a}{z - z_a}$$

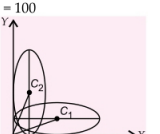
$$\Rightarrow \left(\frac{z - z_a}{\overline{z} - \overline{z}_a}\right)^2 = \frac{z_b - z_a}{\overline{z}_b - \overline{z}_a} \cdot \frac{z_c - z_a}{\overline{z}_c - \overline{z}_a}$$

$$\Rightarrow \left(\frac{z - z_a}{\frac{r^2}{z} - \frac{r^2}{z_a}}\right)^2 = \left(\frac{z_b - z_a}{\frac{r^2}{z_b} - \frac{r^2}{z_a}}\right)\left(\frac{z_c - z_a}{\frac{r^2}{z_c} - \frac{r^2}{z_a}}\right)$$

$$\left(\because z_a, z_b, z_c \text{ and } z \text{ lie on } |z| = r \right.$$
$$\left. \therefore |z_a| = |z_b| = |z_c| = r\right)$$

$$\Rightarrow (z z_a)^2 = (z_a z_b)(z_a z_c) \Rightarrow z^2 = z_b z_c \Rightarrow z_D^2 = z_b z_c$$

16. (d): The locus of the centre of the ellipse is director circle *i.e.* $x^2 + y^2 = 100$



$\angle C_1OC_2 = \theta$

$$\Rightarrow \frac{\pi}{2} - 2\tan^{-1}\left(\frac{5}{5\sqrt{3}}\right) = \frac{\pi}{6}$$

$$\therefore \text{ Arc length} = 10.\frac{\pi}{6} = \frac{5\pi}{3}$$

17. (a): $\frac{1}{c+a} - \frac{1}{b+c} = \frac{1}{a+b} - \frac{1}{c+a}$

$$\Rightarrow \frac{b-a}{b+c} = \frac{c-b}{a+b}$$

$$\Rightarrow 2b^2 = a^2 + c^2 \Rightarrow a^2, b^2, c^2 \in A.P.$$

18. (a): Let the roots be α and 2α

$$\Rightarrow \alpha + 2\alpha = \frac{1-3a}{a^2-5a+3} \text{ and } \alpha \cdot 2\alpha = \frac{2}{a^2-5a+3}$$

$$\Rightarrow 2\left[\frac{1}{9} \times \frac{(1-3a)^2}{(a^2-5a+3)^2}\right] = \frac{2}{a^2-5a+3}$$

$$\Rightarrow 9a^2 - 6a + 1 = 9a^2 - 45a + 27$$

$$\Rightarrow 39a = 26$$

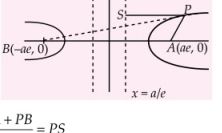
$$\Rightarrow a = \frac{26}{39} = \frac{2}{3}$$

19. (c) : $S_1 = \dfrac{T_1^2 + 3T_1 - 2}{3} = T_1 \Rightarrow T_1^2 = 2$ (as $S_1 = T_1$)

$S_2 - S_1 = \dfrac{T_2^2 - T_1^2 + 3(T_2 - T_1)}{3} = T_2$

$T_2^2 - T_1^2 + 3(T_2 - T_1) = 3T_2 \Rightarrow T_2^2 = T_1^2 + 3T_1 = 2 + 3\sqrt{2}$

20. (b): It turns out that P has to be on the left branch. x-coordinate is found to be $-64/5$



Given $\dfrac{PA + PB}{2} = PS$

$\dfrac{x^2}{4^2} - \dfrac{y^2}{3^2} = 1 \Rightarrow a = 4, b = 3 \Rightarrow \quad e = \dfrac{5}{4}$

21. (c) : Solving C_1 and C_2,

$\Rightarrow A\left(\sqrt{\dfrac{3}{1-k}}, \dfrac{3k}{1-k}\right) \equiv (a, ka^2) \equiv (a, a^2 - 3).$

Tangent to C_2 at A is $y + a^2 - 3 = 2kx$ ... (1)

$\Rightarrow \quad B \equiv (1, -2) \quad (A \neq 1)$

From expression (1), $-2 + a^2 - 3 = 2a\left(1 - \dfrac{3}{a^2}\right)$

or $a^3 - 2a^2 - 5a + 6 = 0 \Rightarrow (a-1)(a-3)(a+2) = 0$

$\Rightarrow \quad a = 3, a = -2, a = 1.$

$\therefore \quad a = 3$

22. (d): (x, y) lies on the circle, with AB as a diameter.

Area $(\Delta ABC) = 3 \quad \Rightarrow \quad \left(\dfrac{1}{2}\right)(AB)(\text{altitude}) = 3$

$\Rightarrow \quad \text{altitude} = \dfrac{3}{\sqrt{2}} \quad \Rightarrow \quad$ no such C exists

($\because C(x, y)$ does not lie on circle as radius of circle $= R = \sqrt{2} <$ Altitude of Δ $\therefore C(x, y)$ does not lie in xy-plane.

23. (a) : The reflection of (α, β) in the line $y = 2x$ is

$(\alpha_1, \beta_1) = \left(\dfrac{4\beta - 3\alpha}{5}, \dfrac{4\alpha + 3\beta}{5}\right) = \alpha_1\beta_1 = 1$

$\Rightarrow \quad 12\alpha^2 - 7\alpha\beta - 12\beta^2 + 25 = 0$

$\equiv 12x^2 + rxy + sy^2 + t = 0$

$\therefore \quad r = -7$

24. (d): a, b, c are in A.P.

$\Rightarrow \quad 2b = a + c \Rightarrow \quad b = \dfrac{a + c}{2} \quad$ ... (1)

$(b - a), (c - b), a$ are in G.P.

$\Rightarrow \quad (c - b)^2 = a(b - a)$

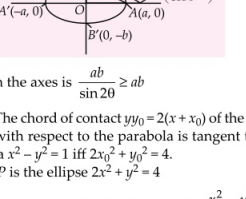
$\Rightarrow \quad \left(c - \dfrac{a + c}{2}\right)^2 = \dfrac{a(c - a)}{2}$

$\Rightarrow \quad c - a = 2a \Rightarrow c = 3a \quad$ ... (2)

From (1) and (2), $\dfrac{a}{1} = \dfrac{b}{2} = \dfrac{c}{3}$

25. (d): Equation of tangent at θ is

$\dfrac{x}{a}\cos\theta + \dfrac{y}{b}\sin\theta = 1$



Area with the axes is $\dfrac{ab}{\sin 2\theta} \geq ab$

26. (c) : The chord of contact $yy_0 = 2(x + x_0)$ of the point $P(x_0, y_0)$ with respect to the parabola is tangent to the hyperbola $x^2 - y^2 = 1$ iff $2x_0^2 + y_0^2 = 4$.

Locus of P is the ellipse $2x^2 + y^2 = 4$

27. (d): Equation of chord of hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ bisected at a given point $P(x_1, y_1)$ is given by

$\dfrac{xx_1}{a^2} - \dfrac{yy_1}{b^2} - 1 = \dfrac{x_1^2}{a^2} - \dfrac{y_1^2}{b^2} - 1$ or $T = S'$

$\therefore \quad$ Equation of chord $T = S'$

$\Rightarrow \quad x(-1) - 2(y)(1) - 1 = (-1)^2 - (2)(1)^2 - 1$

$\Rightarrow \quad -x - 2y - 1 = 1 - 2 - 1$ or $x + 2y - 1 = 0$

$\therefore \quad$ Points of intersection of the chord & axis are $(0, 1/2), (1, 0)$

$\therefore \quad$ Required points (vertices of triangle) are $A(0, 0), B(0, 1/2), C(1, 0)$

$\therefore \quad$ Required area $= \left|\dfrac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 0 & 1/2 & 1 \\ 1 & 0 & 1 \end{vmatrix}\right| = \dfrac{1}{4}$ sq. units

28. (a, c) : $2\cos\left(\alpha + \dfrac{\beta}{2}\right)\cos\dfrac{\beta}{2} = -\cos(\alpha + \beta + \gamma) \quad$ ... (i)

$2\sin\left(\alpha + \dfrac{\beta}{2}\right)\cos\dfrac{\beta}{2} = -\sin(\alpha + \beta + \gamma) \quad$ ... (ii)

$\dfrac{\text{(ii)}}{\text{(i)}} \Rightarrow \tan\left(\alpha + \dfrac{\beta}{2}\right) = \tan(\alpha + \beta + \gamma), \alpha + \beta + \gamma = n\pi + \left(\alpha + \dfrac{\beta}{2}\right)$

$\Rightarrow \alpha+\beta+\gamma=\pi+\alpha+\frac{\beta}{2} \Rightarrow \gamma=\pi-\frac{\beta}{2}$

From (i), $2\cos\left(\alpha+\frac{\beta}{2}\right)\cos\frac{\beta}{2}=-\cos\left(\alpha+\beta+\pi-\frac{\beta}{2}\right)$

$=-\cos\left(\pi+\alpha+\frac{\beta}{2}\right)=\cos\left(\alpha+\frac{\beta}{2}\right)$

$\Rightarrow 2\cos\left(\alpha+\frac{\beta}{2}\right)\cos\frac{\beta}{2}-\cos\left(\alpha+\frac{\beta}{2}\right)=0$

$\cos\left(\alpha+\frac{\beta}{2}\right)\left[2\cos\frac{\beta}{2}-1\right]=0$

Similarly from (ii) $\sin\left(\alpha+\frac{\beta}{2}\right)\left(2\cos\frac{\beta}{2}-1\right)=0$

$\Rightarrow 2\cos\frac{\beta}{2}-1=0 \Rightarrow \cos\frac{\beta}{2}=\frac{1}{2} \Rightarrow \frac{\beta}{2}=\frac{\pi}{3}$

$\Rightarrow \beta=\frac{2\pi}{3}$ and $\gamma=\pi-\frac{\pi}{3}=\frac{2\pi}{3}$

29. (a, b, c) : $\cos x+\cos y=a$

$\cos^2x+\cos^2y+2\cos x\cos y=a^2$ ... (i)

$\cos 2x+\cos 2y=b$

$\Rightarrow 2\cos^2x-1+2\cos^2y-1=b$

$\Rightarrow 2(\cos^2x+\cos^2y)=b+2$

$\Rightarrow \cos^2x+\cos^2y=\frac{b}{2}+1$ ... (ii)

From (i) & (ii) $\cos x\cos y=\frac{a^2}{2}-\left(\frac{b+2}{4}\right)$ ... (iii)

$\cos 3x+\cos 3y=c$

$4\cos^3x-3\cos x+4\cos^3y-3\cos y=c$

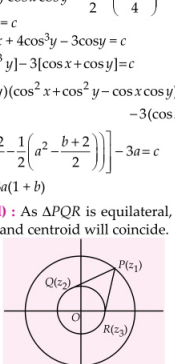
$4[\cos^3x+\cos^3y]-3[\cos x+\cos y]=c$

$4[(\cos x+\cos y)(\cos^2x+\cos^2y-\cos x\cos y)]-3(\cos x+\cos y)=c$

$\Rightarrow 4\left[a\left(\frac{b+2}{2}-\frac{1}{2}\left(a^2-\frac{b+2}{2}\right)\right)\right]-3a=c$

$\Rightarrow 2a^3+c=3a(1+b)$

30. (a, b, c, d) : As ΔPQR is equilateral, orthocentre, circumcentre and centroid will coincide.



$\left|\frac{z_1+z_2+z_3}{3}\right|=1 \Rightarrow |z_1+z_2+z_3|^2=9$

$\Rightarrow (z_1+z_2+z_3)(\bar{z}_1+\bar{z}_2+\bar{z}_3)=9$

$\Rightarrow \left(\frac{4}{\bar{z}_1}+\frac{1}{\bar{z}_2}+\frac{1}{\bar{z}_3}\right)\left(\frac{4}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right)=9$

$(\because |z_1|^2=4, |z_2|^2=1, |z_3|^2=1)$

and $\angle QOR=120°$

31. (b, d): $\log a$, $\log b$, $\log c$ are in A.P.,

$\Rightarrow 2\log b=\log a+\log c \Rightarrow b^2=ac$

$2(\log 2b-\log 3c)=(\log a-\log 2b)+(\log 3c-\log a)$

$\left(\frac{2b}{3c}\right)^2=\frac{a}{2b}\times\frac{3c}{a} \Rightarrow \left(\frac{2b}{3c}\right)^2=\left(\frac{3c}{2b}\right) \Rightarrow \frac{2b}{3c}=1 \Rightarrow 2b=3c$

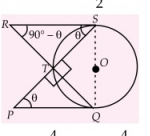
$\Rightarrow b=\frac{3c}{2}$

Since, $b^2=ac \Rightarrow a=\frac{b^2}{c}=\frac{9c^2}{4c}=\frac{9}{4}c, b=\frac{3c}{2}$

Clearly, $a+b>c$, $b+c>a$, $a+c>b$

(If sum of any two numbers is greater than the third then numbers can be assumed as sides of the triangle.)

32. (a, b, c) : $\angle QTS=\angle PTQ=\frac{\pi}{2}$, as QS is diameter



$\frac{QT}{PQ}=\sin\theta, \frac{ST}{RS}=\cos\theta, \frac{4}{PQ}=\tan\theta, \frac{4}{RS}=\cot\theta$

$\therefore \left(\frac{QT}{PQ}\right)^2+\left(\frac{ST}{RS}\right)^2+PQ\cdot RS$

$=\sin^2\theta+\cos^2\theta+(4\cot\theta)(4\tan\theta)=17$

33. (b, c) : The parabola can be written as

$\left(x+\frac{2}{3}\right)^2=2(y-\frac{10}{9})$ *i.e.* $X^2=2Y\left(X=x+\frac{2}{3}, Y=y-\frac{10}{9}\right)$.

A point on the axis is $\left(\frac{-2}{3}, Y\right)$ from which three normals can be drawn if $Y>1$ i.e., $y>\frac{19}{9}>2$

34. (a, b, c, d) : Equation of C_1 is $(x-a)^2+(y-a)^2=a^2$

or $x^2+y^2-2ax-2ay+a^2=0$

Equation of C_2 is $(x-b)^2+(y-b)^2=b^2$

or $x^2+y^2-2bx-2by+b^2=0$

Now circles C_1 and C_2 touches each other if

$C_1C_2=r_1+r_2$

or $\sqrt{(a-b)^2+(a-b)^2}=a+b$

$\Rightarrow \sqrt{2}|(a-b)| = a+b$ or $\sqrt{2}(b-a) = b+a$

$\Rightarrow \dfrac{b+a}{b-a} = \dfrac{\sqrt{2}}{1}$

$\Rightarrow \dfrac{b}{a} = \dfrac{\sqrt{2}+1}{\sqrt{2}-1}$ (using componendo & dividendo)

$\dfrac{b}{a} = \dfrac{(\sqrt{2}+1)^2}{2-1} = \dfrac{3+2\sqrt{2}}{1}$

Also two cicles $x^2 + y^2 + 2g_1x + 2f_1y + d_1 = 0$ & $x^2 + y^2 + 2g_2x + 2f_2y + d_2 = 0$ are orthogonal if $2g_1g_2 + 2f_1f_2 = d_1 + d_2$

$\Rightarrow 2ab + 2ab = a^2 + b^2$

$\Rightarrow \dfrac{a^2+b^2}{ab} = 4$ or $\dfrac{a}{b} + \dfrac{b}{a} = 4$

$\Rightarrow t^2 - 4t + 1 = 0 \quad (t = b/a)$

$\Rightarrow t = \dfrac{4 \pm \sqrt{16-4}}{2} = 2 \pm \sqrt{3} \Rightarrow \dfrac{b}{a} = 2 \pm \sqrt{3}$

$\Rightarrow \dfrac{b}{a} = 2 + \sqrt{3}$(as $2 - \sqrt{3}$ is not included in the question)

Again $C_2 : (x-b)^2 + (y-b)^2 - b^2 = 0$ passes through the centre $C_1(a, a)$

$\therefore \quad (a-b)^2 + (8-b)^2 = b^2$ or $2\left(\dfrac{b-a}{b}\right)^2 = 1$

or $\dfrac{b}{b-a} = \dfrac{\sqrt{2}}{1}$ or $\dfrac{b}{a} = \dfrac{\sqrt{2}}{\sqrt{2}-1}$ using dividendo

or $\dfrac{b}{a} = \sqrt{2}(\sqrt{2}+1) = 2 + \sqrt{2}$

35. (a, c) : Tangent to the circle $x^2 + y^2 = 1$ is $x\cos\theta + y\sin\theta = 1$.

$R(x_0, y_0)$ is the point of intersection of the tangents to the ellipse at P and Q

$\Rightarrow x\cos\theta + y\sin\theta = 1$ and $x_0x + 2y_0y = 4$ represent the same line

$\Rightarrow x_0 = 4\cos\theta$ and $y_0 = 2\sin\theta$

$\Rightarrow \dfrac{x_0^2}{16} + \dfrac{y_0^2}{4} = 1.$

Hence, locus of P is the ellipse $\dfrac{x^2}{16} + \dfrac{y^2}{4} = 1$

36. (a, b, c) : If the parabolas have a common normal of slope m (the only allowed value of m is $m = 0$) then it is given by $y = mx - 2km - km^3$ and

$y = m(x-1) - 2m - m^3 = mx - 3m - m^3$

$\Rightarrow \quad 2km + km^3 = 3m + m^3 \quad \Rightarrow \quad m = 0, m^2 = \dfrac{3-2k}{k-1}.$

If $m^2 < 0$ then only common normal is the axis.

$\Rightarrow \dfrac{3-2k}{k-1} < 0$

$\Rightarrow 3 - 2k < 0$ & $k - 1 > 0$ or $3 - 2k > 0$ & $k - 1 < 0$

$\Rightarrow k > \dfrac{3}{2}$ & $k > 1$ or $k < \dfrac{3}{2}$ and $k < 1$

37. (b, c) : Equation of the ellipse whose axes are along the axes of co-ordinates is given by

$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1.$ ..... (*)

Now * passes through (–4, 1)

$\therefore \quad \dfrac{16}{a^2} + \dfrac{1}{b^2} = 1$ ..... (i)

Also the line $x + 4y - 10 = 0$ tangent to the ellipse now

i.e., $lx + my + n = 0 = x + 4y - 10$ are tangent to ellipse

$\therefore \quad n^2 = a^2l^2 + m^2b^2 \quad \Rightarrow a^2 + 16b^2 = 100$ ..... (ii)

Solving (i) & (ii), we get $a^2 = 20, a^2 = 80$

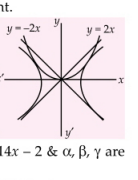
$b^2 = 5, b^2 = 5/4$

Required equation/possible equations of ellipse $\dfrac{x^2}{20} + \dfrac{y^2}{5} = 1$ & $\dfrac{x^2}{80} + \dfrac{y^2}{5/4} = 1$

38. (c) : The circle ABC is the circle described on the point of (1, 1) and (5, –7) as diameter

39. (c, d) : (α, α^2) lie on the parabola $y = x^2$

(α, α^2) must lie between the asymptotes of hyperbola $\dfrac{x^2}{1} - \dfrac{y^2}{4} = 1$ in 1st and 2nd quadrant.



$\therefore$ Asymptotes are $y = \pm 2x$

$\therefore \quad 2\alpha < \alpha^2 \Rightarrow \alpha < 0$ or $\alpha > 2$

and $-2\alpha < \alpha^2$

$\alpha < -2$ or $\alpha > 0$

$\therefore \quad \alpha \in (-\infty, -2)$ or $\alpha \in (2, \infty)$

40. (a) : Given $f(x) = 3x^3 - 13x^2 + 14x - 2$ & α, β, γ are roots of $f(x) = 0$

Clearly $f(0) = -2$, $f(1) = 2$, $f(2) = -2$, $f(3) = 4$.

Here $f(0) f(1) < 0$, $f(1) f(2) < 0$, $f(2) f(3) < 0$

$\Rightarrow \quad 0 < \alpha < 1, 1 < \beta < 2, 2 < \gamma < 3$

$\Rightarrow \quad [\alpha] = 0, [\beta] = 1, [\gamma] = 2$

$\therefore \quad \tan^{-1}[\alpha] + \tan^{-1}[\beta + 1] + \tan^{-1}[\gamma + 2]$

$= 0 + \tan^{-1}2 + \tan^{-1}3$

$= \pi + \tan^{-1}\left(\dfrac{2+3}{1-2\cdot 3}\right) = \pi - \dfrac{\pi}{4} = \dfrac{3\pi}{4}$

41. (c) : Since $0 < \alpha < 1 \quad \Rightarrow \lim\limits_{n \to \infty} \alpha^{\lfloor n \rfloor} \beta^{1/n} = 0$

$\therefore \quad k = 0$

$\therefore \tan^{-1}(k) = \tan^{-1}0 = 0$

42. (d): $\tan^{-1}\alpha + \tan^{-1}\beta + \tan^{-1}\gamma = \tan^{-1}\left(\dfrac{\sum\alpha - \alpha\beta\gamma}{1 - \sum\alpha\beta}\right)$

As α, β, γ are roots of $3x^3 - 13x^2 + 14x - 2 = 0$

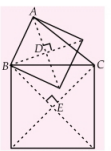
$\therefore \sum\alpha = \alpha+\beta+\gamma = \frac{13}{3}, \sum\alpha\beta = \alpha\beta+\beta\gamma+\gamma\alpha = \frac{14}{3}$ and $\alpha\beta\gamma = \frac{2}{3}$

$$= \tan^{-1}\left(\frac{\frac{13}{3}-\frac{2}{3}}{1-\frac{14}{3}}\right) = \tan^{-1}(-1)$$

Since α, β, γ are positive and real,

$$\Rightarrow \tan^{-1}\alpha + \tan^{-1}\beta + \tan^{-1}\gamma = \frac{3\pi}{4}$$

43. (a): $\frac{z_B - z_D}{z_A - z_D} = \frac{BD}{AD}e^{i\pi/2} = i$



$\Rightarrow z_B - z_D = z_A i - z_D i$

$\Rightarrow z_B - z_A i = z_D(1-i)$

$\Rightarrow z_D = \frac{z_B - z_A i}{1-i}$

Similarly, $z_E = \frac{z_B - z_C i}{1-i}$

$$z_E - z_D = \frac{i(z_A - z_C)}{1-i}$$

Angle between AC and $DE = \text{Arg}\left(\frac{z_C - z_A}{z_E - z_D}\right)$

$$= \text{Arg}\left(\frac{(z_C - z_A)(1-i)}{i(z_A - z_C)}\right) = \text{Arg}(1+i) = \frac{\pi}{4}$$

44. (a): $DE = |z_E - z_D| = \left|\frac{i(z_A - z_C)}{1-i}\right| = \frac{|1-\omega^2|}{\sqrt{2}} = \sqrt{\frac{3}{2}}$

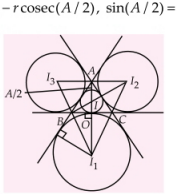
45. (d): $z_E = \frac{z_B - z_C i}{1-i} = \frac{\omega - \omega^2 i}{1-i} = \frac{(\omega+\omega^2)+i(\omega-\omega^2)}{2}$

$$= \frac{-1-\sqrt{3}}{2} \quad (\text{using } 1+\omega+\omega^2 = 0)$$

$$AE = \left|1 + \frac{1+\sqrt{3}}{2}\right| = \frac{3+\sqrt{3}}{2}$$

46. (b): $II_1 = AI_1 - AI$. From $\Delta AI_1 B$

$r_1\text{cosec}(A/2) - r\,\text{cosec}(A/2),\ \sin(A/2) = \frac{I_1 B}{AI_1} = \frac{r_1}{AI_1}$



$= \text{cosec}(A/2)[s\tan(A/2) - (s-a)\tan(A/2)]$

$= \text{cosec}(A/2)\, a\tan(A/2) = a\sec(A/2)$

47. (a): From $\Delta I_1 DC$,

$$\sin(90^\circ - C/2) = \frac{I_1 D}{I_1 C} = \frac{r_1}{I_1 C}$$

$\Rightarrow I_1 C = r_1 \sec(C/2)$

Similarly, $I_2 C = r_2 \sec(C/2)$

$I_1 I_2 = I_1 C + I_2 C = (r_1 + r_2)\sec(C/2)$

$= [4R\sin(A/2)\cos(B/2)\cos(C/2)$

$+ 4R\cos(A/2)\sin(B/2)\cos(C/2)]\sec(C/2)$

$= 4R[\sin(A/2)\cos(B/2) + \cos(A/2)\sin(B/2)]$

$= 4R\sin\left(\frac{A+B}{2}\right) = 4R\cos\left(\frac{C}{2}\right)$

$= \frac{2c}{\sin C}\cos(C/2) \quad (\because c = 2R\sin C)$

as $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$

$= \frac{2c\cos(C/2)}{2\sin(C/2)\cos(C/2)} = c\ \text{cosec}(C/2)$

48. (c): Area of $\Delta I_1 I_2 I_3$ is

$$\frac{1}{2}\times(I_2 I_3)\times AI_1 = \frac{1}{2}(a\,\text{cosec}\,A/2)r_1\text{cosec}(A/2) = \frac{abc}{2r}$$

49. (b): Take B as origin, BC as x-axis and take A as (h, k), C $(4, 0)$

Area of $\Delta ABC = \frac{1}{2}4\times k = 2k$ ... (1)

$D = (2, 0)$ and $E\left(\frac{h+4}{2}, \frac{k}{2}\right)$

$\because$ $AD \perp BE$, slope of AD × slope of $BE = -1$

$\Rightarrow k^2 + (h+4)(h-2) = 0$ ... (2)

Also $AC = 3 \Rightarrow (h-4)^2 + k^2 = 9$ ... (3)

$(2) - (3) \Rightarrow h = \frac{3}{2}$ and $k^2 = \frac{11}{4} \Rightarrow k = \frac{\sqrt{11}}{2}$

From (1), area of $\Delta ABC = \sqrt{11}$

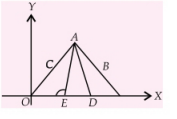
50. (c): Now AD is the bisector.

$\frac{AB}{AC} = \frac{BD}{DC} \Rightarrow c = 2b$

$b + c > a \Rightarrow b + c > 6$

$\therefore \quad b > 2$ (using $c = 2b$)

Again, $\frac{b^2 + 4b^2 - 3b}{4b^2} < 1$



$\Rightarrow b < 6$

$\therefore \quad 2 < b < 6$ and consequently $4 < c < 12$

51. (c): Now, $c^2 = h^2 + k^2$ and $b^2 = (h-6)^2 + k^2$

$$c^2 - b^2 = 12h - 3b \Rightarrow h = \frac{b^2 + 12}{4}$$

Given that $k^2 > 10 \Rightarrow c^2 - h^2 > 10$

$\Rightarrow \quad 4b^2 - \left(\frac{b^2+12}{4}\right)^2 > 10$

or $b^4 - 40b^2 + 304 > 0$

$\Rightarrow \quad (b^2-20)^2 - 96 > 0$

$\Rightarrow \quad (b^2 - 20 + \sqrt{96})(b^2 - 20 - \sqrt{96}) > 0$

$\Rightarrow \quad 20 - 4\sqrt{6} < b^2 < 20 + 4\sqrt{6}$

$\Rightarrow \quad 11.4 < b^2 < 29.6$

$\Rightarrow \quad b = 4$ or 5

52. A-q, B-s, C-r, D-p

(A) In (0, cos1), $\cos^{-1}x > \sin^{-1}x$

Also $\cos^{-1}x > 1$ and $\sin^{-1}x < 1$

$\Rightarrow$ The greatest is $(\cos^{-1}x)^{\cos^{-1}x} = t_4$, the least is $(\sin^{-1}x)^{\cos^{-1}x} = t_2$

and $(\sin^{-1}x)^{\sin^{-1}x} < (\cos^{-1}x)^{\sin^{-1}x}$

$\Rightarrow \quad t_1 < t_3$

So, $t_4 > t_3 > t_1 > t_2$

(B) Similarly, if $\cos 1 < x < \frac{1}{\sqrt{2}}$

$\cos^{-1}x > \sin^{-1}x$ and both are less than 1

So greatest is t_3 and least is t_2 and $t_4 > t_1$

Hence $t_3 > t_4 > t_1 > t_2$

(C) For $\frac{1}{\sqrt{2}} < x < \sin 1$

We have $1 > \sin^{-1}x > \cos^{-1}x$

So the greatest is t_2 and the least is t_3, $t_2 > t_4$

Hence, $t_2 > t_1 > t_4 > t_3$

(D) For $\sin 1 < x < 1$ we have $\sin^{-1}x > 1 > \cos^{-1}x$

So the greatest is t_1 and the least is t_3, $t_2 > t_4$

So $t_1 > t_2 > t_4 > t_3$

53. A-s, B-p, C-p, D-q

(A) Use $\cos C = \frac{a^2+b^2-c^2}{2ab}$

(B) $(y+z)\cos 3\theta - (xyz)\sin 3\theta = 0$ ... (1)

$(xyz)\sin 3\theta = 2z\cos 3\theta + 2y\sin 3\theta$ ... (2)

$(xyz)\sin 3\theta = (y+2z)\cos 3\theta + y\sin 3\theta$ ... (3)

From (1) and (2),

$(y+z)\cos 3\theta = 2z\cos 3\theta + 2y\sin 3\theta$

$\Rightarrow \quad y(\cos 3\theta - 2\sin 3\theta) = z\cos 3\theta$

From (2) and (3)

$2z\cos 3\theta + 2y\sin 3\theta = (y+2z)\cos 3\theta + y\sin 3\theta$

$y\sin 3\theta = y\cos 3\theta \quad \Rightarrow \quad \sin 3\theta = \cos 3\theta$

$\Rightarrow \quad \tan 3\theta = 1 \Rightarrow 3\theta = n\pi + \frac{\pi}{4}, \quad n \in z$

$\Rightarrow \quad \theta = \frac{n\pi}{3} + \frac{\pi}{12} \Rightarrow \theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{9\pi}{12}$ (three values of θ)

(C) $\tan\theta = \cot 5\theta$

$\Rightarrow \quad \frac{\sin\theta}{\cos\theta} - \frac{\cos 5\theta}{\sin 5\theta} = 0 \quad \Rightarrow \quad \cos 6\theta = 0$

$\Rightarrow \quad 4\cos^3 2\theta - 3\cos 2\theta = 0$

$\Rightarrow \quad \cos 2\theta = 0 \quad$ or $\quad \cos 2\theta = \pm\frac{\sqrt{3}}{2}$

$\sin 2\theta = \cos 4\theta \quad \Rightarrow \quad 2\sin^2 2\theta + \sin 2\theta - 1 = 0$

(using $\cos 2n\theta = 1 - 2\sin^2 n\theta$

$\therefore \quad \cos 4\theta = 1 - 2\sin^2 2\theta$)

$\sin 2\theta = -1$ or $1/2$

$\Rightarrow \quad \cos 2\theta = 0$ and $\sin 2\theta = -1$

$2\theta = -\frac{\pi}{2} \quad \Rightarrow \quad \theta = -\frac{\pi}{4}$

and $\cos 2\theta = \pm\frac{\sqrt{3}}{2}$, $\sin 2\theta = \frac{1}{2}$

$\Rightarrow 2\theta = \frac{\pi}{6}, \frac{5\pi}{6} \quad \Rightarrow \quad \theta = \frac{\pi}{12}, \frac{5\pi}{12}$

$\therefore \quad \theta = -\frac{\pi}{4}, \frac{\pi}{12}, \frac{5\pi}{12}$ (Three values of θ)

(D) $\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0 \Rightarrow z^3 = 0 \Rightarrow z = 0$

54. A-r, B-p, C-p, D-q

(A) $\frac{a\bar{b}+\bar{a}b}{|ab|} = \frac{2\mathrm{Re}(a\bar{b})}{|ab|} = \frac{2\mathrm{Re}(a\bar{b})}{|a\bar{b}|} \le 2 \quad (\because \mathrm{Re}(a\bar{b}) \le |a\bar{b}|)$

(B) $t_n = \tan^{-1}\left(\frac{2^{n-1}}{1+2^{2n-1}}\right) = \tan^{-1}\left(\frac{2^n - 2^{n-1}}{1+2^n\cdot 2^{n-1}}\right)$

$= \tan^{-1}2^n - \tan^{-1}2^{n-1}$

$\Rightarrow \quad S_n = \tan^{-1}2 - \tan^{-1}1 + \tan^{-1}4 - \tan^{-1}2 + ...$

$....+ \tan^{-1}2^n - \tan^{-1}2^{n-1}$

$= -\tan^{-1}1 + \tan^{-1}2^n$

$\lim_{n\to\infty} S_n = -\frac{\pi}{4} + \frac{\pi}{2} = \frac{\pi}{4}$

(C) We know that $1 + \omega + \omega^2 + ... + \omega^{2008} + \omega^{2009} = 0$

$\Rightarrow \quad 1 + \omega + \omega^2 + ... + \omega^{2008} = -\omega^{2009} = \frac{-1}{\omega}$

$\Rightarrow \quad \log_2\left(1 + \omega + \omega^2 + ... + \omega^{2008} - \frac{1}{\omega}\right) = \log_2\left|\frac{-1}{\omega} + \frac{-1}{\omega}\right|$

$= \log_2\left|\frac{-2}{\omega}\right| = \log_2 2 - \log_2|\omega| = 1$

(D) If $\frac{\pi}{4} < A < \frac{\pi}{2}$

$0 < \cot A < 1, 0 < \cot^3 A < 1 \quad \Rightarrow \quad \cot A \cot^3 A < 1$

$\tan^{-1}(\cot A) + \tan^{-1}(\cot^3 A) = \tan^{-1}\left(\frac{\cot A + \cot^3 A}{1 - \cot^4 A}\right)$

$$= \tan^{-1}\left(\frac{\cot A\,(1+\cot^2 A)}{1-\cot^4 A}\right) = \tan^{-1}\left(\cot A \cdot \frac{\tan^2 A}{\tan^2 A - 1}\right)$$

$$= \tan^{-1}\left(\frac{-1}{2}\cdot\frac{2\tan A}{1-\tan^2 A}\right) = -\tan^{-1}\left(\frac{1}{2}\tan 2A\right)$$

55. A-r, B-q, C-s, D-p

$M = -1, 1/2, 2$

(A) $\sum \frac{-1}{M_i^2} = \frac{-21}{4}$

(B) $\sum \frac{1}{M_i} = \frac{3}{2}$

(C) $\sum \left|\frac{-1/M_i}{\sqrt{1+M_i^2}}\right| = \frac{5\sqrt{2}+9\sqrt{5}}{10}$

(D) $\sum \sqrt{\left(\frac{1}{M_i^2}\right)^2 + \left(\frac{1}{M_i}\right)^2} = \frac{4\sqrt{2}+9\sqrt{5}}{10}$

56. A-(s, t), B-(p, q, r, s), C-(p, q, r), D-(q, r, s, t)

The other vertices of the triangle are

$(5, 2\sqrt{5})$ and $(5, -2\sqrt{5})$

Therefore, the centroid is $\left(\frac{10}{3}, 0\right)$; the circumcentre is

$\left(\frac{9}{2}, 0\right)$ and the incentre is (3, 0)

57. A-r, B-q, C-s, D-q

(A) $sp + s'p = 2a$ which represents an ellipse

(B) $\alpha = 3h - 2, \beta = 3k \Rightarrow \frac{\alpha+2}{3} = h, \frac{\beta}{3} = k$ which represents a circle.

(C) $|sp - s'p| = 2a$ which represents a hyperbola.

(D) Locus is a circle with the given diagonal as diameter which represents a circle.

58. (2): $2\tan^{-1}\left(\frac{b}{a}\right) = \frac{\pi}{3} \Rightarrow \frac{b}{a} = \frac{1}{\sqrt{3}}$

$e^2 = 1 + \frac{1}{3} = \frac{4}{3}$

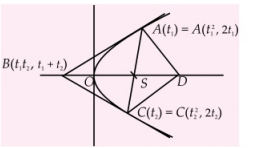
$\frac{1}{e'^2} + \frac{1}{e^2} = 1 \Rightarrow \frac{1}{e'^2} + \frac{3}{4} = 1 \Rightarrow \frac{1}{e'^2} = \frac{1}{4} \Rightarrow e' = 2$

59. (0): $a \in \left(0, \frac{1}{3}\right)$

60. (8): If t_1 & t_2 are parameters of end points of focal chord of parabola $y^2 = 4ax$ then $t_1 t_2 = -1$

Clearly $ABCD$ is a rectangle

Co-ordinate of $B(t_1t_2, (t_1 + t_2))$



$AB = |(t_2 - t_1)|\sqrt{1+t_1^2}$

$BC = (t_2 - t_1)\sqrt{1+t_2^2}$

$\text{Area} = AB \times BC = (t_2 - t_1)^2\sqrt{(1+t_1^2)(1+t_2^2)}$

$= (t_2 - t_1)^2\sqrt{2 + t_1^2 + t_2^2} = \left(t_1 + \frac{1}{t_1}\right)^2\sqrt{2 + t_1^2 + \frac{1}{t_1^2}}$

$= \left(t_1 + \frac{1}{t_1}\right)^3 \geq 2^3 = 8 \left(\text{as } t_1 + \frac{1}{t_1} \geq 2\right)$

$\therefore$ Least value of $\left(t_1 + \frac{1}{t_1}\right)^3 = 8$

61. (1): The given curve represents the point $\left(-\frac{1}{2}, 2\right)$

$\therefore$ Minimum distance = 1.

62. (1): The point E dividing $\overline{AB}$ externally in the ratio 2 : 1 is (9, 0). Since P lies on the circle described on $\overline{DE}$ as diameter, co-ordinates of P are of the form $(5 + 4\cos\theta, 4\sin\theta)$.

$\therefore$ Maximum length of the altitude drawn from point P to the base $AB = |4\sin\theta|_{\max} = 4$.

63. (0): Combined equation of the tangents drawn from (0, 0) to the circle is $(x^2 + y^2 - 2rx - 2hy + h^2)h^2 = (-rx - hy + h^2)^2$ here coefficient of (using $SS' = T^2$)

x^2 + coefficient of $y^2 = 0$

$\Rightarrow (h^2 - r^2) + (h^2 - r^2) = 0 \Rightarrow \frac{h}{r} = \pm 1$

$\Rightarrow \frac{h}{r} = -1$ or $\frac{h}{r} = 1 \quad \therefore S = \frac{h}{r} + \frac{h}{r} = -1 + 1 = 0$

64. (8): According to the given condition,

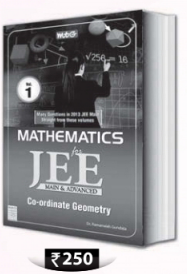
$200 < \frac{9}{2}(2(12 - d) + 8d) < 220$ or $\frac{184}{54} < d < \frac{224}{54}$

or $d > 3$ & $d < 5$

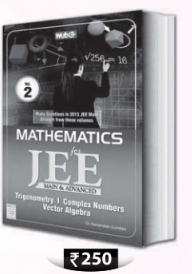
$\Rightarrow d = 4$

$\therefore$ First term $= 12 - d = 8$

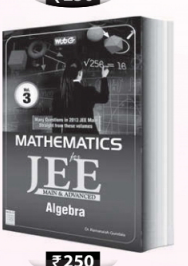
MATHEMATICS FOR JEE MAIN & ADVANCED



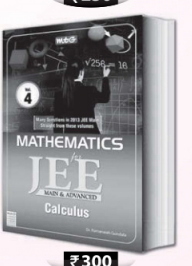
₹250



₹250



₹250



₹300

- Exclusive Theory with solved examples
- 4000+ MCQs

Buy online at www.mtg.in

MTG BOOKS

For more information or to place your order:
Call at : 0124-4951200 @ Mail at : info@mtg.in
Log on to: www.mtg.in

Mock Test PAPER 6

West Bengal JEE 2014

The entire syllabus of Mathematics of WB-JEE is being divided into six modules, in each module there will be a Mock Test Paper (MTP) and two MTP in the whole syllabus which will be published in subsequent issues.

The syllabus for module break-up is given below :

Module I : Algebra - I, **Module - II :** Trigonometry, **Module - III :** Co-ordinate geometry of two dimensions, **Module - IV :** Differential calculus and its applications, **Module - V :** Algebra - II, **Module - VI :** Integral calculus and its applications, Differential equations.

CONTENTS :

Integral calculus : Integration as a reverse process of differentiation, indefinite integral of standard functions, Integration by parts, Integration by substitution and partial fraction.

Definite integral as a limit of a sum with equal subdivisions. Fundamental theorem of integral calculus and its applications. Properties of definite integrals.

Differential Equations : Formulation and solution of differential equations of the forms.

1) $dy/dx = f(x) \times g(y)$
2) $dy/dx = f(y/x)$
3) $dy/dx = (ax + by)/(cx + dy)$
4) $dy/dx = (a_1x + b_1y + c_1)/(a_2x + b_2y + c_2)$ / $(a_1/a_2 = b_1/b_2)$
5) $dy/dx + p(x)\ y = Q(x)$
6) $d^2y / dx^2 + p_1 dy/dx + p_2 y = 0$ with p_1 and p_2 constants.
7) $d^2y/dx^2 = f(x)$

Application of Calculus : Tangents and normals, conditions of tangency, Determination of monotonicity, maxima and minima, Differential coefficient as a measure of rate, Motion in a straight line with constant acceleration, Geometric interpretation of definite integral as area, calculation of area bounded by elementary curves and straight lines, Area of the region included between two elementary curves.

This paper is useful for JEE Main & Advanced and other Engineering Entrance Exams.

MODULE - VI : INTEGRAL CALCULUS & ITS APPLICATIONS, DIFFERENTIAL EQUATIONS

Time : 40 mins **Marks : 30**

SECTION-1

This section contains 18 questions. Q. 1 to Q. 18 carry one mark each, for which only one option is correct. Any wrong answer will lead to deduction of 1/3 marks.

1. $\int \tan^4 x\,dx = \alpha x + \beta \tan x + \gamma \tan^3 x + C$, then

$\alpha + \beta + \gamma =$

(a) 1 (b) 1/2 (c) 1/3 (d) –1

2. $\int \frac{(x^m)^x \log_e(ex)}{1+(x^{2m})^x} = \left(1 - \frac{A}{B}\right)\tan^{-1}(x^m)^x + C.$

Then which is correct?

(a) $A = m - 1$ (b) $B = m$

(c) $B - A = 1$ (d) All of these

3. If $\int \frac{dx}{\sqrt{1-\tan^2 x}} = \frac{1}{k}\sin^{-1}(k \sin x) + C$, then

$k^2(1 + k^2)(1 + k^4)(1 + k^2 + k^{12}) =$

(a) 2010 (b) 2012 (c) 2013 (d) 2014

4. $\int \frac{dx}{x\sqrt{x^2 + 2x - 1}} =$

(a) $\cos^{-1}\left(\frac{1-x}{\sqrt{2}x}\right) + C$ (b) $\cos^{-1}\left(\frac{1+x}{\sqrt{2}x}\right) + C$

(c) $\sin^{-1}\left(\frac{1-x}{\sqrt{2}x}\right) + C$ (d) $\sin^{-1}\left(\frac{1+x}{\sqrt{2}x}\right) + C$

5. $\int e^x \left(\log x + \frac{1}{x^2}\right) dx = f(x)g(x) + C$, then

(a) $f(x) = e^x, g(x) = \log x$

(b) $f(x) = \log x + \frac{1}{x},\ g(x) = e^x$

(c) $f(x) = \log x - \frac{1}{x},\ g(x) = e^x$

(d) none of these

By : Sankar Ghosh, HOD (Math), Takshyashila. Mob : 09831244397.

6. $\int \frac{dx}{\cos 2x + 3\sin^2 x} = \frac{1}{a}\tan^{-1}(b\tan x) + C$, then ab =

(a) 0 (b) 1 (c) 2 (d) 3

7. $\int_0^{\pi/4} \{\tan^n(x-[x]) + \tan^{n-2}(x-[x])\}dx =$

(a) $\frac{1}{n-1}$ (b) $\frac{1}{n+1}$ (c) $\frac{1}{n-2}$ (d) $\frac{1}{n+2}$

8. The value of $\int_0^{100\pi} |\sin x|\, dx =$

(a) 100 (b) 200 (c) 300 (d) 400

9. $\lim_{n\to\infty} \frac{1}{n}\sum_{r=1}^{n}\left(\frac{r}{n+r}\right) =$

(a) log 2 (b) 1 + log 2
(c) 1 – log 2 (d) 0

10. Let $f(x) = \max\{2-x, 2, 1+x\}$, then $\int_{-1}^{1} f(x)\, dx =$

(a) 0 (b) 2 (c) 9/2 (d) 3

11. If $A = \int_1^{\sin\theta} \frac{t\,dt}{1+t^2}$ and $B = \int_1^{\operatorname{cosec}\theta} \frac{dt}{t(1+t^2)}$, then the value of $\begin{vmatrix} A & A^2 & B \\ e^A \cdot e^B & B^2 & -1 \\ 1 & A^2+B^2 & -1 \end{vmatrix}$ is

(a) sinθ (b) cosecθ
(c) 0 (d) 1

12. Let $\frac{d}{dx}(f(x)) = \frac{e^{\sin x}}{x}$, $x > 0$.

If $\int_1^4 \frac{2e^{\sin x^2}}{x}dx = f(k) - f(1)$, then one of the possible value of k is

(a) –4 (b) 0 (c) 2 (d) 16

13. The area of the plane region bounded by the curves $x + 2y^2 = 0$ and $x + 3y^2 = 1$ is equal to

(a) 4/3 (b) 5/3 (c) 1/3 (d) 2/3

14. The area bounded by the curve $y = \frac{1}{2}(2 - 3x - 2x^2)$ below the line $y = x + 1$ and the x-axis in square units is

(a) $\frac{37}{14}$ (b) $\frac{17}{48}$ (c) $\frac{37}{48}$ (d) $\frac{29}{48}$

15. The integrating factor of the differential equation $\frac{dy}{dx} + \frac{y}{(1-x)\sqrt{x}} = 1 - \sqrt{x}$ is

(a) $\frac{1-\sqrt{x}}{1+\sqrt{x}}$ (b) $\frac{1+x}{1-x}$

(c) $\frac{1+\sqrt{x}}{1-\sqrt{x}}$ (d) $\frac{\sqrt{x}}{1-\sqrt{x}}$

16. The solution of the differential equation $xdy - ydx = \sqrt{x^2+y^2}\, dx$ is

(a) $y - \sqrt{x^2+y^2} = Cx$ (b) $y + \sqrt{x^2+y^2} = Cx^2$

(c) $x + \sqrt{x^2+y^2} = Cy^2$ (d) $x - \sqrt{x^2+y^2} = Cy^2$

17. The solution of the differential equation $\frac{dy}{dx} + \frac{y}{x} = \sin x$ is

(a) $x(y - \cos x) = \sin x + C$
(b) $xy\cos x = \sin x + C$
(c) $x(y + \cos x) = \sin x + C$
(d) $x(y + \cos x) = \cos x + C$

18. If $\int_0^x t^2 f(t)dt = x + \int_x^1 f(t)dt$, then the value of $f(1/2)$ =

(a) 1/5 (b) 4/5 (c) 3/5 (d) 2/5

SECTION-II

This section contains 9 questions numbered 19 to 27. Each question has four choices of which one is correct. Each question carries +2 marks for correct answer and –2/3 for wrong answer.

19. If $\int \frac{dx}{5\cos^2 x + 4\sin x(\sin x + \cos x)} = \alpha^{-1}\tan^{-1}\left(\frac{1}{\beta} + \gamma\tan x\right) + k$, then α, β, γ are in

(a) A.P. (b) G.P.
(c) H.P. (d) none of these

20. $\int \frac{dx}{(2x-7)\sqrt{x^2-7x+12}} =$

(a) $2\sec^{-1}(2x-7) + C$ (b) $\sec^{-1}(2x-7) + C$

(c) $\frac{1}{2}\sec^{-1}(2x-7) + C$ (d) none of these

21. If $\int e^{ax}\sin bx = \frac{e^{3x}}{13}f(x) + C$, then $f''(x) : f(x)$ is

(a) always positive (b) always negative
(c) always constant (d) none of these

22. If $\int \frac{4e^x + 6e^{-x}}{9e^x - 4e^{-x}}\,dx = Ax + B\log(9e^{2x} - 4) + C$, then $A + B =$

(a) $-\frac{9}{35}$ (b) $-\frac{19}{36}$ (c) $\frac{19}{36}$ (d) $\frac{9}{35}$

23. Suppose $S = \frac{1}{n^4}\prod_{r=1}^{2n}(n^2 + r^2)^{1/n}$, then $\lim_{n\to\infty} \log S =$

(a) $\int_0^2 \log(2 + x^2)dx$ (b) $\int_0^1 \log(1 + x^2)dx$

(c) $2\int_0^1 \log(1 + x^2)dx$ (d) $\int_0^2 \log(x^2 - 4x + 5)dx$

24. The order and degree, respectively of the differential equation whose solution is $ax^2 + by^2 = 1$, a and b being arbitrary constants are

(a) 1, 1 (b) 2, 1 (c) 2, 2 (d) 1, 2

25. If $(x^2 + y^2)\,dy = xy\,dx$, $y(1) = 1$ and $y(x_0) = e$, then $x_0 =$

(a) $\sqrt{2(e^2 - 1)}$ (b) $\sqrt{2(e^2 + 1)}$

(c) $\sqrt{3}e$ (d) $\sqrt{\frac{e^2 + 1}{2}}$

26. The area of the region bounded by the curves $y = x^2$, $y = |2 - x^2|$ and $y = 2$ which lies to the right side of the line $x = 1$ is

(a) $\frac{4}{5}(5 + 3\sqrt{2})$ (b) $\frac{3}{5}(5 + 3\sqrt{2})$

(c) $\frac{4}{3}(5 - 3\sqrt{2})$ (d) $\frac{3}{4}(5 - 3\sqrt{2})$

27. The area common to the curves $x^2 + y^2 = 2$ and $x^2 + 4y^2 = 4$ is

(a) $\tan^{-1} 2\sqrt{2}$ (b) $2\tan^{-1} 2\sqrt{2}$

(c) $4\tan^{-1}\sqrt{2}$ (d) $4\tan^{-1} 2\sqrt{2}$

SECTION-III

This section contains 3 multiple choice questions numbered 28 to 30. Each question has four choices, of which more than one option may be correct and each question carries +2 marks for correct answer and –2/3 for wrong answer. For partial correct response without any incorrect marking :

$$\text{Marks awarded} = 2 \times \frac{\text{No. of correct response}}{\text{total no. of correct options}}$$

28. Let $y = f(x)$ be a curve in the first quadrant such that the triangle formed by the co-ordinate axes and the tangent at any point on the curve has area 2. If $y(1) = 1$, then $y(2) =$

(a) 0 (b) 1 (c) 2 (d) 1/2

29. For which of the following values of m, is the area bounded by the curves $y = x - x^2$ and $y = mx$ equals 9/2 ?

(a) –4 (b) –2 (c) 2 (d) 4

30. The area of the region bounded by the curve $y = e^x$ and lines $x = 0$ and $y = e$ is

(a) $e - 1$ (b) $\int_1^e \ln(e + 1 - y)\,dy$

(c) $e - \int_0^1 e^x\,dx$ (d) $\int_1^x \ln y\,dy$

SOLUTIONS

1. (c): $\int \tan^4 x\,dx = \int \tan^2 x(\sec^2 x - 1)dx$

$= \int \tan^2 x \sec^2 x\,dx - \int \tan^2 x\,dx$

$= \int \tan^2 x\,d(\tan x) - \int(\sec^2 x - 1)dx$

$= \frac{1}{3}\tan^3 x - (\tan x - x) + C = \frac{1}{3}\tan^3 x - \tan x + x + C$

$\therefore\ \alpha = 1, \beta = -1, \gamma = 1/3.$

$\therefore\ \alpha + \beta + \gamma = \frac{1}{3}$

2. (d): $\int \frac{(x^m)^x \log_e(ex)}{1 + (x^{2m})^x}dx = \int \frac{x^{mx}(\log x + 1)}{1 + (x^{mx})^2}dx$

[Put $x^{mx} = z \Rightarrow mx \log x = \log z$

$\Rightarrow m\left[x \cdot \frac{1}{x} + \log x\right]dx = \frac{1}{z}dz$

$\Rightarrow zm(1 + \log x)dx = dz$

$\Rightarrow x^{mx}(1 + \log x)dx = \frac{dz}{m}$]

$= \frac{1}{m}\int \frac{dz}{1 + z^2}$

$= \left(1 - \frac{m-1}{m}\right)\tan^{-1} z + C$

$= \left(1 - \frac{m-1}{m}\right)\tan^{-1}(x^{mx}) + C$

$= \left(1 - \frac{m-1}{m}\right)\tan^{-1}(x^m)^x + C$

On comparing, $A = m - 1$, $B = m$ and $B - A = 1$.

3. (a): $\int \frac{dx}{\sqrt{1 - \tan^2 x}} = \int \frac{\cos x}{\sqrt{\cos^2 x - \sin^2 x}}dx$

$= \int \frac{\cos x\,dx}{\sqrt{1 - 2\sin^2 x}} = \int \frac{d(\sin x)}{\sqrt{1 - (\sqrt{2}\sin x)^2}}$

$= \frac{1}{\sqrt{2}}\sin^{-1}(\sqrt{2}\sin x) + C$

$= \frac{1}{k}\sin^{-1}(k\sin x) + C$

$\therefore \quad k = \sqrt{2}$

Now, $k^2(1+k^2)(1+k^4)(1+k^2+k^{12}) = 2010$

4. (a): $\int \frac{dx}{x\sqrt{x^2+2x-1}} \quad \left[\text{Put } x = \frac{1}{t} \Rightarrow dx = -\frac{1}{t^2}dt\right]$

$= \int \frac{-\frac{1}{t^2}dt}{\frac{1}{t}\sqrt{\frac{1}{t^2}+\frac{2}{t}-1}} = -\int \frac{dt}{\sqrt{1+2t-t^2}} = -\int \frac{dt}{\sqrt{2-(t-1)^2}}$

$= \cos^{-1}\left(\frac{t-1}{\sqrt{2}}\right) + C = \cos^{-1}\left(\frac{1-x}{\sqrt{2}x}\right) + C$

5. (c): $\int e^x \left(\log x + \frac{1}{x^2}\right)dx = \int e^x \log x\, dx + \int e^x \cdot \frac{1}{x^2}dx$

$= e^x \log x - \int \frac{1}{x}e^x dx + \int e^x \cdot \frac{1}{x^2}dx$

$= e^x \log x - \left\{\frac{1}{x}e^x - \int -\frac{1}{x^2}\cdot e^x dx\right\} + \int e^x \cdot \frac{1}{x^2}dx$

$= e^x \log x - \frac{1}{x}e^x - \int \frac{1}{x^2}e^x dx + \int \frac{1}{x^2}e^x dx + C$

$= e^x \log x - \frac{1}{x}e^x + C = \left(\log x - \frac{1}{x}\right)e^x + C$

$\therefore \quad f(x) = \log x - \frac{1}{x}$ and $g(x) = e^x$

6. (c): $\int \frac{dx}{\cos 2x + 3\sin^2 x} = \int \frac{dx}{\cos^2 x - \sin^2 x + 3\sin^2 x}$

$= \int \frac{dx}{\cos^2 x + 2\sin^2 x} = \int \frac{\sec^2 x\, dx}{1 + 2\tan^2 x}$

$= \int \frac{\sec^2 x\, dx}{1 + (\sqrt{2}\tan x)^2} \quad \left[\text{Put } \sqrt{2}\tan x = t \Rightarrow \sqrt{2}\sec^2 x\, dx = dt\right]$

$= \frac{1}{\sqrt{2}}\int \frac{dt}{1+t^2} = \frac{1}{\sqrt{2}}\tan^{-1} t + C = \frac{1}{\sqrt{2}}\tan^{-1}(\sqrt{2}\tan x) + C$

On comparing, $a = \sqrt{2}, b = \sqrt{2}$

$\therefore \quad ab = 2$

7. (a): $\int_0^{\pi/4} \{\tan^n(x-[x]) + \tan^{n-2}(x-[x])\}dx$

$= \int_0^{\pi/4} (\tan^n x + \tan^{n-2} x)dx \quad \left[\because \text{In } 0 < x < \frac{\pi}{4},\ [x] = 0\right]$

$= \int_0^{\pi/4} \tan^{n-2}x(1+\tan^2 x)dx = \int_0^{\pi/4} \tan^{n-2} x\sec^2 x\, dx$

$= \int_0^{\pi/4} \tan^{n-2} x\, d(\tan x) = \frac{\tan^{n-1} x}{n-1}\Big|_0^{\pi/4} = \frac{1}{n-1}$

8. (b): $\int_0^{100\pi} |\sin x|\, dx = 100\int_0^{\pi} |\sin x|\, dx$

$[\because f(x) = |\sin x| \text{ is a periodic function of peroid } \pi]$

$= 100\int_0^{\pi} \sin x\, dx \qquad [\because \sin x \geq 0\ \forall\ x \in [0, \pi]$

$= 100[-\cos x]_0^{\pi} = 100[-(-1-1)] = 200$

9. (c): $\lim_{n\to\infty} \frac{1}{n}\sum_{r=1}^{n}\left(\frac{r}{n+r}\right) = \lim_{n\to\infty} \frac{1}{n}\sum_{r=1}^{n}\left(\frac{r/n}{1+\frac{r}{n}}\right)$

$= \int_0^1 \frac{x}{1+x}dx \quad \left[\because \int_0^1 f(x)dx = \lim_{n\to\infty} n\frac{1}{r=1}\sum^{r} f\left(\frac{r}{n}\right)\right]$

$= \int_0^1 \left(1 - \frac{1}{1+x}\right)dx = [x - \log(1+x)]_0^1 = 1 - \log 2$

10. (c): In (–1, 0), $2 - x$ is maximum among $2 - x$, 2, $1 + x$.

In (0, 1), 2 is maximum.

$\therefore \quad \int_{-1}^{1} f(x)dx = \int_{-1}^{0} (2-x)dx + \int_0^1 2\, dx = \frac{9}{2}$

11. (c): $A = \int_1^{\sin\theta} \frac{tdt}{1+t^2}$ and $B = \int_1^{\text{cosec}\theta} \frac{dt}{t(1+t^2)}$

Put $t = \frac{1}{z} \Rightarrow dt = -\frac{1}{z^2}dz$

$\therefore \quad B = \int_1^{\sin\theta} \frac{-\frac{1}{z^2}dz}{\frac{1}{z}\left(1+\frac{1}{z^2}\right)} = -\int_1^{\sin\theta} \frac{zdz}{z^2+1} = -\int_1^{\sin\theta} \frac{tdt}{1+t^2}$

$\left[\because \int_a^b f(x)dx = \int_a^b f(t)dt\right]$

$\Rightarrow \quad B = -A$

$\therefore \quad \begin{vmatrix} A & A^2 & B \\ e^A\cdot e^B & B^2 & -1 \\ 1 & A^2+B^2 & -1 \end{vmatrix} = \begin{vmatrix} A & A^2 & -A \\ 1 & A^2 & -1 \\ 1 & 2A^2 & -1 \end{vmatrix}$

$= A\begin{vmatrix} 1 & A & -1 \\ 1 & A^2 & -1 \\ 1 & 2A^2 & -1 \end{vmatrix} = 0$

12. (d): $\int_1^4 \frac{2e^{\sin x^2}}{x}dx$ (Put $z = x^2 \Rightarrow dz = 2x\,dx$)

x	1	4
z	1	16

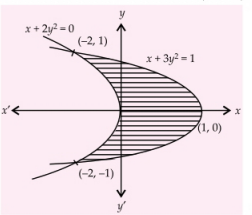
$= \int_1^4 \frac{2xe^{\sin x^2}}{x^2}dx$

$= \int_1^{16} \frac{e^{\sin z}}{z}dz$

$= [f(z)]_1^{16} = f(16) - f(1)$

$\therefore$ $f(k) = f(16)$. $\therefore$ The possible value of k is 16.

13. (a): The given curves are $x + 2y^2 = 0$ and $x + 3y^2 = 1$. Now solving the above two equations, we get $y = \pm 1$ and $x = -2$. Thus the curves intersect at $(-2, \pm 1)$

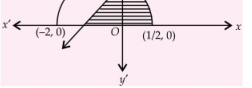


$\therefore$ Required area $= \int_{-1}^{1} [(1 - 3y^2) - (-2y^2)]\,dy = \frac{4}{3}$

14. (c): The given curve is $y = \frac{1}{2}(2 - 3x - 2x^2)$

$\Rightarrow \left(x + \frac{3}{4}\right)^2 = -\left(y - \frac{25}{16}\right)$ ... (i)

and the line $y = x + 1$ ... (ii)



The area enclosed by the curves (i) and (ii) are shown above in the diagram.

$\therefore$ Required area $= \frac{1}{2}\cdot 1\cdot 1 + \int_0^{1/2} \frac{1}{2}(2 - 3x - 2x^2)\,dx$

$= \frac{1}{2} + \frac{1}{2}\left(\frac{13}{24}\right) = \frac{1}{2} + \frac{13}{48} = \frac{37}{48}$ sq. units

15. (c): The given equation is

$\frac{dy}{dx} + \frac{y}{(1-x)\sqrt{x}} = 1 - \sqrt{x}$

Here, I.F. $= e^{\int \frac{dx}{(1-x)\sqrt{x}}}$

$\int \frac{dx}{(1-x)\sqrt{x}}$. Put $x = z^2 \Rightarrow dx = 2z\,dz$

$\Rightarrow \int \frac{2z\,dz}{(1-z^2)z} = 2\cdot\frac{1}{2\cdot 1}\log\left|\frac{1+z}{1-z}\right| = \log\left|\frac{1+\sqrt{x}}{1-\sqrt{x}}\right| + C$

$= e^{\log\left|\frac{1+\sqrt{x}}{1-\sqrt{x}}\right|} = \left|\frac{1+\sqrt{x}}{1-\sqrt{x}}\right|$

16. (b): The given differential equation is

$x\,dy - y\,dx = \sqrt{x^2 + y^2}\,dx$

$\Rightarrow \frac{x\,dy - y\,dx}{x^2} = \frac{\sqrt{x^2 + y^2}}{x^2}dx$

$\Rightarrow d\left(\frac{y}{x}\right) = \frac{1}{x}\cdot\sqrt{1 + \left(\frac{y}{x}\right)^2}\,dx$

$\Rightarrow \int \frac{d(y/x)}{\sqrt{1 + \left(\frac{y}{x}\right)^2}} = \int \frac{dx}{x}$

$\Rightarrow \log\left|\frac{y}{x} + \frac{\sqrt{x^2 + y^2}}{x}\right| = \log x + \log C$

$\Rightarrow \log\left|y + \sqrt{x^2 + y^2}\right| - \log x = \log x + \log C$

$\Rightarrow \log\left|y + \sqrt{x^2 + y^2}\right| = 2\log x + \log C$

$\Rightarrow \log\left|y + \sqrt{x^2 + y^2}\right| = \log(x^2 C)$

$\Rightarrow y + \sqrt{x^2 + y^2} = x^2 C$

17. (c): The given differential equation is,

$\frac{dy}{dx} + \frac{y}{x} = \sin x$ ... (i)

Here I.F. $= e^{\int \frac{1}{x}dx} = e^{\log x} = x$

Multiplying (i) both sides by I.F., we get

$x\left[\frac{dy}{dx} + \frac{y}{x}\right] = x\sin x \Rightarrow \frac{d}{dx}(xy) = x\sin x$

$\Rightarrow \int d(xy) = \int x\sin x\,dx$

$\Rightarrow xy = -x\cos x + \sin x + C$

$\Rightarrow x(y + \cos x) = \sin x + C$

18. (b): Given $\int_0^x t^2 f(t)dt = x + \int_x^1 f(t)dt$

Differentiating both the sides with respect to x, we get

$$x^2 f(x) = 1 - f(x)$$

Putting $x = \frac{1}{2}$, we get $\frac{1}{4}f\left(\frac{1}{2}\right) = 1 - f\left(\frac{1}{2}\right)$

$$\Rightarrow \quad f\left(\frac{1}{2}\right) = \frac{1}{1+\frac{1}{4}} = \frac{4}{5}$$

19. (b): $\int \frac{dx}{5\cos^2 x + 4\sin x(\sin x + \cos x)}$

$$= \int \frac{\sec^2 x\, dx}{5 + 4\tan^2 x + 4\tan x} \qquad (\text{Put } z = \tan x)$$

$$= \int \frac{dz}{4z^2 + 4z + 5}$$

$$= \frac{1}{4}\int \frac{dz}{z^2 + z + \frac{5}{4}} = \frac{1}{4}\int \frac{dz}{(z^2) + 2\cdot z\cdot\frac{1}{2} + \left(\frac{1}{2}\right)^2 + \frac{5}{4} - \frac{1}{4}}$$

$$= \frac{1}{4}\int \frac{dz}{\left(z + \frac{1}{2}\right)^2 + 1} = \frac{1}{4}\tan^{-1}\left(\frac{1}{2} + z\right) + C$$

$$= \frac{1}{4}\tan^{-1}\left(\frac{1}{2} + \tan x\right) + C$$

On comparing, we get,

$\therefore \quad \alpha = 4, \beta = 2, \gamma = 1$, thus α, β, γ are in G.P.

20. (b): $\int \frac{dx}{(2x-7)\sqrt{x^2 - 7x + 12}} \quad \left[\text{Put } 2x - 7 = \frac{1}{z}\right]$

$$= -\frac{1}{2}\int \frac{\frac{1}{z^2}dz}{\frac{1}{z}\sqrt{\frac{1}{4}\left(\frac{1}{z} + 7\right)^2 - \frac{7}{2}\left(\frac{1}{z} + 7\right) + 12}}$$

$$= -\frac{1}{2}\int \frac{\frac{1}{z}dz}{\sqrt{\frac{1}{4}\left(\frac{1+7z}{z}\right)^2 - 7\left(\frac{1+7z}{2z}\right) + 12}}$$

$$= -\frac{1}{2}\times 2\int \frac{dz}{\sqrt{(7z+1)^2 - 14z(1+7z) + 48z^2}}$$

$$= -\int \frac{dz}{\sqrt{1 - z^2}} = -\sin^{-1}z + C = -\sin^{-1}\left(\frac{1}{2x-7}\right) + C$$

$$= -\operatorname{cosec}^{-1}(2x-7) + C = \sec^{-1}(2x-7) + C$$

Alternative approach

$$\int \frac{dx}{(2x-7)\sqrt{x^2 - 7x + 12}}$$

$$= \int \frac{dx}{(2x-7)\sqrt{\frac{1}{4}[(2x-7)^2 - 1]}}$$

$$= 2\int \frac{dx}{(2x-7)\sqrt{(2x-7)^2 - 1}}$$

$$= 2\cdot\frac{1}{2}\sec^{-1}(2x-7) + C$$

$$= \sec^{-1}(2x-7) + C \qquad \left[\because \int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1}x\right]$$

$$\begin{vmatrix} x^2 - 7x + 12 \\ \frac{1}{4}(4x^2 - 28x + 48) \\ = \frac{1}{4}[(2x)^2 - 2\cdot 2x\cdot 7 \\ \qquad + 7^2 + 48 - 49] \\ = \frac{1}{4}[(2x-7)^2 - 1^2] \end{vmatrix}$$

21. (c): $\int e^{ax}\sin bx\, dx = \frac{e^{3x}}{13}f(x) + C$

But $\int e^{ax}\sin bx\, dx = \frac{e^{ax}}{a^2 + b^2}(a\sin bx - b\cos bx)$

On comparing

$\therefore \quad a = 3, b = 2$ and $f(x) = a\sin bx - b\cos bx$

i.e. $f(x) = 3\sin 2x - 2\cos 2x$

$\therefore \quad f'(x) = 6\cos 2x + 4\sin 2x$

$f''(x) = -12\sin 2x + 8\cos 2x$

$= -4(3\sin 2x - 2\cos 2x) = -4f(x)$

$\therefore \quad f''(x) : f(x) = -4$ which is a constant.

22. (b): $\int \frac{4e^x + 6e^{-x}}{9e^x - 4e^{-x}}dx \quad [\text{Put } t = e^x]$

$$= \int \frac{4t + \frac{6}{t}}{9t - \frac{4}{t}}\cdot\frac{dt}{t} = \int \frac{4t^2 + 6}{t(9t^2 - 4)}dt = \int \frac{4t^2 + 6}{t(3t+2)(3t-2)}dt$$

Let $\frac{4t^2 + 6}{t(3t+2)(3t-2)} = \frac{A}{t} + \frac{B}{3t+2} + \frac{C}{3t-2}$

$\Rightarrow \quad 4t^2 + 6 = A(3t+2)(3t-2) + Bt(3t-2) + Ct(3t+2)$

Putting $t = 0$

$6 = -4A \quad \Rightarrow \quad A = -3/2$

Putting $t = -2/3$,

$$4\cdot\frac{4}{9} + 6 = B\left(-\frac{2}{3}\right)(-4)$$

$$\therefore \quad \frac{70}{9} = \frac{8}{3}B \Rightarrow B = \frac{35}{12}$$

Putting $t = 2/3$, $\frac{70}{9} = C\frac{2}{3}\left(3\cdot\frac{2}{3} + 2\right)$.

$$\therefore \quad C = \frac{35}{12}$$

$$\therefore \int \frac{4e^x+6e^{-x}}{9e^x-4e^{-x}}dx = -\frac{3}{2}\int\frac{dt}{t}+\frac{35}{12}\int\frac{dt}{3t+2}+\frac{35}{12}\int\frac{dt}{3t-2}$$

$$= -\frac{3}{2}\log t - \frac{35}{36}\log(3t+2)+\frac{35}{36}\log(3t-2)+C$$

$$= -\frac{3}{2}x+\frac{35}{36}\log(3e^x+2)+\frac{35}{36}\log(3e^x-2)+C$$

$$= -\frac{3}{2}x+\frac{35}{36}\log(9e^{2x}-4)+C$$

$$\therefore \quad A+B = -\frac{3}{2}+\frac{35}{36} = -\frac{19}{36}$$

23. (d): Here $S = \frac{1}{n^4}\prod_{r=1}^{2n}(n^2+r^2)^{1/n}$

$$\log S = \frac{1}{n}\sum_{r=1}^{2n}\log(n^2+r^2)-4\log n$$

$$= \frac{1}{n}\sum_{r=1}^{2n}\log\left\{n^2\left(1+\left(\frac{r}{n}\right)^2\right)\right\}-4\log n$$

$$= \frac{1}{n}\cdot 4n\log(n)+\frac{1}{n}\sum_{r=1}^{2n}\log\left(1+\left(\frac{r}{n}\right)^2\right)-4\log n$$

$$= \frac{1}{n}\sum_{r=1}^{2n}\log\left(1+\left(\frac{r}{n}\right)^2\right)$$

$$\therefore \lim_{n\to\infty}\log S = \lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{2n}\log\left(1+\left(\frac{r}{n}\right)^2\right)$$

$$= \int_0^2 \log(1+x^2)dx$$

$$\lim_{n\to\infty}\log S = \int_0^2 \log(x^2-4x+5)dx$$

$$\left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$$

24. (b): The given equation is

$$ax^2+by^2 = 1 \qquad \text{... (i)}$$

Differentiating (i) with respect to x, we get

$$2ax+2by\frac{dy}{dx}=0 \qquad \text{... (ii)}$$

Differentiating again, we get

$$2a+2b\left(y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2\right)=0$$

$$i.e.,\ a+b\left(y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2\right)=0 \qquad \text{...(iii)}$$

Eliminating a and b,

$$\begin{vmatrix} x^2 & y^2 & -1 \\ x & y & 0 \\ 1 & y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2 & 0 \end{vmatrix} = 0$$

$$\Rightarrow \quad xy\frac{d^2y}{dx^2}+x\left(\frac{dy}{dx}\right)^2-y=0$$

$\therefore$ Order is 2 and degree is 1.

25. (c): $(x^2+y^2)\,dy = xy\,dx \ \Rightarrow \ \frac{dy}{dx}=\frac{xy}{x^2+y^2}$

Put $y = vx$

$$\therefore \quad v+x\frac{dv}{dx}=\frac{v}{1+v^2} \ \Rightarrow \ x\frac{dv}{dx}=\frac{v}{1+v^2}-v=-\frac{v^3}{1+v^2}$$

$$\Rightarrow \int\frac{1+v^2}{v^3}dv = -\int\frac{dx}{x} \ \Rightarrow \ -\frac{1}{2v^2}+\log v = -\log x+C$$

$$\Rightarrow \quad -\frac{x^2}{2y^2}+\log\frac{y}{x} = -\log x+C$$

Now, $y(1) = 1$.

$$\therefore \quad -\frac{1}{2}=C$$

$$\therefore \quad -\frac{x^2}{2y^2}+\log\frac{y}{x} = -\log x-\frac{1}{2}$$

$$\Rightarrow \quad \log\frac{y}{x}+\log x = \frac{x^2}{2y^2}-\frac{1}{2}$$

Given that $y(x_0) = e$

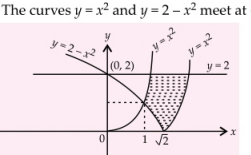
$$\therefore \quad \log\left(\frac{e}{x_0}\right)+\log(x_0) = \frac{x_0^2}{2e^2}-\frac{1}{2}$$

$$\Rightarrow \quad \log e+\frac{1}{2} = \frac{x_0^2}{2e^2} \ \Rightarrow \ 3 = \frac{x_0^2}{e^2} \quad [\because \log e = 1]$$

$$\Rightarrow \quad x_0^2 = 3e^2.$$

$$\therefore \quad x_0 = \sqrt{3}e$$

26. (c): The curves $y = x^2$ and $y = 2 - x^2$ meet at (1, 1)



MtG

JEE ADVANCED EXPLORER

13 years Solved Papers with detailed Solutions

10 Model Test Papers



Send DD/MO in favour of MTG Books.
Available at all leading bookshops throughout the country.

₹ 395 •

Buy online at www.**mtg.in**

Available at leading bookshops throughout India.

MtG BOOKS

For more infromation or to place your order :
Call at : 0124 - 4951200 @ Mail at : info@mtg.in
Log on to : www.mtg.in

Area $= \int_0^2 \sqrt{y+2}\,dy - \int_0^1 \sqrt{2-y}\,dy - \int_0^2 \sqrt{y}\,dy$

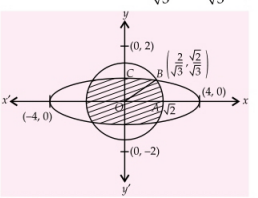
$= \frac{4}{3}(5-3\sqrt{2})$

27. (d): The given curves are

$x^2 + y^2 = 2$ ... (i)

and $x^2 + 4y^2 = 4$ *i.e.* $\frac{x^2}{4} + \frac{y^2}{1} = 1$ ... (ii)

Solving (i) and (ii), we get $x = \frac{2}{\sqrt{3}}$, $y = \frac{\sqrt{2}}{\sqrt{3}}$



The required area is shown in diagram.

$\therefore$ area $OABC = \frac{1}{2}\int_0^{2/\sqrt{3}} \sqrt{4-x^2}\,dx + \int_{2/\sqrt{3}}^{\sqrt{2}} \sqrt{2-x^2}\,dx$

$= \tan^{-1} 2\sqrt{2}$

$\therefore$ Required area $= 4\tan^{-1} 2\sqrt{2}$.

28. (a, d): The tangent at $P(x, y)$ is

$Y - y = y_1(X - x) = 0$

It meets x-axis at $A\left(x - \frac{y}{y_1}, 0\right)$ and y-axis at $B(0, y - xy_1)$

$\therefore$ $\Delta OAB = 2 \Rightarrow OA \cdot OB = 4$

$\Rightarrow \left(x - \frac{y}{p}\right)(y - xp) = 4$, where $p = \frac{dy}{dx} = y_1$

$\Rightarrow (y - xp)^2 = -4p \Rightarrow y - xp = 2\sqrt{-p}$

$\Rightarrow y = xp + 2\sqrt{-p}$

General solution is $y = Cx + 2\sqrt{-C}$ ... (1)

$x = 1, y = 1 \Rightarrow C = -1, y = 2 - x$.

$\therefore$ $y(2) = 0$

Differentiating (1) with respect to C

$0 = x - \frac{1}{\sqrt{-C}}$ ... (2)

Eliminating C from (1) and (2),

$y = -\frac{1}{x} + \frac{2}{x} = \frac{1}{x}$. $\therefore$ $y(2) = \frac{1}{2}$

29. (b, d): The given curves are $y = x - x^2$ and $y = mx$. They meet at $x = 0$ and $x = 1 - m$

Area $\Rightarrow \int_0^{1-m} (x - x^2 - mx)dx = \pm\frac{9}{2}$

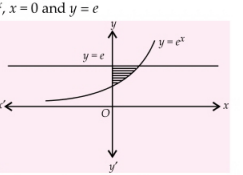
$\Rightarrow \int_0^{1-m} \{(1-m)x - x^2\}dx = \pm\frac{9}{2}$

$\Rightarrow \frac{(1-m)^3}{2} - \frac{(1-m)^3}{3} = \pm\frac{9}{2}$

$\Rightarrow (1-m)^3 = \pm 27 \Rightarrow 1 - m = \pm 3$

$\therefore$ $m = -2, 4$

30. (b, c): The given curves are $y = e^x$, $x = 0$ and $y = e$



Area, $A = \int_1^e x\,dy = \int_1^e \log y\,dy = \int_1^e \log(e+1-y)\,dy$

$\left[\because \int_a^b f(x)dx = \int_a^b f(a+b-x)dx\right]$

Further, $A = e \times 1 - \int_0^1 e^x dx = e - \int_0^1 e^x dx$

Form IV

1.	Place of Publication	: New Delhi
2.	Periodicity of its publication	: Monthly
3.	Printer's and Publisher's Name	: Mahabir Singh
	Nationality	: Indian
	Address	: Mathematics Today, 406, Taj Apartment, New Delhi - 110029.
4.	Editor's Name	: Anil Ahlawat
	Nationality	: Indian
	Address	: Mathematics Today, 19, National Media Centre, Gurgaon Haryana - 122002
5.	Name and address of individuals who own the newspapers and partners or shareholders holding more than one percent of the total capital	: Mahabir Singh 406, Taj Apartment New Delhi

I, Mahabir Singh, hereby declare that particulars given above are true to the best of my knowledge and belief.

Mahabir Singh
Publisher

PRACTICE PAPER 2014

JEE Main

Exam on 6th April 2014

1. If $f : R \to R$ is defined by $f(x) = x - [x] - \frac{1}{2} \forall\, x \in R$ where $[x]$ denotes the greatest integer function, then $\left\{x \in R : f(x) = \frac{1}{2}\right\}$ is

(a) Z, the set of all integers
(b) N, the set of all natural numbers
(c) ϕ, the empty set
(d) R, the set of all real numbers

2. If $f : R \to R$ and $f(x) = ax + \sin x + a$, then
(a) $f(x)$ is one-one onto function if $a \in R$
(b) $f(x)$ is one-one onto function if $a \in R - \{-1, 1\}$
(c) $f(x)$ is one-one onto function if $a \in R - \{0\}$
(d) $f(x)$ is one-one onto function if $a \in R - \{-1\}$

3. The equation $2x^2 - (1 + 2a)x + 1 + a = 0$ has integral roots when a assumes the integral values
(a) $-1, 0$ (b) $-1, 1$
(c) $-2, 0$ (d) $-1, 2$

4. If α and β are the roots of the equation $x^2 - 2x + 4 = 0$, then $2^{n+1}\cos\frac{n\pi}{3}$ is equal to
(a) $\alpha^{-n} + \beta^{-n}$ (b) $\alpha^{\frac{1}{n}} + \beta^{\frac{1}{n}}$
(c) $\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)^{2n}$ (d) $\alpha^n + \beta^n$

5. If $P = \begin{bmatrix} \cos\frac{\pi}{6} & \sin\frac{\pi}{6} \\ -\sin\frac{\pi}{6} & \cos\frac{\pi}{6} \end{bmatrix}$, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP^T$ then $P^T Q^{2011} P =$
(a) $\begin{bmatrix} 1 & \frac{\sqrt{3}}{2} \\ 0 & 2011 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2011 \\ 0 & 1 \end{bmatrix}$
(c) $\begin{bmatrix} \frac{\sqrt{3}}{2} & 2011 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 1 & 2011 \end{bmatrix}$

6. A five digit number divisible by 3 is to be formed using the numerals, 0, 1, 2, 3, 4 and 5 without repetition. The total number of ways this can be done is
(a) 216 (b) 240
(c) 600 (d) 3125

7. If $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = 0$, then a, b, c are
(a) equal (b) in A.P.
(c) in G.P. (d) in H.P.

8. Let $P(n) : n^2 + n$ is an odd integer and $P(K) \Rightarrow P(K + 1)$ is true, $P(n)$ is true for all
(a) $n > 2$ (b) $n > 1$
(c) n (d) none of these

9. The coefficient of x^{24} in the expansion of $(1 + x^2)^{12}(1 + x^{12})(1 + x^{24})$ is
(a) ${}^{12}C_6$ (b) ${}^{12}C_6 + 2$
(c) ${}^{12}C_6 + 4$ (d) ${}^{12}C_6 + 6$

10. A G.P. consist of an even number of terms. If the sum of all the terms is 5 times the sum of the terms occupying the odd places, the common ratio will be
(a) 2 (b) 3
(c) 4 (d) 5

11. In the interval $(-3, 3)$ the function $f(x) = \frac{x}{3} + \frac{3}{x}, x \neq 0$ is
(a) increasing
(b) decreasing
(c) neither increasing nor decreasing
(d) partly increasing and partly decreasing

12. If $\cos^{-1}\frac{y}{b} = n\log\frac{x}{n}$, then $x^2 y_2 =$
(a) $xy_1 + n^2 y$ (b) $xy_1 - n^2 y$
(c) $-xy_1 + n^2 y$ (d) $-xy_1 - n^2 y$

By : Sankar Ghosh, HOD (Math), Takshyashila. Mob : 09831244397.

13. The function $f(x)=\begin{cases}|x-3|, \text{if } x \geq 1\\ \frac{x^2}{4}-\frac{3x}{2}+\frac{13}{4}, \text{if } x<1\end{cases}$ is

(a) continuous and differentiable at $x = 3$
(b) continuous at $x = 3$, but not differentiable at $x = 3$
(c) continuous and differentiable every where
(d) continuous at $x = 1$ but not differentiable at $x = 1$

14. The equation of the line passing through (1, 2, 3) and perpendicular to $3x + 4y - 5z = 6$ is

(a) $\frac{x-1}{3}=\frac{2-y}{4}=\frac{3-z}{5}$
(b) $\frac{x-1}{3}=\frac{y-2}{4}=\frac{3-z}{5}$
(c) $\frac{x-3}{1}=\frac{y-4}{2}=\frac{z+5}{3}$
(d) $\frac{1-x}{3}=\frac{y-2}{4}=\frac{z-3}{-5}$

15. Let $\vec{a}=\hat{i}-\hat{k}, \vec{b}=x\hat{i}+\hat{j}+(1-x)\hat{k}$ and $\vec{c}=y\hat{i}+x\hat{j}+(1+x-y)\hat{k}$. Then $[\vec{a}\ \vec{b}\ \vec{c}]$ depends on

(a) only x (b) only y
(c) neither x nor y (d) both x and y

16. Ellipses are described with a line segment AB as the fixed major axis. The locus of an end of a latus rectum is

(a) straight line (b) parabola
(c) circle (d) ellipse

17. The angle between the lines whose direction cosines satisfy the equations $l + m + n = 0$, $l^2 + m^2 - n^2 = 0$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{3}$

18. The solution of the equation $7\cos^2 x + \sin x \cos x - 3 = 0$ is given by

(a) $n\pi+\frac{\pi}{2}(n\in I)$ (b) $n\pi-\frac{\pi}{4}(n\in I)$
(c) $n\pi+\tan^{-1}\frac{4}{3}(n\in I)$ (d) $n\pi+\frac{3\pi}{4}, k\pi+\tan^{-1}\frac{4}{3}$

19. If $I=\int\frac{dx}{3(1-x^2)-(5+4x)\sqrt{1-x^2}}$, then I is equal to [where $\theta = \cos^{-1}x$]

(a) $\frac{2}{\tan\theta-5}+c$ (b) $\frac{-2}{\tan\left(\frac{\theta}{2}\right)-3}+c$
(c) $\sin^{-1}\left(\frac{2\theta}{\sin^2\theta}\right)$ (d) $\tan\theta - 2$

20. The area bounded by the curves $y=\sqrt{5-x^2}$ and $y = |x - 1|$ is

(a) $\left(\frac{5\pi}{4}-2\right)$ square units
(b) $\left(\frac{5\pi-2}{4}\right)$ square units
(c) $\left(\frac{5\pi-1}{4}\right)$ square units
(d) $\left(\frac{\pi}{2}-5\right)$ square units

21. Which of the following statement is not true?

(a) Mumbai is the capital of Rajasthan or Maharashtra.
(b) $\sqrt{3}$ is a rational number or an irrational number.
(c) 125 is a multiple of 7 or 8.
(d) A rectangle is a quadrilateral or a regular hexagon.

22. The mean and variance of n observations $x_1, x_2, x_3, ..., x_n$ are 5 and 0 respectively. If $\Sigma x_i^2 = 400$, then the value of n is equal to

(a) 80 (b) 25
(c) 20 (d) 16

23. If $P(A) = 0.2$, $P(B) = 0.4$ and $P(A \cap B) = 0.08$, then $P(B/\bar{A})$ equals

(a) 0.4 (b) 0.2
(c) 0.8 (d) none of these

24. The equation of the curve for which the square of the ordinate is twice the rectangle contained by the abscissa and the x-intercept of the normal and passing through (2, 1)is

(a) $x^2 + y^2 - x = 0$ (b) $4x^2 + 2y^2 - 9y = 0$
(c) $2x^2 + 4y^2 - 9x = 0$ (d) $4x^2 + 2y^2 - 9x = 0$

25. $\int_0^{2010}(x-[x])dx$ equals

(a) 1005 (b) 2010
(c) 4020 (d) none of these

Directions : Question numbers 26 to 30 are Assertion-Reason type questions. Each of the questions contains two statements Statement-1 (Assertion) and Statement-2 (Reason). Each of these questions also have four alternative choices, only one of which is the correct answer. You have to select the correct choice.

(a) Statement-1 is true, Statement-2 is true ; Statement-2 is a correct explanation Statement-1.
(b) Statement-1 is true, Statement-2 is true ; Statement-2 is not correct explanation for Statement - 1.
(c) Statement-1 is true, Statement-2 is false.
(d) Statement-1 is false, Statement-2 is true.

26. **Statement-1 :** $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1+\tan^n x}dx = \frac{\pi}{6}, n \in R$

Statement-2 : $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

27. **Statement-1 :** If S (3, 4) and S' (9, 12) are the foci of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1(a>b)$ and (1, –4) is the foot of the perpendicular drawn from S to a tangent then the eccentricity of the ellipse is $\frac{5}{13}$.

Statement-2 : The locus of the foot of the perpendicular drawn from the focus to the tangent at any point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1(a>b)$ is the auxiliary circle $x^2+y^2=a^2$ which is concentric with the ellipse.

28. **Statement-1 :** Let $|x| \le 1$, then the value of $\sin^{-1}(\cos(\sin^{-1}x)) + \cos^{-1}(\sin(\cos^{-1}x))$ is $\frac{\pi}{2}$

Statement-2 : For $|x| \le 1$, the value of $\cos(\sin^{-1}x)$ and $\sin(\cos^{-1}x)$ are equal.

29. **Statement-1 :** If $x = a(1-\cos\theta)$, $y = a(\theta+\sin\theta)$ then $\left(\frac{dy}{dx}\right)_{(\theta=\pi/2)} = 1$

Statement-2 : If $x = f(t)$ and $y = g(t)$, then $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ and $\frac{dx}{dt} \neq 0$

30. **Statement-1 :** If both roots of the equation $2x^2 - x + a = 0$ ($a \in R$), lies in (1, 2), then $-1 < a \le \frac{1}{8}$.

Statement-2 : If $f(x) = 2x^2 - x + a$ then $D \ge 0$, $f(1) > 0$, $f(2) > 0$ yield $-1 < a \le \frac{1}{8}$.

SOLUTIONS

1. **(c) :** The set $\left\{x \in R : f(x) = \frac{1}{2}\right\}$ contains no elements since $f(x) = \frac{1}{2}$ gives $x - [x] - \frac{1}{2} = \frac{1}{2}$

$\Rightarrow x - [x] = 1$ which is not true since $x - [x] \in [0, 1]$,

i.e. the given set is the empty set.

2. **(b):** If $a \neq 0$, Range of $f(x) = R$. Also $f'(x) = a + \cos x$. If $a > 1$, $f'(x) > 0$ the function is increasing and if $a < -1$, $f'(x) < 0$, $\therefore$ the function is decreasing.
So $f(x)$ to be one-one, onto it must be monotonic and is possible if $a < -1$ or $a > 1$.
[Remember : A function is said to be one-one, when it is strictly increasing or strictly decreasing *i.e.* $f'(x) > 0\ \forall\ x \in$ domain or $f'(x) < 0\ \forall\ x \in$ domain]

3. **(d):** Here the given equation is
$2x^2 - (1 + 2a)x + 1 + a = 0$
Here the discriminant = $(1 + 2a)^2 - 4 \cdot 2\ (1 + a)$
$= 4a^2 + 4a + 1 - 8 - 8a = 4a^2 - 4a + 1 - 8$
$= (2a - 1)^2 - 8$ = (integer) when $a = -1, 2$

4. **(d):** The given equation is $x^2 - 2x + 4 = 0$

$$x = \frac{2 \pm \sqrt{4 - 4 \cdot 1 \cdot 4}}{2 \cdot 1} = 1 \pm \sqrt{3}i$$

Now, $2^{n+1}\cos\frac{n\pi}{3} = 2^n \cdot 2\cos\frac{n\pi}{3}$

$$= 2^n\left[\cos\frac{n\pi}{3} + i\sin\frac{n\pi}{3} + \cos\frac{n\pi}{3} - i\sin\frac{n\pi}{3}\right]$$

$$= 2^n\left(\cos\frac{n\pi}{3} + i\sin\frac{n\pi}{3}\right) + 2^n\left(\cos\frac{n\pi}{3} - i\sin\frac{n\pi}{3}\right)$$

$$= \alpha^n + \beta^n \quad \left[\because\ \alpha = 2\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)\right.$$

$$\therefore\ \alpha^n = 2^n\left(\cos\frac{n\pi}{3} + i\sin\frac{n\pi}{3}\right)$$

$$\left.\text{Similarly, } \beta^n = 2^n\left(\cos\frac{n\pi}{3} - i\sin\frac{n\pi}{3}\right)\right]$$

5. **(b):** We know, if P is orthogonal matrix,
$P^TP = PP^T = I$
Now, $Q^{2011} = (PAP^T)\ (PAP^T)\ldots\ldots(PAP^T) = PA^{2011}P^T$
$\therefore\ P^TQ^{2011}P = P^T.\ PA^{2011}\ P^TP$
$P^TQ^{2011}P = A^{2011}$

Now, $A^2 = \begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix} = \begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}$

$$A^3 = \begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix} = \begin{bmatrix}1 & 3\\ 0 & 1\end{bmatrix} \Rightarrow A^{2011} = \begin{bmatrix}1 & 2011\\ 0 & 1\end{bmatrix}$$

6. **(a) :** A number is said to be divisible by 3, if the sum of digits in it is a multiple of 3.

Here, the sum of the given digits is

$0 + 1 + 2 + 3 + 4 + 5 = 15$

So to make a five digit number divisible by 3, we are to exclude either 0 or 3. if we exclude 0, then 5! = 120 number of ways are possible. Again if we exclude 3, then the number of ways of making five digit number is (5! – 4!) = 96, because 0 cannot be placed in the first place from left, as it will give a number of four digits.

$\therefore$ The required number of ways = 120 + 96 = 216

7. (b): $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = 0$

$C'_1 = C_1 - C_2$ and $C'_2 = C_2 - C_3$

$\Rightarrow \begin{vmatrix} -1 & 2-a & x+a \\ -1 & 3-b & x+b \\ -1 & 4-c & x+c \end{vmatrix} = 0$

$R'_1 = R_1 - R_2$ and $R'_2 = R_2 - R_3$

$\Rightarrow \begin{vmatrix} 0 & b-a-1 & a-b \\ 0 & c-b-1 & b-c \\ -1 & 4-c & x+c \end{vmatrix} = 0$

$\Rightarrow -1[b-a-1)(b-c)-(c-b-1)(a-b)] = 0$

$\Rightarrow -[(b-a)(b-c)-(b-c)+(b-a)(c-b)-(b-a)] = 0$

$\Rightarrow -[-b+c-b+a] = 0 \Rightarrow 2b = a + c$

$\therefore$ a, b, c are in A.P.

8. (d): Here $P(1) = 1 + 1 = 2$, is not an odd number, therefore $P(1)$ is not true. Therefore $P(n)$ cannot be true for $n \in N$.

9. (b): The given expansion is

$(1+x^2)^{12}(1+x^{12})(1+x^{24}) = (1+x^2)^{12}(1+x^{12}+x^{24}+x^{36})$

$\therefore$ Coefficient of x^{24} is ${}^{12}C_0 + {}^{12}C_6 + {}^{12}C_{12} = {}^{12}C_6 + 2$

10. (c): Let the G.P. be a, ar, ar^2, ..., ar^{2n-1}, where r is the common ratio.

Now, $S_{2n} = 5[a + ar^2 + ... + n$ terms$]$

$\Rightarrow \dfrac{a}{1-r}(1-r^{2n}) = 5\cdot\dfrac{a}{1-r^2}\left(1-(r^2)^n\right)$

$\Rightarrow 5 = 1 + r \Rightarrow r = 4$

11. (b): $f(x) = \dfrac{x}{3} + \dfrac{3}{x}$

$\therefore f'(x) = \dfrac{1}{3} - \dfrac{3}{x^2} < 0 \ \forall\ x \in (-3, 3)$ and $x \neq 0$

$\therefore$ $f(x)$ is decreasing on $(-3, 3)$

12. (d): $\cos^{-1}\dfrac{y}{b} = n\log\dfrac{x}{n}$

On differentiating, we get,

$$-\frac{1}{\sqrt{1-\dfrac{y^2}{b^2}}}\cdot\frac{y_1}{b} = \frac{n}{x}$$

or, $\dfrac{y_1^2}{b^2 - y^2} = \dfrac{n^2}{x^2}$ or, $x^2y_1^2 = n^2(b^2 - y^2)$

Differentiating again $2x^2 y_1 y_2 + 2x y_1^2 = -2n^2 y y_1$

Dividing both sides by $2y_1$, $x^2y_2 + x y_1 = -n^2 y$

13. (b): Here $f(x) = \begin{cases} |x-3|, \text{if } x \geq 1 \\ \dfrac{x^2}{4} - \dfrac{3x}{2} + \dfrac{13}{4}, \text{if } x < 1 \end{cases}$

or, $f(x) = 3 - x$, when $1 \leq x < 3$

$x - 3$, when $x \geq 3$

$\dfrac{x^2}{4} - \dfrac{3}{2}x + \dfrac{13}{4}$, when $x < 1$

Now, first we check differentiability at $x = 3$.

From definition $f'(3) = \underset{h\to 0}{\text{Lt}}\ \dfrac{f(3+h) - f(3)}{h}$

$= \underset{h\to 0}{\text{Lt}}\ \dfrac{f(3+h)}{h}$ $[\because f(3) = 0]$

Now $Rf'(3) = \underset{h\to 0^+}{\text{Lt}}\ \dfrac{f(3+h)}{h}$

$= \underset{h\to 0^+}{\text{Lt}}\ \dfrac{3+h-3}{h}\left[\begin{array}{l}\because\ h \to 0^+ \\ \therefore\ 3+h > 3\end{array}\right] = 1$

and $Lf'(3) = \underset{h\to 0^-}{\text{Lt}}\ \dfrac{f(3+h)}{h}$

$= \underset{h\to 0^-}{\text{Lt}}\ \dfrac{3-(3+h)}{h}\left[\begin{array}{l}\because\ h \to 0^- \\ \therefore\ 3+h > 3\end{array}\right]$

$\therefore$ $Lf'(3) = -1$ $\because$ $Rf'(3) \neq Lf'(3)$

$\therefore$ $f(x)$ is not differentiable at $x = 3$

Now R.H.L. $= \underset{x\to 3^+}{\text{Lt}}\ f(x) = \underset{x\to 3^+}{\text{Lt}}\ x - 3 = 0$

L.H.L. $= \underset{x\to 3^-}{\text{Lt}}\ f(x) = \underset{x\to 3^-}{\text{Lt}}\ 3 - x = 0$

Also, $f(3) = 0$ $\therefore$ $f(x)$ is continuous at $x = 3$

14. (b): The equation of a line passing through the point (1, 2, 3) is

$$\frac{x-1}{l} = \frac{y-2}{m} = \frac{z-3}{n} \qquad ...(i)$$

Since the line (i) is perpendicular to the plane

$$3x + 4y - 5z = 6 \qquad ...(ii)$$

$\therefore$ The line (i) is parallel to the normal of the plane (ii)

$\therefore$ (i) becomes $\dfrac{x-1}{3} = \dfrac{y-2}{4} = \dfrac{3-z}{5}$

15. (c): Given that $\vec{a} = \hat{i} - \hat{k}$

$\vec{b} = x\hat{i} + \hat{j} + (1-x)\hat{k}$ and $\vec{c} = y\hat{i} + x\hat{j} + (1+x-y)\hat{k}$

Now, $[\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} 1 & 0 & -1 \\ x & 1 & 1-x \\ y & x & 1+x-y \end{vmatrix}$

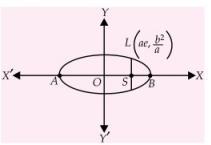
$= 1\,(1 + x - y - x + x^2) + 0 - 1(x^2 - y) = 1$

$\therefore\ [\vec{a} \cdot \vec{b} \cdot \vec{c}] = 1$ (independent of x and y)

16. (b) Let the equation of the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

L, the end point of latus rectum has co-ordinate $\left(ae, \frac{b^2}{a}\right)$

$\therefore\ x = ae,\ y = \frac{b^2}{a} = a(1 - e^2)$



$\therefore\ y = a\left(1 - \frac{x^2}{a^2}\right) = \frac{a^2 - x^2}{a}$

or, $ay = a^2 - x^2 \quad \Rightarrow \quad x^2 = a(a - y)$

Thus the locus of L is parabola.

17. (d): We have $l + m + n = 0$ ...(i)

and $l^2 + m^2 - n^2 = 0$ ...(ii)

Now, $l^2 + m^2 - (l + m)^2 = 0$ or $lm = 0$

$\therefore$ either $l = 0$ or $m = 0$

Now, $l + m + n = 0$

$l + 0 \cdot m + 0 \cdot n = 0$

$\therefore\ \frac{l}{0} = \frac{m}{1} = \frac{n}{-1}$

$\therefore$ direction ratios of one line are 0, 1 and –1

Again $l + m + n = 0$

$0 \cdot l + m + 0 \cdot n = 0$

$\therefore\ \frac{l}{1} = \frac{m}{0} = \frac{n}{-1}$

$\therefore$ direction ratios of the second line are 1, 0, –1

Let θ be the angle between the lines.

$\therefore\ \cos\theta = \frac{0(1) + 1(0) + (-1)(-1)}{\sqrt{0^2 + 1^2 + (-1)^2}\sqrt{(-1)^2 + 0^2 + 1^2}}$

$\Rightarrow\ \cos\theta = \frac{1}{2} = \cos\frac{\pi}{3} \quad \therefore\ \theta = \frac{\pi}{3}$

18. (b): $7\cos^2 x + \sin x \cos x - 3 = 0$

$\Rightarrow\ 7\cos^2 x + \sin x \cos x - 3(\cos^2 x + \sin^2 x) = 0$

$\Rightarrow\ 4\cos^2 x + \sin x \cos x - 3\sin^2 x = 0$

$\Rightarrow\ 3\sin^2 x - \sin x \cos x - 4\cos^2 x = 0$

$\Rightarrow\ (\sin x + \cos x)(3\sin x - 4\cos x) = 0$

If $\sin x + \cos x = 0$, then $\tan x = -1$

or, $\tan x = \tan\left(\frac{-\pi}{4}\right)$

$\therefore\ x = n\pi - \frac{\pi}{4},\ n \in I$

If $3\sin x - 4\cos x = 0$

then, $\tan x = \frac{4}{3}$

But $\tan x = \frac{4}{3}$ does not satisfy the given equation, so it cannot be a solution.

19. (b): $\int \frac{dx}{3(1 - x^2) - (5 + 4x)\sqrt{1 - x^2}}$

Put $\cos\theta = x$

$\therefore\ -\sin\theta\, d\theta = dx$

$= \int \frac{-\sin\theta\, d\theta}{3\sin^2\theta - (5 + 4\cos\theta)\sin\theta}$

$= \int \frac{-d\theta}{3\sin\theta - 5 - 4\cos\theta} = \int \frac{d\theta}{4\cos\theta - 3\sin\theta + 5}$

Put $\tan\frac{\theta}{2} = z$

$= \int \frac{2\frac{dz}{1 + z^2}}{4\left(\frac{1 - z^2}{1 + z^2}\right) - 3\left(\frac{2z}{1 + z^2}\right) + 5}$

$= \int \frac{2\,dz}{4(1 - z^2) - 6z + 5(1 + z^2)}$

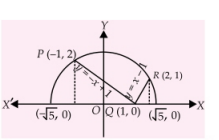
$= 2\int \frac{dz}{z^2 - 6z + 9} = 2\int \frac{dz}{(z - 3)^2} = -\frac{2}{z - 3} + c$

$= -\frac{2}{\tan\frac{\theta}{2} - 3} + c$

20. (b): The given curves are

$y = \sqrt{5 - x^2}$ ...(i)

$y = |x - 1|$ ...(ii)



The equation (i) represents a semi circle above the x-axis and (ii) gives two straight lines, which are

$$y=\begin{cases}x-1, \text{ when } x>1\\ 1-x, \text{ when } x<1\end{cases}$$

Clearly from the diagram, the required area is $PQRP$

$$=\int_{-1}^{2}\sqrt{5-x^2}\,dx-\int_{-1}^{1}(1-x)dx-\int_{1}^{2}(x-1)dx$$

$$=\left[\frac{x}{2}\sqrt{5-x^2}+\frac{5}{2}\sin^{-1}\frac{x}{\sqrt{5}}\right]_{-1}^{2}-\left[x-\frac{x^2}{2}\right]_{-1}^{1}-\left[\frac{x^2}{2}-x\right]_{1}^{2}$$

$$=\frac{5\pi}{4}-\frac{1}{2} \text{ square units.}$$

21. (c) : (a) The component statements of "Mumbai is the capital of Rajasthan or Maharashtra" are

p : Mumbai is the capital of Rajasthan.

q : Mumbai is the capital of Maharashtra.

Here we find p is false and q is true, so the compound statement is true.

(b) The component statements of "$\sqrt{3}$ is a rational or an irrational number" are

$p=\sqrt{3}$ is a rational number.

$q=\sqrt{3}$ is an irrational number

Here, we find p is false and q is true, so the compound statement is true. Similarly it can be proved that the statement (c) is false and (d) is true.

22. (d): Given $\frac{1}{n}\sum_{i=1}^{n}x_i=5,\ \frac{1}{n}\sum_{i=1}^{n}x_i^2-\bar{x}^2=0$

$$\Rightarrow\ \frac{1}{n}\sum_{i=1}^{n}x_i^2=\bar{x}^2=25\ \Rightarrow\ 25n=400$$

$$\Rightarrow\ n=\frac{400}{25}=16$$

23. (a) : Given $P(A)=0.2$

$P(B)=0.4$ and $P(A\cap B)=0.08$

Now, $P(B/\bar{A})=\dfrac{P(B\cap\bar{A})}{P(\bar{A})}=\dfrac{P(B)-P(A\cap B)}{1-P(A)}$

$$=\frac{0.4-0.08}{1-0.2}=\frac{0.32}{0.80}=0.4$$

24. (d): Equation of normal at (x_1, y_1) is

$$y-y_1=-\frac{dx_1}{dy_1}(x-x_1)$$

$\therefore$ x intercept $=y_1\dfrac{dy_1}{dx_1}+x_1$ (putting $y=0$)

$$\therefore\ y_1^2=2x_1\left(y_1\frac{dy_1}{dx_1}+x_1\right)$$

$$\text{or, } \frac{y_1^2}{2x_1}-x_1=y_1\frac{dy_1}{dx_1} \text{ or, } \frac{dy_1}{dx_1}=\frac{y_1^2-2x_1^2}{2x_1y_1}$$

$$v+x_1\frac{dv}{dx_1}=\frac{v^2-2}{2v}$$

$\therefore$ Put $y_1=vx_1$, $\dfrac{dy_1}{dx_1}=v+x_1\dfrac{dv}{dx_1}$

$$\text{or, } x_1\frac{dv}{dx_1}=\frac{v^2-2-2v^2}{2v}=-\frac{(2+v^2)}{2v}$$

$$\text{or, } \int\frac{2v}{v^2+2}dv+\int\frac{dx_1}{x_1}=\log c$$

or $\log(v^2+2)+\log x_1=\log c$

or, $x_1(v^2+2)=c$ or, $y_1^2+2x_1^2-c\,x_1=0$ ...(i)

When this passes through (2, 1), $c=\dfrac{9}{2}$

Now changing to current co-ordinate, (i) becomes

$4x^2+2y^2-9x=0$

25. (a): $\int_{0}^{2010}(x-[x])dx$

Here $f(x)=x-[x]$

Now $f(x+1)=x+1-[x+1]=x+1-([x]+1)=x-[x]$

$f(x+1)=f(x)$

hence $f(x)$ is a periodic function of period 1.

$$\therefore\ \int_{0}^{2010}(x-[x])dx=2010\int_{0}^{1}(x-[x])dx$$

$$=2010\int_{0}^{1}x\,dx\quad\left[\because\ \int_{0}^{1}[x]dx=0\right]$$

$$=2010\times\frac{1}{2}=1005$$

26. (a): $I=\int_{\pi/6}^{\pi/3}\dfrac{1}{1+\tan^n x}\,dx$

$$I=\int_{\pi/6}^{\pi/3}\frac{\cos^n x}{\sin^n x+\cos^n x}\,dx$$

$$=\int_{\pi/6}^{\pi/3}\frac{\cos^n\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)dx}{\sin^n\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)+\cos^n\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}$$

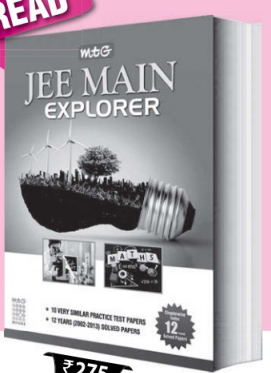
$$=\int_{\pi/6}^{\pi/3}\frac{\cos^n\left(\frac{\pi}{2}-x\right)}{\cos^n\left(\frac{\pi}{2}-x\right)+\sin^n\left(\frac{\pi}{2}-x\right)}$$

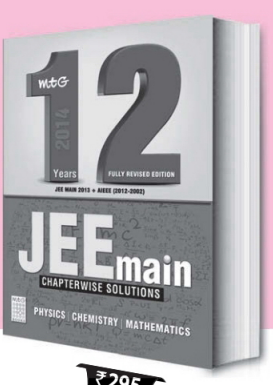
$$I=\int_{\pi/6}^{\pi/3}\frac{\sin^n x}{\sin^n x+\cos^n x}dx$$

BEST TOOLS FOR SUCCESS IN

JEE Main

READ





10 very Similar Practice Test Papers

12 JEE MAIN 2013 + AIEEE (2012-2002)

Buy online at www.mtg.in

Available at all leading bookshops throughout India.

For more information or to place your order :
Call at : 0124 - 4951200 @ Mail at : info@mtg.in
Log on to : www.mtg.in

$\therefore \quad 2I = \int_{\pi/6}^{\pi/3} dx = \frac{\pi}{3} - \frac{\pi}{6}$

$I = \frac{\pi}{6}$

$\therefore \int_{\pi/6}^{\pi/3} \frac{1}{1+\tan^n x} dx = \frac{\pi}{6}$

Thus Statement-1 and Statement-2 both are true Statement-2 is the correct explanation of Statement-1.

27. (a): $S(3, 4)$ and $S'(9, 12)$

$\therefore \quad SS' = 10$ and centre $(6, 8)$

Equation of the auxiliary circle is

$(x-6)^2 + (y-8)^2 = a^2$ .. (i)

Circle (i) is passing through $(1, -4)$

$\therefore \quad a^2 = 13^2$

Now, $SS' = 10 \Rightarrow 2ae = 10 \Rightarrow e = \frac{5}{13}$

Statement - 2 is the definition of the auxiliary circle.

28. (a): Here, $\cos(\sin^{-1} x) = \sin(\cos^{-1} x)$

$= \sqrt{1-x^2}, \forall\, |x| \le 1$

Now, $\sin^{-1}(\cos(\sin^{-1}x)) + \cos^{-1}(\sin(\cos^{-1}x))$

$= \sin^{-1}(\sqrt{1-x^2}) + \cos^{-1}(\sqrt{1-x^2})$

$= \frac{\pi}{2} \qquad [\forall\ \sqrt{1-x^2} \ge 0 \Rightarrow |x| \le 1]$

Thus both Statement-1 and 2 are true and Statement -2 is the correct explanation of Statement - 1

29. (a): Given $x = a(1 - \cos\theta) \Rightarrow \frac{dx}{d\theta} = a\sin\theta$

and $y = a(\theta + \sin\theta) \Rightarrow \frac{dy}{d\theta} = a(1+\cos\theta)$

Now, $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}, \left(\frac{dx}{d\theta} \ne 0\right)$

$= \frac{1+\cos\theta}{\sin\theta} = \frac{2\cos^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}} = \cot\frac{\theta}{2}$

$\therefore \left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{2}} = \cot\frac{\pi}{4} = 1$

Thus both the Statements-1 and 2 are true and Statement-2 is the proper explanation of Statement-1.

30. (a): Given that both the roots of the equation $2x^2 - x + a = 0$ lies in the interval $(1, 2)$, then we have the following three conditions:

(i) $D \ge 0 \Rightarrow 1 - 8a \ge 0 \Rightarrow a \le \frac{1}{8}$

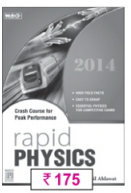
(ii) $f(1) > 0 \Rightarrow 2 - 1 + a > 0 \Rightarrow a > -1$

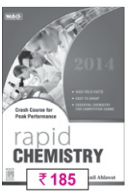
(iii) $f(2) > 0 \Rightarrow 8 - 2 + a > 0 \Rightarrow a > -6$

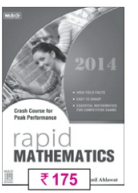
So both the Statements-1 and 2 are true and Statement-2 is the correct explanation of the Statement-1.

MTG A Series of Quick Reference Books

Cover complete syllabus in point form • A Handy High Yield Facts Book
Easy to Grasp • Essential for all Competitive Examinations









Buy ONLINE at www.mtg.in

Buy ONLINE at www.mtg.in

MTG BOOKS
Plot 99, Sector 44 Institutional Area, Gurgaon (HR) - 122 003
Tel.: (0124) 4951200 e-mail : info@mtg.in web : www.mtg.in

PRACTICE PAPER 2014

BITSAT

Exam from 14th May to 8th June

1. If $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + ...\infty}}}$, then $\frac{dy}{dx} =$

(a) $\frac{2y-1}{\cos x}$ (b) $\frac{\cos x}{2y-1}$

(c) $\frac{2x-1}{\cos y}$ (d) $\frac{\cos y}{2y-1}$

2. The equation of the hyperbola referred to its axis as axis of co-ordinate and whose distance between the foci is 16 and eccentricity is $\sqrt{2}$, is

(a) $x^2 - y^2 = 16$ (b) $x^2 - y^2 = 32$

(c) $x^2 - 2y^2 = 16$ (d) $x^2 - y^2 = -16$

3. If a_1, a_2, a_3, a_4 are the coefficients of any four consecutive terms in the expansion of $(1 + x)^n$, then $\frac{a_1}{a_1 + a_2} + \frac{a_3}{a_3 + a_4} =$

(a) $\frac{a_2}{a_2 + a_3}$ (b) $\frac{a_2}{2(a_2 + a_3)}$

(c) $\frac{2a_2}{a_2 + a_3}$ (d) $\frac{2a_3}{a_2 + a_3}$

4. $\frac{d}{dx}[\tan^{-1}(\log_{10} x)] =$

(a) $\frac{1}{[1 + (\log_{10} x)]^2 \cdot x \log_e 10}$

(b) $\frac{1}{1 + (\log_{10} x)^2}$

(c) $\frac{1}{[1 + (\log_{10} x)^2] \cdot x \log_{10} e}$

(d) none of these

5. If e, e' be the eccentricities of two conics $S = 0$ and $S' = 0$ and if $e^2 + e'^2 = 3$, then both S and S' are

(a) hyperbolas (b) ellipses

(c) parabolas (d) none of these

6. If $f(x) = \begin{cases} \int_0^x (1 + |1 - t|)\,dt, & x > 2 \\ 5x - 7, & x \le 2 \end{cases}$, then

(a) $f(x)$ is not continuous at $x = 2$

(b) f is differentiable everywhere

(c) RHL at $x = 2$ doesn't exist

(d) f is continuous but not differentiable at $x = 2$

7. The length of normal at θ on the curve $x = a\cos^3\theta$, $y = a\sin^3\theta$ is

(a) $|a\sin^2\theta|$ (b) $|a\sin^2\theta\tan\theta|$

(c) $|a\sin^2\theta\cos\theta|$ (d) $|a\sin^3\theta\tan\theta|$

8. If A, B, C are the angles which a directed line makes with the positive directions of the co-ordinate axes, then $\sin^2 A + \sin^2 B + \sin^2 C =$

(a) 1 (b) 2

(c) 3 (d) none of these

9. Sum of the series $\frac{1^3}{1} + \frac{1^3 + 2^3}{1 + 3} + \frac{1^3 + 2^3 + 3^3}{1 + 3 + 5}$ ... to 16 terms is

(a) 346 (b) 446

(c) 546 (d) none of these

10. If $f(x) = \begin{cases} x, & x < 0 \\ 1, & x = 0 \\ x^2, & x > 0 \end{cases}$ then, $\lim_{x \to 0} f(x) =$

(a) 0 (b) 1

(c) doesn't exist (d) 2

11. The tangent at any point on the curve $x^4 + y^4 = a^4$ cuts off intercepts p and q on the co-ordinate axes, then the value of $p^{-4/3} + q^{-4/3}$ is equal to

(a) $a^{-4/3}$ (b) $a^{-1/2}$

(c) $a^{1/2}$ (d) none of these

12. $\int_0^1 \frac{dx}{(x^2 + 1)^{3/2}} =$

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) 1 (d) $\frac{-1}{\sqrt{2}}$

By : **Vidyalankar Institute**, Pearl Centre, Senapati Bapat Marg, Dadar (W), Mumbai - 28. Tel.: (022) 24306367

13. The sum of the series $\frac{1}{3\times7}+\frac{1}{7\times11}+\frac{1}{11\times15}+\ldots\infty$ is
(a) 1/3 (b) 1/6 (c) 1/9 (d) 1/12

14. If $[x]$ denotes the greatest integer, less than or equal to x, then the value of
$\lim_{x\to0}(1-x+[x-1]+[1-x])$ is
(a) 0 (b) 1
(c) doesn't exist (d) –1

15. $\int x^n \log x\, dx =$
(a) $\frac{x^{n+1}}{n}[n\log x-1]+C$
(b) $\frac{x^{n+1}}{n+1}[(n+1)\log x-1]+C$
(c) $\frac{x^{n+1}}{(n+1)^2}[(n+1)\log x-1]+C$
(d) none of these

16. The orthogonal trajectories of the family of curves $a^{n-1}y = x^n$ are given by
(a) $x^n + n^2y$ = constant (b) $ny^2 + x^2$ = constant
(c) $n^2x + y^n$ = constant (d) $n^2x - y^n$ = constant

17. If each observation of a raw data, whose variance is σ^2, is multiplied by λ, then the variance of the new set is
(a) σ^2 (b) $\lambda^2\sigma^2$ (c) $\lambda+\sigma^2$ (d) $\lambda^2+\sigma^2$

18. If the line $y = x\sqrt{3}$ cuts the curve
$x^3+y^3+3xy+5x^2+3y^2+4x+5y-1=0$ at the points A, B and C, then $OA\cdot OB\cdot OC$ (where O is (0, 0)) is
(a) $\frac{4}{13}(3\sqrt{3}-1)$ (b) $3\sqrt{3}+1$
(c) $\frac{1}{\sqrt{3}}(2+7\sqrt{3})$ (d) none of these

19. A candidate appearing for the competitive test finds the following information on seeing the question paper :
(i) the total time is 3 hours
(ii) the paper has two sections - section A and section B.
(iii) to answer atleast two questions from section A and atleast three questions from section B.
(iv) A question of first section carries 10 marks and that of section B carries 15 marks.
(v) time to answer question of section A is 15 minutes and that of section B is 25 minutes.
(vi) The maximum number of questions to be answered is 10.
The maximum marks he can secure is
(a) 115 (b) 125
(c) 100 (d) none of these

20. Equation of a line which is parallel to the line common to the pair of lines given by $6x^2 - xy - 12y^2 = 0$ and $15x^2 + 14xy - 8y^2 = 0$ and the sum of whose intercepts on the axes is 7, is
(a) $2x - 3y = 42$ (b) $3x + 4y = 12$
(c) $5x - 2y = 10$ (d) none of these

21. The roots of $\begin{vmatrix} x & a & b & 1\\ \lambda & x & b & 1\\ \lambda & \mu & x & 1\\ \lambda & \mu & \upsilon & 1\end{vmatrix}=0$ are independent of
(a) λ, μ, υ (b) a, b
(c) $\lambda, \mu, \upsilon, a, b$ (d) none of these

22. The incentre of the triangle formed by the coordinate axes and $3x + 4y = 12$ is
(a) (1/2, 1/2) (b) (1, 1)
(c) (1, 1/2) (d) (1/2, 1)

23. $\int \frac{xe^x}{(x+1)^2}dx =$
(a) $e^x/(x+1)$ (b) $e^x/(x+1)^2$
(c) $-e^x/(x+1)^3$ (d) none of these

24. The equation of the circle passing through (1, 0) and (0, 1) and having smallest possible radius is
(a) $x^2+y^2-x-y=0$ (b) $x^2+y^2+x+y=0$
(c) $x^2+y^2-2x-y=0$ (d) $x^2+y^2-x-2y=0$

25. If α, β are roots of $x^2 + px + 1 = 0$ and γ, δ are the roots $x^2 + qx + 1 = 0$, the value of $(\alpha-\gamma)(\beta-\gamma)(\alpha+\delta)(\beta+\delta)$ is
(a) p^2-q^2 (b) q^2-p^2 (c) p^2 (d) q^2

26. A line meets the co-ordinate axes in A and B. A circle is circumscribed about the ΔAOB. The distance from the end points of the side AB to the line touching the circle at origin O are equal to p and q respectively. The diameter of the circle is
(a) $p(p+q)$ (b) $q(p+q)$
(c) $p+q$ (d) $\frac{p+q}{2}$

27. Area between the curves $y = x^3$ and $y = \sqrt{x}$ is
(a) 5/12 (b) 5/3
(c) 5/4 (d) none of these

28. The equation of the directrix of the parabola $x^2 - 4x - 3y + 10 = 0$ is
(a) $y = -5/4$ (b) $y = 5/4$
(c) $y = -3/4$ (d) $x = 5/4$

29. If $a\cos^2(3\alpha) + b\cos^4\alpha = 16\cos^6\alpha + 9\cos^2\alpha$ is an identity, then
(a) $a = 1, b = 24$ (b) $a = 3, b = 24$
(c) $a = 4, b = 2$ (d) $a = 7, b = 18$

30. Tangents are drawn to the circle $x^2 + y^2 = 25$ from the point (13, 0). They include an angle
(a) $\tan^{-1}(5/12)$ (b) $\tan^{-1}(12/5)$
(c) $2\tan^{-1}(5/12)$ (d) none of these

31. $1 + 4 + 13 + 40 + ... + n$ terms is
(a) $\left(\frac{3^{n+1} - 3 - 2^n}{4}\right)$ (b) $\left(\frac{3^{n+1} + 3 - 2^n}{4}\right)$
(c) $\left(\frac{3^{n+1} + 3 + 2^n}{4}\right)$ (d) $\left(\frac{3^{n-1} - 3 + 2^n}{4}\right)$

32. The eccentric angle of a point on the ellipse $\frac{x^2}{6} + \frac{y^2}{2} = 1$ whose distance from the centre of the ellipse is 2, is
(a) $\pi/4$ (b) $3\pi/2$ (c) $5\pi/3$ (d) $7\pi/6$

33. Two teams are to play a series of 5 matches between them. A match ends in a win or loss or draw for a team. A number of people forecast the result of each match and no two people make the same forecast for the series of matches. The smallest group of people in which one person forecasts correctly for all the matches will contain 'n' people, where 'n' is
(a) 81 (b) 243
(c) 486 (d) none of these

34. Odds 8 to 5 against a person who is 40 years old living till he is 70 and 4 to 3 against another person now 50 till he will be living 80. Probability that one of them will be alive next 30 years is
(a) 59/91 (b) 44/91 (c) 51/91 (d) 32/91

35. Solution to the differential equation $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$, where
(a) $\frac{y-x}{1+xy} = K$ (b) $xy = K$
(c) $x^2 - y^2 = K$ (d) none of these

36. If z is a non-real root of $\sqrt[7]{-1}$, then $z^{86} + z^{175} + z^{289}$ is equal to
(a) 0 (b) –1 (c) 3 (d) 1

37. The number of ways in which 6 different balls can be put in two boxes of different sizes so that no box remains empty is
(a) 62 (b) 64
(c) 36 (d) none of these

38. The number of even proper divisors of 1008 is
(a) 23 (b) 24
(c) 22 (d) none of these

39. The solution of the differential equation $2x + \frac{dy}{dx} - y = 3$; given $y(0) = -1$ represents
(a) straight line (b) circle
(c) parabola (d) ellipse

40. If the three consecutive coefficients in the expansion of $(1 + x)^n$ are 28, 56 and 70, then 'n' equals
(a) 6 (b) 4 (c) 8 (d) 10

41. If $f(x) = A\sin\left(\frac{\pi x}{2}\right) + B$, $f'\left(\frac{1}{2}\right) = \sqrt{2}$ and $\int_0^1 f(x)dx = \frac{2A}{\pi}$, then the constant A and B are, respectively
(a) $\pi/2$ and $\pi/2$ (b) $2/\pi$ and $3/\pi$
(c) 0 and $-4/\pi$ (d) $4/\pi$ and 0

42. $\lim_{x\to\infty} \frac{\log[x]}{x}$, where $[x]$ is the greatest integer less than or equal to x is
(a) 1 (b) –1
(c) does not exist (d) 2

43. If at any point (x, y) on a curve, subtangent and subnormal are of equal length, then the length of tangent is
(a) $\sqrt{2y}$ (b) $|\sqrt{2}y|$
(c) $|y|$ (d) none of these

44. If a straight line in space is equally inclined to the co-ordinate axes, the cosine of its angle of inclination to any one of the axes is
(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{\sqrt{2}}$

45. Let $\vec{v} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{w} = \hat{i} + 3\hat{k}$. If $\vec{u}$ is a unit vector, then the maximum value of the scalar triple product $[\vec{u}\ \vec{v}\ \vec{w}]$ is
(a) –1 (b) $\sqrt{10} + \sqrt{6}$
(c) $\sqrt{59}$ (d) $\sqrt{60}$

SOLUTIONS

1. (b): $y = \sqrt{\sin x + y} \Rightarrow y^2 = y + \sin x$

$\Rightarrow 2y \cdot \frac{dy}{dx} = \cos x + \frac{dy}{dx} \quad \therefore \quad \frac{dy}{dx} = \frac{\cos x}{2y - 1}$

2. (b): $2ae = 16,\ e = \sqrt{2} \Rightarrow ae = 8, e^2 = 2$

$\Rightarrow \quad a^2(2) = 64 \quad \text{or} \quad a^2 = 32$

Now, $b^2 = a^2(e^2 - 1) = 32(2 - 1) = 32$

$\therefore \quad \dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1 \;\Rightarrow\; x^2 - y^2 = 32$

3. (c) : a_1, a_2, a_3, a_4 = coefficients in T_{r+1}, T_{r+2}, T_{r+3}, T_{r+4} for $(1+x)^n$

$a_1 = {}^nC_r$, $a_2 = {}^nC_{r+1}$, $a_3 = {}^nC_{r+2}$, $a_4 = {}^nC_{r+3}$

$$\therefore \quad \frac{a_1}{a_1 + a_2} = \frac{1}{1 + \dfrac{a_2}{a_1}}$$

But, $\dfrac{a_2}{a_1} = \dfrac{{}^nC_{r+1}}{{}^nC_r} = \dfrac{n-r}{r+1}$

$$\Rightarrow \quad 1 + \frac{a_2}{a_1} = \frac{n+1}{r+1}$$

Similarly, $1 + \dfrac{a_4}{a_3} = \dfrac{n+1}{r+3}$, $1 + \dfrac{a_3}{a_2} = \dfrac{n+1}{r+2}$

4. (a) : $D = \dfrac{1}{1 + (\log_{10} x)^2} \cdot \dfrac{d}{dx}[\log_{10} x]$

$$= \frac{1}{1 + (\log_{10} x)^2} \cdot \frac{1}{x \ln(10)}$$

5. (a) : (I) Both are ellipses, then $e, e' < 1 \;\Rightarrow ee' = 1$

$\therefore$ (b) false.

(II) Both are parabolas, then $e, e' = 1 \;\Rightarrow$ (c) is false.

(III) For hyperbolas, then $e, e' > 1$. $\therefore \; e^2 + e'^2 = 3$ is possible.

6. (d): The function for $x > 2$ can be redefined as;

$$f(x) = \int_0^1 1 + (1-t)dt + \int_1^x 1 + (t-1)dt$$

$$= 2(1) - \frac{1}{2} + x + \frac{x^2}{2} - \frac{1}{2} - x = \frac{x^2}{2} + 1$$

$$\text{R.H.L.} = \lim_{h \to 0^+} \left\{ \frac{(2+h)^2}{2} + 1 \right\} = 3$$

$$\text{L.H.L.} = \lim_{h \to 0^-} \{5(2-h) - 7\} = 3$$

$\because$ L.H.L. = R.H.L.

$\Rightarrow$ $f(x)$ is continuous at $x = 2$

Now,

$$\text{L.H.D.} = \lim_{h \to 0} \frac{f(2) - f(2-h)}{h} = \lim_{h \to 0} \frac{3 - 5(2-h) + 7}{h} = 5$$

$$\text{R.H.D.} = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h} = \lim_{h \to 0} \frac{\dfrac{(2+h)^2}{2} + 1 - 3}{h}$$

$$= \lim_{h \to 0} \left\{ \frac{h^2 + 4h + 4 + 2 - 6}{2h} \right\} = \lim_{h \to 0} \left[\frac{h}{2} + 2 \right] = 2$$

$\because$ L.H.D. $\neq$ R.H.D. $\therefore f(x)$ is non-differentiable at $x = 2$.

7. (b): $\dfrac{dy}{dx} = \dfrac{dy/d\theta}{dx/d\theta} = \dfrac{3a\sin^2\theta\cos\theta}{-3a\cos^2\theta\sin\theta} = -\tan\theta$

$$\therefore \quad N = |\, y\sqrt{1 + (y')^2}\,| = |\, a\sin^3\theta\sec\theta\,| = |\, a\sin^2\theta\tan\theta\,|$$

8. (b): $\sum \cos^2 A = 1 \;\Rightarrow\; \sum (1 - \sin^2 A) = 1$

$$\Rightarrow \quad \sum \sin^2 A = 2$$

9. (b): $T_n = \dfrac{\sum r^3}{\sum (2r-1)} = \dfrac{\left(\dfrac{n(n+1)}{2}\right)^2}{2\left[\dfrac{n(n+1)}{2}\right] - n}$

$$= \left(\frac{n+1}{2}\right)^2 = \frac{n^2 + 2n + 1}{4}$$

$$\therefore \quad S_n = \sum_{r=1}^{n} T_r = \frac{1}{4}\sum_{r=1}^{n} r^2 + \frac{1}{2}\sum_{r=1}^{n} r + \frac{1}{4}\sum_{r=1}^{n} 1$$

$$= \frac{1}{4}\frac{n(n+1)(2n+1)}{6} + \frac{1}{2}\frac{n(n+1)}{2} + \frac{n}{4}$$

Put $n = 16 \;\Rightarrow\; S_n = 446$

10. (a)

11. (a): $(X - x)\left(\dfrac{\partial f}{\partial x}\right) + (Y - y)\left(\dfrac{\partial f}{\partial y}\right) = 0$

$$(X - x)(4x^3) + (Y - y)4y^3 = 0$$

or $\quad Xx^3 + Yy^3 = x^4 + y^4 = a^4 \qquad \ldots (1)$

If intercepts are p and q, then $(p, 0)$ and $(0, q)$ satisfy (1)

$\therefore \quad px^3 = a^4, \; qy^3 = a^4$

$$p^{-4/3} + q^{-4/3} = (a^4)^{-4/3}\,(x^4 + y^4) = a^{-4/3}$$

12. (b): Put $x = \tan\theta \;\Rightarrow\; dx = \sec^2\theta\, d\theta$

Also, when $x = 0$, $\theta = 0$ and when $x = 1$, $\theta = \pi/4$

$$\therefore \quad I = \int_0^{\pi/4} \frac{\sec^2\theta\, d\theta}{\sec^3\theta} = \int_0^{\pi/4} \cos\theta\, d\theta = [\sin\theta]_0^{\pi/4} = \frac{1}{\sqrt{2}}$$

13. (d): $S = \dfrac{1}{4}\left[\left(\dfrac{1}{3} - \dfrac{1}{7}\right) + \left(\dfrac{1}{7} - \dfrac{1}{11}\right) + \ldots\infty\right]$

$$S_\infty = \frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$$

14. (a): $\text{L.H.L.} = \lim_{h \to 0} \{1 - (-h) + [-h - 1] + [1 + h]\}$

$$= 1 + (-2) + (1) = 0$$

$$\text{R.H.L.} = \lim_{h \to 0} \{1 - h + [h - 1] + [1 - h]\}$$

$$= 1 + (-1) + (0) = 0$$

15. (c) : $\dfrac{x^{n+1}}{n+1}\log x - \dfrac{1}{(n+1)}\int x^{n+1} \cdot \dfrac{1}{x}\, dx$

(Applying by parts)

$$= \frac{x^{n+1}}{n+1}\log x - \frac{1}{(n+1)^2}x^{n+1} + C$$

16. (b): Differentiating $a^{n-1}y = x^n$ ... (1)

we get, $a^{n-1}\frac{dy}{dx} = nx^{n-1}$ ... (2)

Using (2) in (1), we get

$$nx^{n-1}\frac{dx}{dy}y = x^n \quad \text{or} \quad n\frac{dx}{dy}y = x$$

Replacing $\frac{dy}{dx}$ by $-\frac{dx}{dy}$, we get $ny = -x\frac{dx}{dy}$

$\Rightarrow \quad nydy + xdx = 0 \quad \Rightarrow \quad \frac{ny^2}{2} + \frac{x^2}{2} = \text{constant}$

$\Rightarrow \quad ny^2 + x^2 = \text{constant}$

which is the required family of orthogonal trajectory.

17. (b): We know that $\text{Var}(aX) = a^2\,\text{var}(X)$

So, $\text{Var}(\lambda X) = \lambda^2\,\text{var}(X) = \lambda^2\sigma^2$

18. (a): Putting $y = x\sqrt{3}$ in the equation of curve

$$x^3(3\sqrt{3}+1) + x^2(3\sqrt{3}+14) + (4+5\sqrt{3})x - 1 = 0$$

If the roots of this equation are x_1, x_2 and x_3

$$\Rightarrow \quad x_1x_2x_3 = \frac{1}{3\sqrt{3}+1} = \frac{3\sqrt{3}-1}{26}$$

Also, for a point A, coordinates are $(x_1, x_1\sqrt{3})$

$$\therefore \quad OA = \sqrt{x_1^2 + 3x_1^2} = 2x_1$$

$$\Rightarrow \quad OA \cdot OB \cdot OC = (2x_1)(2x_2)(2x_3) = 8x_1x_2x_3$$

$$= \frac{4}{13}(3\sqrt{3}-1)$$

19. (a): Let x_1, x_2 denote the number of questions to be attempted from section A, B respectively. Then mathematically the problem can be written as

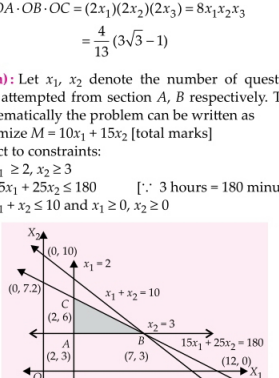
Maximize $M = 10x_1 + 15x_2$ [total marks]

subject to constraints:

$x_1 \geq 2,\ x_2 \geq 3$

$15x_1 + 25x_2 \leq 180$ [$\because$ 3 hours = 180 minutes]

$x_1 + x_2 \leq 10$ and $x_1 \geq 0,\ x_2 \geq 0$



Consider a set of rectangular cartesian axes OX_1, OX_2 in the plane. Each point has co-ordinates of the type (x_1, x_2) and conversely. Clearly each point (x_1, x_2) which satisfies the conditions $x_1 \geq 0,\ x_2 \geq 0$ lies in the first quadrant only. Thus our search for the desired number pair (x_1, x_2) will be restricted to the first quadrant only. Also since,

$x_1 \geq 2,\ x_2 \geq 3$

$15x_1 + 25x_2 \leq 180$

$x_1 + x_2 \leq 10$

$\therefore$ The desired point (x_1, x_2) lies somewhere in the convex region ABC which is shown (shaded) in the figure. This is the feasible region *i.e.* solution space of the problem. The extreme points are

$A(2, 3)$, $B(7, 3)$, $C(2, 6)$

At $A(2, 3)$, $M = 10(2) + 15(3) = 65$

At $B(7, 3)$, $M = 10(7) + 15(3) = 115$

At $C(2, 6)$, $M = 10(2) + 15(6) = 110$

Since the maximum value occurs at B.

$\therefore \quad x_1 = 7,\ x_2 = 3$ is the optimal solution and 115 is the maximum marks.

20. (b): $6x^2 - xy - 12y^2 = 0$

$6x^2 - 9xy + 8xy - 12y^2 = 0$

$3x(2x - 3y) + 4y(2x - 3y) = 0$

$\Rightarrow \quad (3x + 4y)(2x - 3y) = 0$

$15x^2 + 14xy - 8y^2 = 0 \quad \Rightarrow \quad (3x + 4y)(5x - 2y) = 0$

Hence equation of common line is $3x + 4y = c$

$$\therefore \quad \frac{c}{3} + \frac{c}{4} = 7 \quad \Rightarrow \quad c\left(\frac{7}{12}\right) = 7 \quad \Rightarrow \quad c = 12$$

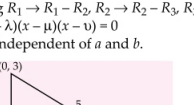
21. (b): $\Delta = \begin{vmatrix} x & a & b & 1 \\ \lambda & x & b & 1 \\ \lambda & \mu & x & 1 \\ \lambda & \mu & \upsilon & 1 \end{vmatrix}$

Applying $R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$, $R_3 \to R_3 - R_4$

Then $(x - \lambda)(x - \mu)(x - \upsilon) = 0$

$\therefore$ x is independent of a and b.

22. (b):



Now, the incentre is given by

$$\left(\frac{\sum ax_1}{\sum a}, \frac{\sum ay_1}{\sum a}\right) = \left(\frac{0+0+3\times4}{12}, \frac{0+0+4\times3}{12}\right)$$

$= (1, 1)$

23. (a): $I = \int \frac{e^x(x+1-1)}{(x+1)^2}dx$

$= \int e^x\left[\frac{1}{x+1} - \frac{1}{(1+x)^2}\right]dx = e^x \cdot \frac{1}{x+1}$

{$\because$ Its of the form $e^x(f(x)+f'(x))$}

24. (a): Let the circle be $x^2+y^2+2gx+2fy+c=0$

Its radius $R = \sqrt{g^2+f^2-c}$ ... (1)

Since (1) passes through (1, 0), (0, 1)

$\therefore \quad 1+2g+c=0$ ... (2)

and $1+2f+c=0$ ... (3)

(2) – (3) gives $g=f$ and $c=-(1+2g)$

$\therefore \quad R = \sqrt{g^2+g^2+1+2g} = \sqrt{2g^2+2g+1}$

For R to be maximum or minimum.

$$\frac{d}{dg}(R^2)=0 \Rightarrow 4g+2=0 \Rightarrow g=-\frac{1}{2}$$

At $g=-\frac{1}{2}$, $\frac{d^2}{dg^2}(R^2)=4>0$

$\therefore \quad R$ is minimum at $g=-\frac{1}{2}$

Also when $g=-\frac{1}{2}$, $f=-\frac{1}{2}$ and

$1-1+c=0. \quad \therefore \quad c=0$

$\therefore$ Required circle is $x^2+y^2-x-y=0$

25. (b): $\alpha+\beta=-p$, $\alpha\beta=1$, $\gamma+\delta=-q$, $\gamma\delta=1$

$E = [\alpha\beta-\gamma(\alpha+\beta)+\gamma^2][\alpha\beta+(\alpha+\beta)\delta+\delta^2]$

$= [1+p\gamma+\gamma^2][1-p\delta+\delta^2]$

$= [-q\gamma+p\gamma][-q\delta-p\delta] = (q+p)(q-p)(\gamma\delta) = (q^2-p^2)\cdot 1$

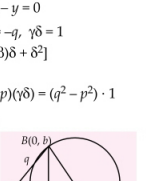
26. (c): Equation of the circle is

$x^2+y^2-ax-by=0$

Tangent at (0, 0) is

$ax+by=0$

$\Rightarrow \quad p=\frac{a^2}{\sqrt{a^2+b^2}}, q=\frac{b^2}{\sqrt{a^2+b^2}}$

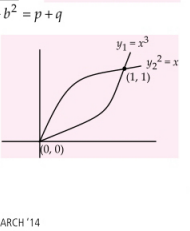


$\therefore \quad$ Diameter $AB = \sqrt{a^2+b^2} = p+q$

27. (a): Required area

$= \int_0^1 (y_1-y_2)dx$

$= \int_0^1 (\sqrt{x}-x^3)dx = \frac{5}{12}$



28. (b): The given equation can be written as

$(x-2)^2 = 3(y-2)$

Shifting the origin at (2, 2), this equation reduces to $X^2 = 3Y$, where $x = X+2$, $y = Y+2$.

The directrix of this parabola with reference to new axes is

$Y=-a$, where $a=\frac{3}{4} \Rightarrow y-2=-\frac{3}{4} \Rightarrow y=\frac{5}{4}$

29. (a): $16(\cos\alpha)^6 + 9(\cos\alpha)^2 = (4\cos^3\alpha)^2 + 9\cos^2\alpha$

$= (\cos3\alpha+3\cos\alpha)^2 + 9\cos^2\alpha$

$= (\cos3\alpha)^2 + 18\cos^2\alpha + 6\cos\alpha\cdot\cos3\alpha$

$= (\cos3\alpha)^2 + 18\cos^2\alpha + 3(\cos2\alpha+\cos4\alpha)$

$= (\cos3\alpha)^2 + 18\cos^2\alpha + 3\cos4\alpha + 3(2\cos^2\alpha-1)$

$= (\cos3\alpha)^2 + 24\cos^2\alpha + 3(\cos4\alpha-1)$

$= (\cos3\alpha)^2 + 24\cos^2\alpha - 6\sin^2 2\alpha$

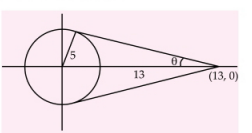
$= (\cos3\alpha)^2 + 24(\cos\alpha)^4$

$= a(\cos3\alpha)^2 + b(\cos\alpha)^4$ (given)

For it to be an identity : $a=1$, $b=24$

30. (c): Equation of line is $y=m(x-13)$

When, $p=r \Rightarrow m=\pm 5/12$



The common tangents are equally aligned to the axis.

$\therefore \quad$ Angle between them is $2\tan^{-1}\left(\frac{5}{12}\right)$.

31. (a): Put $n=1$; it only satisfies option (a).

32. (a): Let θ be the eccentric angle of the point P.

Then the coordinates of P are $(\sqrt{6}\cos\theta, \sqrt{2}\sin\theta)$.

The centre of the ellipse is at the origin. It is given that $OP=2$.

$\therefore \quad OP=2 \Rightarrow \sqrt{6\cos^2\theta+2\sin^2\theta}=2;$

$\Rightarrow \quad 6\cos^2\theta+2\sin^2\theta=4 \Rightarrow 3\cos^2\theta+\sin^2\theta=2$

$\Rightarrow \quad 2\sin^2\theta=1 \Rightarrow \sin^2\theta=\frac{1}{2} \Rightarrow \sin\theta=\pm\frac{1}{\sqrt{2}}$

$\Rightarrow \quad \theta=\pm\frac{\pi}{4}$

33. (b): The smallest number of people

= total number of possible forecasts

= total number of possible results

$= 3\times3\times3\times3\times3=243$

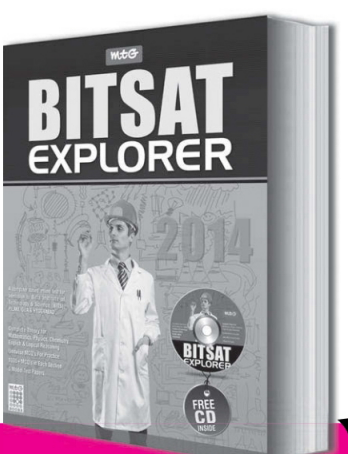
mtg

Crack BITSAT with ease

The BITSAT isn't a tough exam. It is only different from the others you will appear for. The book explores everything you need to crack BITSAT like a genius!

FREE CD INSIDE with

5 Model Test Papers





Complete Theory for

Mathematics, Physics, Chemistry
Logical Reasoning & English

Chapterwise MCQs for practice

1000+ MCQs in each section

5 Model Test Papers

₹ 525

Buy online at www.mtg.in

Available at all leading bookshops throughout India.

mtG BOOKS

For more information or to place your order :
Call at : 0124 - 4951200 @ Mail at : info@mtg.in
Log on to : www.mtg.in

34. (b): Probability that person A will die in 30 years $= \frac{8}{8+5}$

$P(A) = \frac{8}{13}$, $P(\bar{A}) = \frac{5}{13}$, $P(B) = \frac{4}{7}$, $P(\bar{B}) = \frac{3}{7}$

There are two ways in which 1 person is alive after 30 years. $\bar{A}B$ and $A\bar{B}$ are independent events.

$\therefore$ Required probability $= P(\bar{A})P(B) + P(A)P(\bar{B}) = \frac{44}{91}$

35. (a): $\int \frac{dy}{1+y^2} = \int \frac{dx}{1+x^2}$

$\Rightarrow \tan^{-1}y = \tan^{-1}x + c$

$\Rightarrow \tan^{-1}y - \tan^{-1}x = \tan^{-1}K$

$\Rightarrow \tan^{-1}\left(\frac{y-x}{1+xy}\right) = \tan^{-1}K \Rightarrow \frac{y-x}{1+xy} = K$

36. (b): $z^7 = -1$.

$\therefore$ Expression $= (z^7)^{12} \cdot z^2 + (z^7)^{25} + (z^7)^{41} \cdot z^2$

$= (-1)^{12} \cdot z^2 + (-1)^{25} + (-1)^{41} \cdot z^2$

$= z^2 - 1 - z^2 = -1$

37. (a): Each ball can be put in 2 ways. (Either in one box or the other)

$\therefore$ 6 balls can be put in 2^6 ways.

But, in two of the ways one box is empty.

So, total ways $= 2^6 - 2 = 62$

38. (a): $1008 = 2^4 \times 3^2 \times 7$

$\therefore$ The required number of proper divisors = total number of selection of at least one 2's and any number of 3's and 7's

$= 4 \times (2+1) \times (1+1) - 1$

39. (a): $\frac{dy}{dx} - y = 3 - 2x$

$\Rightarrow e^{-x}\frac{dy}{dx} - y \cdot e^{-x} = (3-2x)e^{-x}$

Buy MTG Books Online from www.mtg.in

get 25% on orders above Rs. 999/-

Apply coupon code - DISCOUNT25

OFF*

MTG Books are also available on

*offer not applicable on combos

www.flipkart.com | www.amazon.in | www.snapdeal.com

$\Rightarrow \frac{d(ye^{-x})}{dx} = (3-2x)e^{-x}$

$\Rightarrow \int d(ye^{-x}) = \int (3-2x)\,e^{-x}dx$

$\Rightarrow y = [-3 + 2(x+1)] + c$

$\therefore$ $y(0) = -3 + 2 + c = -1 \Rightarrow c = 0$

$\therefore$ $y + 3 = 2x + 2$ is the solution which represents a straight line.

40. (c): $\frac{{}^nC_{r+1}}{{}^nC_r} = \frac{56}{28} \Rightarrow \frac{n-r}{r+1} = 2 \Rightarrow n = 3r+2$

Similarly, $\frac{{}^nC_{r+2}}{{}^nC_{r+1}} = \frac{70}{56} \Rightarrow \frac{n-(r+1)}{r+2} = \frac{5}{4}$

$\Rightarrow \frac{(3r+2)-(r+1)}{r+2} = \frac{5}{4}$

$\therefore$ $r = 2$ and $n = 8$

41. (d): $f'(x) = A\frac{\pi}{2}\cos\left(\frac{\pi x}{2}\right)$

$\Rightarrow \sqrt{2} = f'(1/2) = A\frac{\pi}{2}\left(\frac{1}{\sqrt{2}}\right) \Rightarrow A = \frac{4}{\pi}$

Now, $\int_0^1 f(x)dx = \frac{2A}{\pi} \Rightarrow \frac{2A}{\pi} = \frac{4}{\pi}\int_0^1 \sin\left(\frac{\pi}{2}x\right)dx + \int_0^1 B\,dx$

$\Rightarrow \frac{2A}{\pi} = -\frac{4}{\pi}\frac{\cos(\pi x/2)}{\pi/2}\Big|_0^1 + B$

$\Rightarrow \frac{2A}{\pi} = \frac{4}{\pi}\times\frac{2}{\pi} + B \Rightarrow B = 0$

42. (d): Assume $x = n + R$, $n \in N$ and R is a fraction < 1

$\therefore L = \lim_{n\to\infty} \frac{\log n}{n+R}\left(\frac{\infty}{\infty}\right)$

Apply L'Hospital's rule : $L = \lim_{n\to\infty}\left(\frac{1/n}{1}\right) = 0$

43. (b): Given $yy' = \frac{y}{y'} \Rightarrow y'^2 = 1$

Length of tangent is $\frac{y}{y'}\sqrt{1+(y')^2} = |\sqrt{2}y|$

44. (c): $\cos^2A + \cos^2B + \cos^2C = 1$

$\Rightarrow \cos A = \cos B = \cos C = \frac{1}{\sqrt{3}}$

45. (c): $[\vec{u}\ \vec{v}\ \vec{w}] = \vec{u}\cdot(\vec{v}\times\vec{w}) \le |\vec{u}|\,|\vec{v}\times\vec{w}| = |\vec{v}\times\vec{w}|$

But $\vec{v}\times\vec{w} = 3\hat{i} - 7\hat{j} - \hat{k} \Rightarrow |\vec{v}\times\vec{w}| = \sqrt{59}$

$\therefore$ $[\vec{u}\ \vec{v}\ \vec{w}] \le \sqrt{59}$

SAMPLE PAPER

CBSE Board 2014

Time : 3 hrs. Marks : 100

GENERAL INSTRUCTIONS

(i) All questions are compulsory.

(ii) The question paper consists of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, section B comprises of 12 questions of four marks each and section C comprises of 7 questions of six marks each.

(iii) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

(iv) There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculators is not permitted. You may ask for logarithmic tables, if required.

SECTION - A

1. Find the principal value of $\sin^{-1}\left(\frac{1}{2}\right)$.

2. Write the range of the principal branch of $\sec^{-1}(x)$ defined on the domain $R - (-1, 1)$.

3. Find x, if $\begin{bmatrix} 5 & 3x \\ 2y & z \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 12 & 6 \end{bmatrix}^T$

4. Differentiate e^{-3x} w.r.t. x.

5. Determine the order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right) = \sqrt{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}}$.

6. If A is a square matrix satisfying $A^2 = I$, then what is the inverse of A?

7. Find the projection of $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ on $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$.

8. Find the direction cosines of a line whose direction ratios are 2, – 6, 3.

9. Let $\vec{a}$ and $\vec{b}$ be two given vectors such that $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = \sqrt{6}$. Find the angle between $\vec{a}$ and $\vec{b}$.

10. A square matrix A, of order 3, has $|A| = 5$, find $|A \cdot \text{adj} A|$

SECTION - B

11. Let $A = \{x \in Z : 0 \le x \le 12\}$. Show that the relation $R = \{(a, b) : |a - b| \text{ is a multiple of } 4\}$ is (i) reflexive (ii) symmetric (iii) transitive.

12. Prove that : $\cos\{\tan^{-1}\{\sin(\cot^{-1}x)\}\} = \sqrt{\frac{x^2+1}{x^2+2}}$

OR

If $y = \cot^{-1}\sqrt{\cos x} - \tan^{-1}\left(\sqrt{\cos x}\right)$, prove that $\sin y = \tan^2\frac{x}{2}$.

13. Prove that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc + bc + ca + ab$

14. Differentiate the following function with respect to x : $\log\left(\frac{a + b\sin x}{a - b\sin x}\right)$

15. If $xy = 4$, prove that $x\left(\frac{dy}{dx} + y^2\right) = 3y$

16. If the function $f(x)$ given by

$$f(x) = \begin{cases} 3ax + b, & \text{if } x > 1 \\ 11, & \text{if } x = 1 \\ 5ax - 2b, & \text{if } x < 1 \end{cases}$$

is continuous at $x = 1$, find the values of a and b.

OR

Differentiate $\tan^{-1}\left\{\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right\}$, with respect to $\cos^{-1}x^2$.

17. Evaluate: $\int x\tan^{-1}x\,dx$.

OR

$$\int\left(\frac{\sin x}{1+\sin x}\right)dx$$

18. Evaluate : $\int\frac{(x^2+4)}{(x^4+16)}dx$.

19. Prove that : $\int_0^{\pi} x\sin^3 x\,dx=\frac{2\pi}{3}$.

20. If $\vec{a}\cdot\vec{b}=\vec{a}\cdot\vec{c}$, $\vec{a}\times\vec{b}=\vec{a}\times\vec{c}$ and $\vec{a}\neq\vec{0}$, then prove that $\vec{b}=\vec{c}$.

21. Find the equation of the plane passing through the points (0, 0, 0) and (3, –1, 2) and parallel to the line $\frac{x-4}{1}=\frac{y+3}{-4}=\frac{z+1}{7}$.

OR

Find the length and the coordinates of the foot of the perpendicular from the point (7, 14, 5) to the plane $2x+4y-z=2$.

22. A clever student used a biased coin so that the head is 3 times as likely to occur as tail. If the coin is tossed twice, find the probability distribution and mean of numbers of tails. Is this a good tendency? Justify your answer.

SECTION-C

23. If the sum of the length of the hypotenuse and a side of a right-angled triangle is given, show that the area of the triangle is maximum when the angle between them is $(\pi/3)$.

OR

Show that the cone of greatest volume which can be inscribed in a given sphere is such that three times its altitude is twice the diameter of the sphere. Find the volume of the largest cone inscribed in a sphere of radius R.

24. Find the area of the region
$\{(x, y) : 0\le y\le(x^2+1),\ 0\le y\le(x+1),\ 0\le x\le 2\}$.

25. Using matrices, solve the following system of equations:

$$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4;\ \frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1;\ \frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2.$$

26. Solve $\frac{dy}{dx}-2y=\cos 3x$.

27. Show that the lines $\frac{x}{1}=\frac{y-2}{2}=\frac{z+3}{3}$ and $\frac{x-2}{2}=\frac{y-6}{3}=\frac{z-3}{4}$ are coplanar. Also, find the equation of the plane containing these lines.

28. A company manufactures two types of stickers A : "SAVE ENVIRONMENT" and B : "BE COURTEOUS". Type A requires 5 minutes each for cutting and 10 minutes each for assembling. Type B requires 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours and 20 minutes available for cutting and 4 hours available for assembling in a day. He earns a profit of ₹ 50 on each type A and ₹ 60 on each type B. How many stickers of each type should the company manufacture in a day to maximize profit? Give your views about "SAVE ENVIRONMENT" and "BE COURTEOUS".

29. Two bags A and B contain 4 white and 3 yellow bulbs and 2 white and 2 yellow bulbs respectively. From bag A, two bulbs are drawn at random and then transferred to bag B. A bulb is then drawn from bag B and is found to be a yellow bulb. What is the probability that the transferred bulbs were 1 white and 1 yellow? Write two advantages of using CFL bulbs over incandescent bulbs.

OR

In an examination, 10 questions of true-false type are asked. A student tosses a fair coin to determine his answer to each question. If the coin falls heads, he answers 'true' and if it falls tails, he answers 'false'. Find the probability that he answers at most 7 questions correctly. Give two advantages of having an exam.

SOLUTIONS

1. $\therefore\quad \sin^{-1}\left(\frac{1}{2}\right)=$ An angle in $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ whose sine is $\frac{1}{2}$

$\Rightarrow\quad \sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{6}$

2. The range is $\left[0,\frac{\pi}{2}\right)\cup\left(\frac{\pi}{2},\pi\right]$

3. $\begin{bmatrix} 5 & 3x \\ 2y & z \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 12 & 6 \end{bmatrix}^T \Rightarrow \begin{bmatrix} 5 & 3x \\ 2y & z \end{bmatrix} = \begin{bmatrix} 5 & 12 \\ 4 & 6 \end{bmatrix}$

On comparing, we get $3x = 12 \Rightarrow x = 4$

4. Let $y = e^{-3x}$

$\Rightarrow \quad \frac{dy}{dx} = -3e^{-3x}$.

5. The given equation can be written as

$$\left(\frac{d^2y}{dx^2}\right)^2 = 1 + \left(\frac{dy}{dx}\right)^2.$$

Clearly, it is an equation of order 2 and degree 2.

6. $A^{-1} = A$

7. Projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

$$= \frac{(2\hat{i} - \hat{j} + \hat{k})\cdot(\hat{i} - 2\hat{j} + \hat{k})}{\sqrt{1^2 + (-2)^2 + 1^2}} = \frac{(2+2+1)}{\sqrt{6}} = \frac{5}{\sqrt{6}}.$$

8. Here, $a = 2$, $b = -6$, $c = 3$.

$\therefore$ d.c.'s are $\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}$,

i.e., $\frac{2}{\sqrt{2^2+(-6)^2+3^2}}, \frac{-6}{\sqrt{2^2+(-6)^2+3^2}}, \frac{3}{\sqrt{2^2+(-6)^2+3^2}}$,

i.e., $\frac{2}{7}, \frac{-6}{7}, \frac{3}{7}$.

9. Let θ be the angle between $\vec{a}$ and $\vec{b}$. Then,

$\vec{a}\cdot\vec{b} = \sqrt{6} \Rightarrow |\vec{a}||\vec{b}|\cos\theta = \sqrt{6}$

$\Rightarrow \quad (\sqrt{3})(2)\cos\theta = \sqrt{6}$

$\Rightarrow \quad \cos\theta = \frac{\sqrt{6}}{2\sqrt{3}} = \frac{1}{\sqrt{2}} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$.

Hence, the required angle is $\frac{\pi}{4}$.

10. $A\cdot\text{adj } A = |A| \cdot I$

$|A\cdot\text{adj } A| = ||A| \cdot I|$

$\Rightarrow \quad |A\cdot\text{adj}A| = |A|^3 \cdot |I| = (5)^3 \cdot 1 = 125$

11. (i) $|a - a| = 0$, which is a multiple of 4. So, $a\ R\ a$. So, R is reflexive.

(ii) $a\ R\ b \Rightarrow |a - b|$ is a multiple of 4

$\Rightarrow \quad |-(b - a)|$ is a multiple of 4

$\Rightarrow \quad |b - a|$ is a multiple of $4 \Rightarrow b\ R\ a$.

$\therefore \quad R$ is symmetric.

(iii) $aRb,\ bRc \Rightarrow |a - b|$ is a multiple of 4 and $|b - c|$ is a multiple of 4.

Let $|a - b| = 4k_1$ and $|b - c| = 4k_2$. Then,

$|a - c| = |(a - b) - (c - b)| = |4k_1 - 4k_2| = 4|(k_1 - k_2)|$

$= 4|k_1 - k_2|$, which is a muliple of 4.

So, R is transitive.

12. We have,

$$\sin(\cot^{-1}x) = \sin\left\{\sin^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)\right\} = \frac{1}{\sqrt{1+x^2}}$$

$\therefore \quad \cos[\tan^{-1}\{\sin(\cot^{-1}x)\}]$

$$= \cos\left\{\tan^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)\right\}$$

$$= \cos\left\{\cos^{-1}\left(\frac{\sqrt{1+x^2}}{\sqrt{2+x^2}}\right)\right\} = \sqrt{\frac{1+x^2}{2+x^2}} = \sqrt{\frac{x^2+1}{x^2+2}}$$

OR

We have, $y = \cot^{-1}(\sqrt{\cos x}) - \tan^{-1}(\sqrt{\cos x})$

$\Rightarrow \quad y = \frac{\pi}{2} - \tan^{-1}(\sqrt{\cos x}) - \tan^{-1}(\sqrt{\cos x})$

$\Rightarrow \quad y = \frac{\pi}{2} - 2\tan^{-1}(\sqrt{\cos x})$

$$\Rightarrow \quad y = \frac{\pi}{2} - \cos^{-1}\left\{\frac{1-(\sqrt{\cos x})^2}{1+(\sqrt{\cos x})^2}\right\}$$

$$\left[\because \quad 2\tan^{-1}x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right]$$

$$\Rightarrow \quad y = \frac{\pi}{2} - \cos^{-1}\left(\frac{1-\cos x}{1+\cos x}\right)$$

$$\Rightarrow \quad y = \frac{\pi}{2} - \cos^{-1}\left(\tan^2\frac{x}{2}\right) \quad \Rightarrow \quad \cos^{-1}\left(\tan^2\frac{x}{2}\right) = \frac{\pi}{2} - y$$

$$\Rightarrow \quad \tan^2\frac{x}{2} = \cos\left(\frac{\pi}{2} - y\right) \quad \Rightarrow \quad \tan^2\frac{x}{2} = \sin y.$$

13. Let $\Delta = \begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$.

Taking a, b and c common from C_1, C_2 and C_3 respectively, we get,

$$\Delta = abc \begin{vmatrix} 1+1/a & 1/b & 1/c \\ 1/a & 1+1/b & 1/c \\ 1/a & 1/b & 1+1/c \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2 + C_3$

$$\Delta = abc \begin{vmatrix} 1+1/a+1/b+1/c & 1/b & 1/c \\ 1+1/a+1/b+1/c & 1+1/b & 1/c \\ 1+1/a+1/b+1/c & 1/b & 1+1/c \end{vmatrix}$$

Taking $\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$ common from C_1

$$\Rightarrow \quad \Delta = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix} 1 & 1/b & 1/c \\ 1 & 1+1/b & 1/c \\ 1 & 1/b & 1+1/c \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$\Rightarrow \quad \Delta = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix} 1 & 1/b & 1/c \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Expanding along C_1

$$\Rightarrow \quad \Delta = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\cdot 1\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

$$\Rightarrow \quad \Delta = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \text{ or } \Delta = (abc+bc+ca+ab)$$

14. Let $y = \log\left(\frac{a+b\sin x}{a-b\sin x}\right)$.

Then, $y = \log(a+b\sin x) - \log(a-b\sin x)$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{d}{dx}\{\log(a+b\sin x)\} - \frac{d}{dx}\{\log(a-b\sin x)\}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{a+b\sin x}\times\frac{d}{dx}(a+b\sin x) - \left\{\frac{1}{a-b\sin x} \times \frac{d}{dx}(a-b\sin x)\right\}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{a+b\sin x}(0+b\cos x) - \frac{1}{a-b\sin x}(0-b\cos x)$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{b\cos x}{a+b\sin x} + \frac{b\cos x}{a-b\sin x}$$

$$= b\cos x\left\{\frac{1}{a+b\sin x} + \frac{1}{a-b\sin x}\right\}$$

$$\Rightarrow \quad \frac{dy}{dx} = b\cos x\left\{\frac{a-b\sin x + a+b\sin x}{(a+b\sin x)(a-b\sin x)}\right\} = \frac{2ab\cos x}{a^2-b^2\sin^2 x}$$

15. We have $xy = 4$ ...(i)

Differentiating (i) w.r.t. x, we get

$$x\frac{dy}{dx} + y = 0$$

Adding $3y$ both the sides, we get

$$x\frac{dy}{dx} + 4y = 3y$$

$$\Rightarrow \quad x\frac{dy}{dx} + xy \times y = 3y \qquad \{\text{from(i)}\}$$

$$\Rightarrow \quad x\left(\frac{dy}{dx} + y^2\right) = 3y$$

16. We have,

$$\text{(L.H.L. at } x = 1) = \lim_{x\to 1^-} f(x)$$

$$= \lim_{x\to 1^-}(5ax - 2b) = 5a - 2b$$

$$\text{(R.H.L. at } x = 1) = \lim_{x\to 1^+} f(x)$$

$$= \lim_{x\to 1^+}(3ax + b) = 3a + b$$

and, $f(1) = 11$.

Since $f(x)$ is continuous at $x = 1$. Therefore,

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(1)$$

$$\Rightarrow \quad 5a - 2b = 3a + b = 11$$

$$\Rightarrow \quad 5a - 2b = 11 \text{ and } 3a + b = 11$$

$$\Rightarrow \quad a = 3 \text{ and } b = 2$$

OR

Let $u = \tan^{-1}\left\{\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right\}$ and $v = \cos^{-1}x^2$.

Putting $x^2 = \cos\theta$, we get

$$u = \tan^{-1}\left\{\frac{\sqrt{1+\cos\theta}-\sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta}+\sqrt{1-\cos\theta}}\right\}$$

$$\Rightarrow \quad u = \tan^{-1}\left\{\frac{\sqrt{2\cos^2(\theta/2)}-\sqrt{2\sin^2(\theta/2)}}{\sqrt{2\cos^2(\theta/2)}+\sqrt{2\sin^2(\theta/2)}}\right\}$$

$$\Rightarrow \quad u = \tan^{-1}\left\{\frac{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}\right\} \Rightarrow u = \tan^{-1}\left\{\frac{1-\tan\frac{\theta}{2}}{1+\tan\frac{\theta}{2}}\right\}$$

$$\Rightarrow \quad u = \tan^{-1}\left\{\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)\right\}$$

$$\Rightarrow \quad u = \frac{\pi}{4} - \frac{1}{2}\theta = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x^2$$

$$[\because \quad x^2 = \cos\theta \Rightarrow \theta = \cos^{-1}x^2]$$

$$\therefore \quad \frac{du}{dx} = -\frac{1}{2}\times\frac{-2x}{\sqrt{1-x^4}} = \frac{x}{\sqrt{1-x^4}}$$

and $v = \cos^{-1}x^2 \Rightarrow \frac{dv}{dx} = \frac{-2x}{\sqrt{1-x^4}}$

So, $\frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}} = -\frac{1}{2}$

17. Integrating by parts, taking $\tan^{-1}x$ as the first function, we get

$$\int x\tan^{-1}x\,dx = (\tan^{-1}x)\cdot\int x dx - \int\left\{\frac{d}{dx}(\tan^{-1}x)\cdot\int x dx\right\}dx$$

$$= (\tan^{-1}x)\cdot\frac{x^2}{2} - \int\frac{1}{(1+x^2)}\cdot\frac{x^2}{2}dx$$

$$= \frac{x^2\tan^{-1}x}{2} - \frac{1}{2}\int\left(1-\frac{1}{(1+x^2)}\right)dx$$

[on dividing x^2 by $1+x^2$]

$$= \frac{x^2\tan^{-1}x}{2} - \frac{1}{2}\int dx + \frac{1}{2}\int\frac{1}{(1+x^2)}dx$$

$$= \frac{x^2\tan^{-1}x}{2} - \frac{x}{2} + \frac{1}{2}\tan^{-1}x + C$$

$$= \frac{1}{2}(1+x^2)\tan^{-1}x - \frac{1}{2}x + C.$$

OR

$$\int\left(\frac{\sin x}{1+\sin x}\right)dx = \int\frac{(1+\sin x)-1}{(1+\sin x)}dx$$

$$\int\left(1-\frac{1}{1+\sin x}\right)dx = \int dx - \int\frac{1}{(1+\sin x)}dx$$

$$= \int dx - \int\frac{1}{(1+\sin x)}\times\frac{(1-\sin x)}{(1-\sin x)}dx$$

$$= \int dx - \int\frac{(1-\sin x)}{\cos^2 x}dx = \int dx - \int\left(\frac{1}{\cos^2 x} - \frac{\sin x}{\cos^2 x}\right)dx$$

$$= \int dx - \int\sec^2 x\,dx + \int\sec x\tan x\,dx = x - \tan x + \sec x + C.$$

18. We have

$$\int\left(\frac{x^2+4}{x^4+16}\right)dx = \int\frac{\left(1+\frac{4}{x^2}\right)}{\left(x^2+\frac{16}{x^2}\right)}dx$$

[dividing numerator and denominator by x^2]

$$= \int\frac{\left(1+\frac{4}{x^2}\right)}{\left(x-\frac{4}{x}\right)^2+8}dx = \int\frac{dt}{t^2+8}$$

$$\left[\text{putting } \left(x-\frac{4}{x}\right)=t \Rightarrow \left(1+\frac{4}{x^2}\right)dx = dt\right]$$

$$= \int\frac{dt}{t^2+(\sqrt{8})^2} = \frac{1}{\sqrt{8}}\tan^{-1}\left(\frac{t}{\sqrt{8}}\right)+C$$

$$= \frac{1}{\sqrt{8}}\tan^{-1}\left(\frac{x-\frac{4}{x}}{\sqrt{8}}\right)+C = \frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{x^2-4}{2\sqrt{2}x}\right)+C.$$

19. Let $I = \int_0^{\pi} x\sin^3 x\,dx$ ...(i)

Then, $I = \int_0^{\pi}(\pi-x)\sin^3(\pi-x)dx$

or $I = \int_0^{\pi}(\pi-x)\sin^3 x\,dx$ ...(ii)

Adding (i) and (ii), we get

$$2I = \int_0^{\pi}\pi\sin^3 x\,dx = \pi\int_0^{\pi}\sin^2 x\cdot\sin x\,dx$$

$$= \pi\int_0^{\pi}(1-\cos^2 x)\sin x\,dx$$

$$= -\pi\int_1^{-1}(1-t^2)dt, \text{ where } \cos x = t$$

$$[x=0 \Rightarrow t=1 \text{ and } x=\pi \Rightarrow t=-1]$$

$$= \pi\int_{-1}^{1}(1-t^2)dt = \pi\left[t-\frac{t^3}{3}\right]_{-1}^{1} = \frac{4\pi}{3}.$$

Hence, $I = \frac{2\pi}{3} \Rightarrow \int_0^{\pi} x\sin^3 x\,dx = \frac{2\pi}{3}$.

20. $\vec{a}\cdot\vec{b} = \vec{a}\cdot\vec{c}$ and $\vec{a}\neq\vec{0}$

$\Rightarrow$ $\vec{a}\cdot\vec{b} - \vec{a}\cdot\vec{c} = 0$ and $\vec{a}\neq\vec{0}$

$\Rightarrow$ $\vec{a}\cdot(\vec{b}-\vec{c}) = 0$ and $\vec{a}\neq\vec{0}$

$\Rightarrow$ $\vec{b}-\vec{c} = 0$ or $\vec{a}\perp(\vec{b}-\vec{c})$

$\Rightarrow$ $\vec{b} = \vec{c}$ or $\vec{a}\perp(\vec{b}-\vec{c})$ ...(i)

Again, $\vec{a}\times\vec{b} = \vec{a}\times\vec{c}$ and $\vec{a}\neq\vec{0}$

$\Rightarrow$ $(\vec{a}\times\vec{b}) - (\vec{a}\times\vec{c}) = \vec{0}$ and $\vec{a}\neq\vec{0}$

$\Rightarrow$ $\vec{a}\times(\vec{b}-\vec{c}) = \vec{0}$ and $\vec{a}\neq\vec{0}$

$\Rightarrow$ $(\vec{b}-\vec{c}) = \vec{0}$ or $\vec{a}\,||\,(\vec{b}-\vec{c})$

$\Rightarrow$ $\vec{b} = \vec{c}$ or $\vec{a}\,||\,(\vec{b}-\vec{c})$ ...(ii)

From (i) and (ii), we get $\vec{b} = \vec{c}$

[$\because$ $\vec{a}\perp(\vec{b}-\vec{c})$ and $\vec{a}\,||\,(\vec{b}-\vec{c})$ both cannot hold simultaneously].

21. Any plane passes through (0, 0, 0) is

$a(x-0)+b(y-0)+c(z-0) = 0 \Rightarrow ax+by+cz = 0$ ...(i)

If this plane passes through (3, –1, 2), we have

$3a - b + 2c = 0$ ...(ii)

Also, if the plane (i) is parallel to the given line, then the normal to this plane is perpendicular to the given line.

$\therefore$ $a\times 1 + b\times(-4) + c\times 7 = 0 \Rightarrow a - 4b + 7c = 0$ ...(iii)

Cross multiplying (ii) and (iii), we have

$$\frac{a}{(-7+8)} = \frac{b}{(2-21)} = \frac{c}{(-12+1)} = k(\text{say})$$

$\Rightarrow \quad \frac{a}{1} = \frac{b}{-19} = \frac{c}{-11} = k$

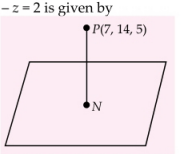
$\Rightarrow \quad a = k,\ b = -19k$ and $c = -11k$.

Substituting $a = k$, $b = -19k$ and $c = -11k$ in (i), we get the required equation of the plane as

$kx - 19ky - 11kz = 0 \Leftrightarrow x - 19y - 11z = 0.$

OR

Any line through $P(7, 14, 5)$ and perpendicular to the plane $2x + 4y - z = 2$ is given by



$$\frac{x-7}{2} = \frac{y-14}{4} = \frac{z-5}{-1} = \lambda \text{(say)}.$$

Any point on this line is

$N(2\lambda + 7, 4\lambda + 14, -\lambda + 5)$.

If N is the foot of the perpendicular from P to the given plane then it must lie on the plane $2x + 4y - z = 2$.

$\therefore \quad 2(2\lambda + 7) + 4(4\lambda + 14) - (-\lambda + 5) = 2 \Rightarrow \lambda = -3.$

Thus, we get the point $N(1, 2, 8)$ on the plane.

Hence, the foot of the perpendicular from $P(7, 14, 5)$ to the given plane is $N(1, 2, 8)$.

Length of the perpendicular from P to the given plane

$= PN = \sqrt{(7-1)^2 + (14-2)^2 + (5-8)^2} = \sqrt{189} = 3\sqrt{21}$ units.

22. $P(H) = 3/4$; $P(T) = 1/4$;

Event: Tails; $n = 2$; $x = 0, 1, 2$

$$P(0) = \frac{3}{4} \times \frac{3}{4} = \frac{9}{16};\ P(1) = \frac{3}{4} \times \frac{1}{4} \times 2 = \frac{6}{16};$$

$$P(2) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$$

$\therefore$ Probability distribution is

x	0	1	2
$P(x)$	$\frac{9}{16}$	$\frac{6}{16}$	$\frac{1}{16}$

$$\text{Mean} = \Sigma x P(x) = 0 \times \frac{9}{16} + 1 \times \frac{6}{16} + 2 \times \frac{1}{16} = 0 + \frac{6}{16} + \frac{2}{16}$$

$$= \frac{8}{16} = \frac{1}{2}$$

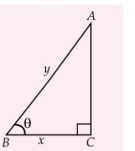
$\therefore \quad \text{Mean} = \frac{1}{2}$

Value :

1. No, it may be good once or twice but not forever.
2. Honesty pays in a long run.

23. Let us consider a right-angled triangle with base = x and hypotenuse = y. Let $x + y = k$, where k is a constant. Let θ be the angle between the base and the hypotenuse. Let A be the area of the triangle. Then,

$$A = \frac{1}{2} \times BC \times AC = \frac{1}{2} x\sqrt{y^2 - x^2}.$$



$$\therefore \quad A^2 = \frac{x^2(y^2 - x^2)}{4} = \frac{x^2[(k-x)^2 - x^2]}{4} \quad [\because\ y = (k - x)]$$

$$\text{or} \quad A^2 = \frac{k^2x^2 - 2kx^3}{4} \qquad \text{...(i)}$$

On differentiating (i), we get

$$2A \cdot \frac{dA}{dx} = \frac{2k^2x - 6kx^2}{4} \qquad \text{...(ii)}$$

$$\text{or} \quad \frac{dA}{dx} = \frac{k^2x - 3kx^2}{4A}$$

Now, $\frac{dA}{dx} = 0 \Rightarrow (k^2x - 3kx^2) = 0 \Rightarrow x = \frac{k}{3}$

[neglecting $x = 0$]

Now, differentiating (ii), we get

$$2\left(\frac{dA}{dx}\right)^2 + 2A \cdot \frac{d^2A}{dx^2} = \frac{2k^2 - 12kx}{4} \qquad \text{...(iii)}$$

Putting $\frac{dA}{dx} = 0$ and $x = \frac{k}{3}$ in (iii)

we get $\frac{d^2A}{dx^2} = \frac{-k^2}{4A} < 0.$

Thus, A is maximum when $x = (k/3)$.

Now, $x = \frac{k}{3} \Rightarrow y = \left(k - \frac{k}{3}\right) = \frac{2k}{3}.$

$\therefore \quad \frac{x}{y} = \cos\theta \Rightarrow \cos\theta = \frac{(k/3)}{(2k/3)} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}.$

Hence, the area is maximum when $\theta = \frac{\pi}{3}$.

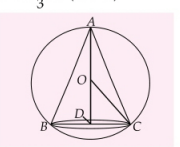
OR

Let R be the radius of the given sphere with centre O, and let V be the volume of the inscribed cone, h be its height and r be the radius of the base.

In the given figure, we have $OD = AD - AO = (h - R)$.

$\therefore \quad R^2 = (h - R)^2 + r^2$ or $r^2 = h(2R - h)$ ...(i)

Now, $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi h^2(2R - h)$ [using (i)].



$\therefore \quad \frac{dV}{dh} = \frac{1}{3}\pi h(4R - 3h)$, and $\frac{d^2V}{dh^2} = \left(\frac{4}{3}\pi R - 2\pi h\right)$.

For maxima or minima, we have $\frac{dV}{dh} = 0$.

Now, $\frac{dV}{dh} = 0 \Rightarrow \frac{1}{3}\pi h(4R - 3h) = 0$

$\Rightarrow \quad h = 0$ or $(4R - 3h) = 0 \Rightarrow h = \frac{4}{3}R$ [$\because \quad h \neq 0$].

And, $\left[\frac{d^2V}{dh^2}\right]_{h=(4/3)R} = -\frac{4\pi R}{3} < 0$.

So, V is maximum when $h = \frac{4}{3}R$ *i.e.*, when $3h = 2(2R)$

i.e., 3 times the height = 2 times the diameter.

Now, volume of the largest cone

$= \frac{1}{3}\pi \times \frac{16R^2}{9} \times \left(2R - \frac{4R}{3}\right) = \frac{32\pi R^3}{81}$

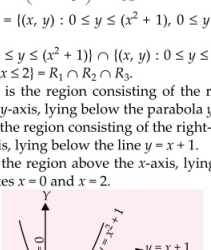
24. Let $R = \{(x, y) : 0 \le y \le (x^2 + 1),\ 0 \le y \le (x + 1),\ 0 \le x \le 2\}$

$= \{(x, y) : 0 \le y \le (x^2 + 1)\} \cap \{(x, y) : 0 \le y \le (x + 1)\} \cap \{(x, y) : 0 \le x \le 2\} = R_1 \cap R_2 \cap R_3$.

Clearly, R_1 is the region consisting of the right-hand side of the y-axis, lying below the parabola $y = x^2 + 1$.

Also, R_2 is the region consisting of the right-hand side of the y-axis, lying below the line $y = x + 1$.

And, R_3 is the region above the x-axis, lying between the ordinates $x = 0$ and $x = 2$.



Thus, $R_1 \cap R_2 \cap R_3$ is the shaded region.

We have, $y = x^2 + 1$ and $y = x + 1$

$\Rightarrow \quad x^2 + 1 = x + 1 \Rightarrow x(x - 1) = 0 \Rightarrow x = 0$ or $x = 1$.

Now, $(x = 0 \Rightarrow y = 1)$ and $(x = 1 \Rightarrow y = 2)$.

Thus, the parabola $y = (x^2 + 1)$ and the line $y = x + 1$ intersect at the points $A(0, 1)$ and $C(1, 2)$.

$\therefore$ Required area = Area of the shaded region

= (area $ODCBA$) + (area $CDEFC$)

$= \int_0^1 (y \text{ of the parabola})\,dx + \int_1^2 (y \text{ of the line})\,dx$

$= \int_0^1 (x^2 + 1)\,dx + \int_1^2 (x + 1)\,dx = \left[\frac{x^3}{3} + x\right]_0^1 + \left[\frac{x^2}{2} + x\right]_1^2$

$= \left(\frac{1}{3} + 1\right) + \left(4 - \frac{3}{2}\right) = \frac{23}{6}$ sq. units.

Hence, the required area is $\frac{23}{6}$ sq. units.

25. Putting $\frac{1}{x} = u$, $\frac{1}{y} = v$ and $\frac{1}{z} = w$, the given equations become,

$2u + 3v + 10w = 4$ ...(i)

$4u - 6v + 5w = 1$ ...(ii)

$6u + 9v - 20w = 2$ ...(iii)

Let $A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}$, $Y = \begin{bmatrix} u \\ v \\ w \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$.

Then, the given system in matrix form is $AY = B$.

Now, $|A| = \begin{vmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{vmatrix} = \begin{vmatrix} 2 & 3 & 10 \\ 0 & -12 & -15 \\ 0 & 0 & -50 \end{vmatrix}$

$[R_2 \to R_2 - 2R_1;\ R_3 \to R_3 - 3R_1]$

$= (-50) \cdot (-24 - 0) = 1200 \neq 0$.

Thus, A is invertible.

So, the given system has a unique solution, $Y = A^{-1}B$.

The minors of the elements of $|A|$ are

$M_{11} = 75$, $M_{12} = -110$, $M_{13} = 72$;

$M_{21} = -150$, $M_{22} = -100$, $M_{23} = 0$;

$M_{31} = 75$, $M_{32} = -30$, $M_{33} = -24$.

So, the cofactors of the elements of $|A|$ are

$A_{11} = 75$, $A_{12} = 110$, $A_{13} = 72$;

$A_{21} = 150$, $A_{22} = -100$, $A_{23} = 0$;

$A_{31} = 75$, $A_{32} = 30$, $A_{33} = -24$.

$\therefore \quad (\text{adj } A) = \begin{bmatrix} 75 & 110 & 72 \\ 150 & -100 & 0 \\ 75 & 30 & -24 \end{bmatrix}^T = \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$.

$\therefore \quad A^{-1} = \frac{1}{|A|} \cdot (\text{adj } A) = \frac{1}{1200} \cdot \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$.

$\therefore \quad Y = A^{-1}B$

$\Rightarrow \quad \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{1200} \cdot \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$

$$= \frac{1}{1200}\cdot\begin{bmatrix} 300+150+150 \\ 440-100+60 \\ 288+0-48 \end{bmatrix} = \frac{1}{1200}\cdot\begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix}$$

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} \frac{600}{1200} \\ \frac{400}{1200} \\ \frac{240}{1200} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$$

$\Rightarrow \quad u = \frac{1}{2}, v = \frac{1}{3}, w = \frac{1}{5} \Rightarrow \frac{1}{x} = \frac{1}{2}, \frac{1}{y} = \frac{1}{3}$ and $\frac{1}{z} = \frac{1}{5}$

Hence $x = 2$, $y = 3$ and $z = 5$.

26. The given equation is of the form

$$\frac{dy}{dx} + Py = Q, \text{ where } P = -2 \text{ and } Q = \cos 3x.$$

Thus, it is a linear equation.

$$\text{IF} = e^{\int P dx} = e^{\int -2dx} = e^{-2x}.$$

So, the required solution is

$$y \times \text{IF} = \int \{Q \times (\text{IF})\} dx + C,$$

$$i.e., \; y \times e^{-2x} = \int e^{-2x} \cos 3x \, dx + C \qquad ...(i)$$

Now, we have

$$I = \int e^{-2x} \cos 3x \, dx$$

$$= e^{-2x} \cdot \frac{\sin 3x}{3} + 2\int e^{-2x} \cdot \frac{\sin 3x}{3} dx$$

[Integrating by parts]

$$= \frac{1}{3} e^{-2x} \sin 3x + \frac{2}{3}\int e^{-2x} \sin 3x \, dx$$

$$= \frac{1}{3} e^{-2x} \sin 3x + \frac{2}{3}\cdot\left[e^{-2x} \cdot \frac{(-\cos 3x)}{3} + 2\int e^{-2x} \frac{(-\cos 3x)}{3} dx \right]$$

[Integrating by parts]

$$= \frac{1}{3} e^{-2x} \sin 3x - \frac{2}{9} e^{-2x} \cos 3x - \frac{4}{9}\int e^{-2x} \cos 3x \, dx$$

$$= \frac{1}{3} e^{-2x} \sin 3x - \frac{2}{9} e^{-2x} \cos 3x - \frac{4}{9} I$$

$$\Rightarrow \quad \left(\frac{13}{9}\right) I = \frac{1}{3} e^{-2x} \sin 3x - \frac{2}{9} e^{-2x} \cos 3x$$

$$\Rightarrow \quad I = \frac{3(e^{-2x} \sin 3x)}{13} - \frac{2(e^{-2x} \cos 3x)}{13}.$$

Putting this value in (i), we get

$$ye^{-2x} = \frac{3(e^{-2x} \sin 3x)}{13} - \frac{2(e^{-2x} \cos 3x)}{13} + C$$

$$\Rightarrow \quad y = \frac{(3\sin 3x - 2\cos 3x)}{13} + Ce^{2x},$$

which is the required solution.

27. We know that the lines

$$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1} \text{ and } \frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$$

are coplanar $\Leftrightarrow \begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0,$

and the equation of the plane containing these lines is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0.$$

Here, $x_1 = 0$, $y_1 = 2$, $z_1 = -3$; $x_2 = 2$, $y_2 = 6$, $z_2 = 3$; $a_1 = 1$, $b_1 = 2$, $c_1 = 3$; $a_2 = 2$, $b_2 = 3$, $c_2 = 4$.

$$\therefore \quad \begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} 2 & 4 & 6 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{vmatrix} = 0.$$

Hence, the two given lines are coplanar.

The equation of the plane containing these lines is

$$\begin{vmatrix} x-0 & y-2 & z+3 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{vmatrix} = 0 \Leftrightarrow \begin{vmatrix} x & y-2 & z+3 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{vmatrix} = 0$$

$$\Rightarrow \quad x(8-9) - (y-2)(4-6) + (z+3)(3-4) = 0$$

$$\Rightarrow \quad -x + 2(y-2) - (z+3) = 0 \Rightarrow x - 2y + z + 7 = 0$$

28. The given problem can be formulated as;

Max. $Z = 50x + 60y$

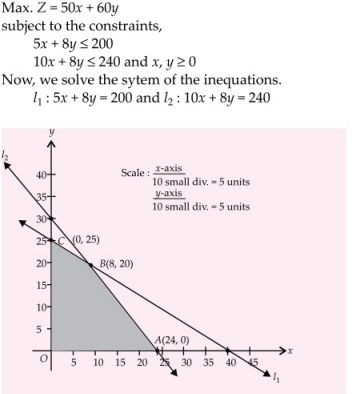
subject to the constraints,

$5x + 8y \le 200$

$10x + 8y \le 240$ and $x, y \ge 0$

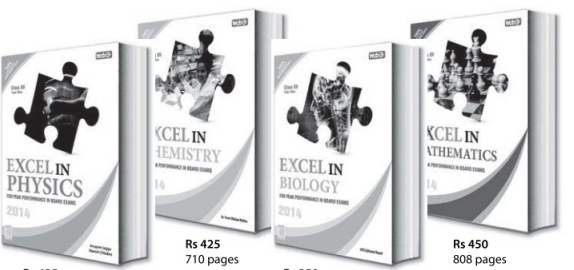
Now, we solve the sytem of the inequations.

$l_1 : 5x + 8y = 200$ and $l_2 : 10x + 8y = 240$



The shaded region in the graph given above is the feasible region.

Concerned about your performance in Class XII Boards?



Rs 425
720 pages

Rs 425
710 pages

Rs 350
496 pages

Rs 450
808 pages

Well, fear no more, help is at hand.....
To excel, studying in right direction is more important than studying hard. Which is why we created the Excel Series. These books – for Physics, Chemistry, Biology & Mathematics – have been put together totally keeping in mind the prescribed syllabus and the pattern of CBSE's Board examinations, so that students prepare and practice with just the right study material to excel in board exams.

Did you know nearly all questions in CBSE's 2013 Board Examination were a part of our Excel books? That too fully solved ?!

HIGHLIGHTS:

- Comprehensive theory strictly based on NCERT, complemented with illustrations, activities and solutions of NCERT questions
- Practice questions & Model Test Papers for Board Exams
- Latest CBSE Board Sample Papers (Solved)
- Previous years' CBSE Board Examination Papers (Solved)

*Scan now with your smartphone or tablet**

Visit www.mtg.in for latest offers and to buy online!

MTG Learning Media

Available at all leading book shops throughout the country.
For more information or for help in placing your order:
Call 0124-4951200 or email info@mtg.in
**Application to read QR codes required*

The coordinates of O, A, B and C are (0, 0), (24, 0), (8, 20) and (0, 25)
So,

Co-ordinates	Max. $Z = 50x + 60y$
$O(0, 0)$	$Z = 50 \times 0 + 60 \times 0 = 0$
$A(24, 0)$	$Z = 24 \times 50 + 60 \times 0 = 1200$
$B(8, 20)$	$Z = 8 \times 50 + 60 \times 20 = \mathbf{1600}$
$C(0, 25)$	$Z = 50 \times 0 + 60 \times 25 = 1500$

$\therefore$ $Z = 50x + 60y$ is maximum when $x = 8$ and $y = 20$

$\therefore$ 8 stickers of 'SAVE ENVIRONMENT' and 20 stickers of 'BE COURTEOUS' for maximum profit of ₹ 1600.

Save environment is a big challenge which is very important and necessary to survive. Be courteous is life skill which everyone must acquire to be compassionate.

29. Let the events are defined as:

E_1 : 2 white bulbs are transferred from A to B

E_2 : 2 yellow bulbs are transferred

E_3 : 1 white and 1 yellow bulb is transferred

A : 1 yellow bulb is drawn from B

$$P(E_1) = \frac{{}^4C_2}{{}^7C_2} = \frac{4\times 3}{7\times 6} = \frac{2}{7},\ P(E_2) = \frac{{}^3C_2}{{}^7C_2} = \frac{3\times 2}{7\times 6} = \frac{1}{7}.$$

$$P(E_3) = \frac{{}^4C_1 \cdot {}^3C_1}{{}^7C_2} = \frac{4}{7}.$$

$$P(A/E_1) = \frac{2}{6} = \frac{1}{3}, P(A/E_2) = \frac{4}{6} = \frac{2}{3}, P(A/E_3) = \frac{3}{6} = \frac{1}{2}$$

$$P(E_3/A) = \frac{P(E_3)\cdot P(A/E_3)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3)}$$

$$= \frac{\frac{4}{7}\times\frac{1}{2}}{\frac{2}{7}\cdot\frac{1}{3} + \frac{1}{7}\cdot\frac{2}{3} + \frac{4}{7}\cdot\frac{1}{2}} = \frac{3}{5}$$

Advantages:-

(1) CFL bulbs can save upto 75% of the energy used by an incandescent bulb.

(2)CFL bulbs burns brightly for 10,000 hours, whereas standard bulbs burns for just 800 to 1,500 hours.

OR

$$P(\text{answer is true}) = \frac{1}{2}$$

$$P(\text{answer is false}) = \frac{1}{2}$$

P(at most 7 correct) = $1 - \{P(8) + P(9) + P(10)\}$

(where $P(8)$ etc. means probability of 8 correct answers)

$$= 1 - \left\{ {}^{10}C_8\left(\frac{1}{2}\right)^8\left(\frac{1}{2}\right)^2 + {}^{10}C_9\left(\frac{1}{2}\right)^9\left(\frac{1}{2}\right) + {}^{10}C_{10}\left(\frac{1}{2}\right)^{10} \right\}$$

$$= 1 - \left\{ {}^{10}C_2 + {}^{10}C_1 + {}^{10}C_0 \right\}\left(\frac{1}{2}\right)^{10}$$

$$= 1 - \{45 + 10 + 1\}\frac{1}{1024} = 1 - \frac{56}{1024} = 1 - \frac{7}{128} = \frac{121}{128}$$

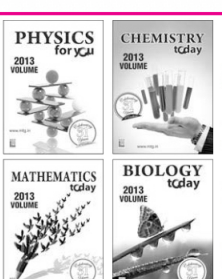
Advantages:-

(1) Examination is an easy tool to regularly assess a student's capability.

(2) Exams promote competition among students.

AVAILABLE BOUND VOLUMES

of your favourite magazines



buy online at www.mtg.in

Volumes of the following years are available:

Physics For You
2013, 2012, 2011, 2010, 2008, 2006

Chemistry Today
2013, 2011, 2010,2009, 2008, 2006

Mathematics Today
2013, 2012, 2011, 2010, 2009, 2008

Biology Today
2013, 2011, 2010, 2009, 2008, 2007, 2006, 2005

Price : ₹ 300 for each volume

POSTAGE FREE!

How to order : Send money by demand draft/money order. Demand Draft should be drawn in favour of **MTG Learning Media (P) Ltd**. Mention the volume you require along with your name and address.

Mail your order to :
Circulation Manager, MTG Learning Media (P) Ltd.
Plot 99, Sector 44 Institutional Area, Gurgaon, (HR)
Tel.: (0124) 4951200
E-mail : info@mtg.in Web : www.mtg.in

MOCK TEST PAPER 2014

JEE Main & Advanced

JEE Main

SECTION-I

COMPREHENSION TYPE

Comprehension-1

In ΔABC with sides a, b and c, a is the average of b and c. Let r be the inradius and R be the circum radius of ΔABC. Then

1. $\angle BAC \in$
(a) $[0, 60°]$ (b) $[60°, 90°]$
(c) $[60°, 120°]$ (d) $[90°, 120°]$

2. Altitude from angle $A = h$, then
(a) $h = 2r$ (b) $h = r$
(c) $h = r/2$ (d) $h = 3r$

3. The distance of circumcentre of ΔABC to side BC is
(a) $R + r$ (b) $R - r$ (c) rR (d) R/r

Comprehension-2

Let $f(x) = [x] + (x - [x])^{[x]}$ for $x \geq \frac{1}{2}$

4. Which of the following statements hold?
(a) $f(x)$ is continuous on $[1, \infty)$
(b) $f(x)$ is discontinuous on $[1, \infty)$
(c) $f(x)$ is discontinuous on $[1/2, 1)$
(d) $f(x)$ is discontinuous on $[1, 3/2]$

5. Which of the following options is true?
(a) $f(x)$ is strictly increasing on $[1, \infty)$
(b) $f(x)$ is strictly decreasing on $[1, \infty)$
(c) $f(x)$ is strictly decreasing on $(1/2, 1)$
(d) $f(x)$ is strictly decreasing on R

Comprehension-3

Let A, B, C be three non-collinear points corresponding to $z_0 = ai$, and $z_1 = \frac{1}{2} + bi$ and $z_2 = 1 + ci$ $(a, b, c \in R)$ respectively. Consider the curve

$Q : z = z_0\cos^4\theta + 2z_1\cos^2\theta \sin^2\theta + z_2\sin^4\theta,\ \theta \in R$

6. The curve represented by z is
(a) circle (b) parabola
(c) ellipse (d) hyperbola

7. The line parallel to AC and passing through midpoint of AB is tangent to curve Q at a point P then x-co-ordinate of P is
(a) 1/4 (b) 1/2 (c) 3/4 (d) 1

SECTION-II

MULTIPLE CORRECT ANSWER TYPE

8. Let t, p, s, y, g be positive numbers with $t \neq p$ such that $p^2 = st$ and t, y, g, p form an A.P. Comment on the nature of the roots of the equation $yx^2 - 2sx + g = 0$
(a) two complex roots
(b) repeated real roots
(c) distinct real roots of same sign
(d) distinct real roots of opposite sign

9. Let $P(-1, 1)$ and $Q(2, 2)$. The line $x + ty + t = 0$ intersects the extension of PQ, then $t \in$
(a) $\left(-3, \frac{-2}{3}\right)$ (b) $(-3, -2)$
(c) $(-2, -1)$ (d) $(-4, -3)$

10. Consider the parabola $y^2 = 4ax$. The tangent at any point P on the parabola intersects its directrix at T. S is the focus and A is the vertex. Then
(a) points T, A, S, P are concyclic
(b) circle with diameter TP passes through S
(c) circle with diameter TP passes through A
(d) all of these

11. Let functions $f, g : R \to R$ be defined by $f(x) = 3x - 1 + |2x + 1|$, $g(x) = \frac{1}{5}(3x + 5 - |2x + 5|)$, then $\forall\ x \in R$
(a) $g(f(x)) = x$ (b) $f(g(x)) = x$
(c) $f(x) = g^{-1}(x)$ (d) $(f(f(x)) = g(g(x))$

SECTION-III

SINGLE ANSWER TYPE

12. Compute $\sum_{n=0}^{\infty} (-1)^n \frac{\cos^3(3^n x)}{3^n}$
(a) $\frac{1}{4}\cos x$ (b) $\frac{3}{4}\cos x$

By : ER. Tapas Kr. Yogi, (BHUBANESWAR). Mob : 09778158718.

(c) $\frac{1}{4}\cos 3x$ (d) $\frac{3}{4}\cos 3x$

13. The equation $\sin(\cos x) = x$ has only one root x_1 in $(0, \pi/2)$ and the equation $\cos(\sin x) = x$ has also only one root x_2 in $(0, \pi/2)$. Then
(a) $x_1 > x_2$ (b) $x_1 < x_2$ (c) $x_1 = x_2$ (d) $x_1 = 2x_2$

14. The locus of a point which moves such that the sum of the squares of its distances from the three vertices of a triangle may be constant, is a circle with centre at
(a) circumcentre (b) centroid
(c) orthocentre (d) incentre

SECTION-IV

INTEGER ANSWER TYPE

15. The number of solutions of the equation $\sin^{-1}[\sin x] = \cos^{-1}[\cos x]$ in $[0, 2\pi]$ is

16. The unit vectors $\hat{a}$ and $\hat{b}$ are perpendicular and the unit vector $\hat{c}$ is inclined at an angle θ to both $\hat{a}$ and $\hat{b}$. If $\hat{c} = \alpha\hat{a} + \beta\hat{b} + \gamma(\hat{a} \times \hat{b})$, then $\beta^2 = \frac{1+\cos 2\theta}{k}$ (α, β, γ are scalars) where $k =$

17. $\lim_{n\to\infty} \{(2+\sqrt{3})^n\} =$

SECTION-V

MATRIX MATCH TYPE

18. Let $\vec{A}(x, y, z) = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ be defined and differentiable at each point (x, y, z) in a certain region of space. Then the curl of $\vec{A}$ written as $\vec{\nabla} \times \vec{A}$ is defined as

$$\vec{\nabla} \times \vec{A} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) \times \left(a_1\hat{i} + a_2\hat{j} + a_3\hat{k}\right),$$

where ∂ represents partial derivatives.

Column-I		Column-II	
(A)	For a scalar function A, $\vec{\nabla} \times (\vec{\nabla} A) =$	(p)	1
(B)	For a vector function $\vec{A}$, $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{A}) =$	(q)	–1
(C)	For a vector function $\vec{A}$, $\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = k\nabla^2\vec{A} + \nabla(\vec{\nabla} \cdot \vec{A})$ where $k =$	(r)	0
		(s)	not defined

19. Consider the line $ax + by + c = 0$

Column-I		Column-II	
(A)	x-intercept is finite and y-intercept is infinite	(p)	$a = 0, b \neq 0, c \neq 0$
(B)	x-intercept is infinite and y-intercept is finite	(q)	$a \neq 0, b \neq 0, c = 0$
(C)	both x and y-intercepts are zero	(r)	$a = 0, b = 0, c \neq 0$
(D)	both x and y-intercepts are infinite	(s)	$a \neq 0, b = 0, c \neq 0$

JEE Advanced

SECTION-I

COMPREHENSION TYPE

Comprehension-1

The probability that event A occurs is 3/4 and the probability that event B occurs is 2/3.

1. For $P(A \cap B)$,
(a) maximum possible value is 2/3
(b) maximum possible value is 1/3
(c) minimum possible value is 5/6
(d) minimum possible value is 5/3

2. For $P(A/B)$, sum of maximum and minimum values =
(a) 13/8 (b) 5/8
(c) 1 (d) 11/8

3. For $P(B/A)$, sum of maximum and minimum values =
(a) 11/9
(b) 13/9
(c) 1
(d) 4/9

Comprehension-2

Let us define f on $(0, 2)$ by setting

$$f(x) = \begin{cases} x^2, & \text{for rational } x \in (0,2) \\ 2x - 1, & \text{for irrational } x \in (0,2) \end{cases}$$

4. Which of the following is true?
(a) f is continuous at $x = 1, 3/2$
(b) f is continuous at $x = 1$ only
(c) f is continuous at $x = 3/2$ only
(d) none of these

5. $f'(1) =$
(a) zero (b) one
(c) two (d) not defined

6. How many points are in the interior of domain of f^{-1}?
(a) zero (b) one (c) two (d) infinite

SECTION-II

SINGLE ANSWER TYPE

7. Suppose f is continuous in $[0, C]$ $(C > 0)$ and is strictly increasing in $[0, C]$ and $f(0) = 0$, then for any $x \in [0, C]$, $\int_0^x f(t)dt + \int_0^{f(x)} f^{-1}(t)dt =$
(a) $xf(x)$ (b) $x - f(x)$
(c) $x + f(x)$ (d) $f(x)$

8. Given A and B are two matrices of 2×2 order with integer entries such that
$A^3 + B^3 = \begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix}$, then $A + B =$
(a) $\begin{bmatrix} 1 & 0 \\ -1 & -2 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix}$
(c) $\begin{bmatrix} -1 & -2 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix}$

9. Three standard, fair six sided dice are rolled simultaneously. The probability that the numbers shown on some two of the dice adds upto give the number shown on the remaining die is
(a) $\frac{5}{72}$ (b) $\frac{5}{24}$ (c) $\frac{5}{12}$ (d) $\frac{5}{48}$

10. The perpendicular distance of a vertex of a unit cube from a diagonal not passing through it is
(a) $\frac{2}{3}$ (b) $\sqrt{\frac{2}{3}}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\frac{2}{\sqrt{3}}$

11. A matrix A (3×3) has only $\{0, 1, -1\}$ as entries. The maximum possible value of the det (A) is
(a) 2 (b) 4 (c) 5 (d) 6

SECTION-III

INTEGER ANSWER TYPE

12. Let S_n be the sum of first n terms of a G.P. If $S_{10} = 10$ and $S_{30} = 70$, then $S_{40} =$

13. Let f be a bijective function defined on $[1, 2] \to [3, 4]$ and if f is an increasing function, then
$\int_1^2 f(t)dt + \int_3^4 f^{-1}(t)dt =$

14. An equilateral triangle is inscribed in the hyperbola $xy = 1$ with one vertex at $(-1, -1)$ and the other two on branch with $x > 0$ then the product of x-co-ordinates of other two vertices is

15. Let n be an odd integer ≥ 5, then the number ${}^nC_1 - 5 \cdot {}^nC_2 + 5^2 \cdot {}^nC_3 - \ldots + 5^{n-1} \cdot {}^nC_n$ can't be 6 or 7 or 8 or 9 is

16. A line l_1 passes through the centre of a circle. A, B, C are three points outside the circle. They lie on a line l_2 perpendicular to l_1 on the same side of l_1 with A farthest from l_1. AP, BQ and CR are tangents to the circle, then $\frac{AB \cdot CR + BC \cdot AP}{CA \cdot BQ} =$

SECTION-IV

DATA SUFFICIENCY TYPE

17. If $A \neq B$ then $(A, B) \neq (B, A)$. Find the number of pairs (A, B) of sets such that $A \cup B = \{a_1, a_2, a_3\}$. Is this data enough to find the answer?
(a) Yes
(b) No. Sets A and B are required.
(c) No. Atleast number of elements in sets A and B are required.
(d) No. At least $n(A \cap B)$ is required

18. Let f be differentiable on $(0, \infty)$ and $\lim_{x \to \infty} f(x) + f'(x) = 0$. Is this data enough to find $\lim_{x \to \infty} f(x)$?
(a) Yes. The required limit is 0.
(b) Yes. The required limit is -1.
(c) No. $f(x)$ definition is needed.
(d) No. Atleast boundary values of $f(x)$ are needed.

19. Given $f(x) = \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots + \frac{x^n}{n!}\right)e^{-x}$
Data given : n is positive integer.
Is this data enough to find the global maximum value?
(a) Yes.
(b) No. It is required if $n \geq 3$
(c) No. It is required whether n is even or odd
(d) For no value of n global maximum value exists.

20. Given, equation of axis, co-ordinates of focus and an equation of tangent to a parabola. Is this data enough to find the equation of parabola?
(a) Yes (b) No

21. Given $x = (\sin\theta)^{\log_a \sin\theta}$, $y = (\cos\theta)^{\log_a \cos\theta}$, $z = (\sin\theta)^{\log_a \cos\theta}$

Conditions $C_1 : 0 < a < 1$

$C_2 : 0 < \theta < \pi/4$

To get the order $x < z < y$

(a) only C_1 is required (b) only C_2 is required

(c) both C_1 and C_2 are required

(d) none of C_1 and C_2 is required

22. Let a, b, c be complex numbers. Consider the two statements.

S_1 : If $a^2 + b^2 > c^2$ then $a^2 + b^2 - c^2 > 0$

S_2 : If $a^2 + b^2 - c^2 > 0$ then $a^2 + b^2 > c^2$

Then

(a) only S_1 is true (b) only S_2 is true

(c) both S_1 and S_2 are true

(d) neither S_1 nor S_2 is true

SOLUTIONS

JEE Main

1. (a) 2. (d) 3. (b)

$2a = b + c$. Use cosine rule.

$$\cos(\angle BAC) = \frac{3b^2 + 3c^2 - 2bc}{8bc} \geq \frac{1}{2}$$

And Area $(\Delta) = \frac{1}{2}ah$, $r = \frac{\Delta}{(3a)/2}$

Distance of circumcentre to $BC = R\cos A$

$$r = 4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$

4. (a) 5. (a)

Redefining $f(x) = 1$ if $x \in \left[\frac{1}{2}, 1\right)$

$= n + (x - n)^n$ if $x \in [n, n+1]$, $n \in N$

6. (b) 7. (b)

Putting $z = x + iy$ and the respective values of z_0, z_1, z_2 and equating real and imaginary parts and eliminating θ, we have

$y = (a + c - 2b)x^2 + 2x(b - a) + a$, $x \in [0, 1]$

8. (a) : Use A.M. $\geq$ G.M. on t, t and s

9. (a, b, c) : In order for the line $x + ty + t = 0$ to intersect the extension of PQ, we must have

$$\frac{1}{3} < \frac{-1}{t} < \frac{3}{2}$$

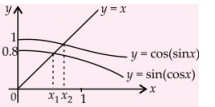
10. (b)

11. (a, b, c) : $f(x) = \begin{cases} 5x, & x \geq \frac{-1}{2} \\ x - 2, & x < \frac{-1}{2}, \end{cases}$ $g(x) = \begin{cases} \frac{x}{5}, & x \geq \frac{-5}{2} \\ x + 2, & x < \frac{-5}{2} \end{cases}$

$g(f(x)) = x = f(g(x))$, $f = g^{-1}$, $g = f^{-1}$

12. (b): Use the identity $4\cos^3 x = \cos 3x + 3\cos x$ repeatedly.

13. (b):



14. (b)

15. (2) : If $\theta = \sin^{-1}[\sin x] = \cos^{-1}[\cos x]$, then $0 \leq \theta \leq \pi/2$. Consider 4 different cases.

$C_1 \rightarrow \quad -1 \leq \sin x < 0$

$C_2 \rightarrow \quad 0 < \sin x < 1$

$C_3 \rightarrow \quad \sin x = 0$

$C_4 \rightarrow \quad \sin x = 1$

The solution set is $\left\{2n\pi, 2n\pi + \frac{\pi}{2}, n \in z\right\}$

16. (2)

17. (1) : Use binomial theorem,

Let $(2 + \sqrt{3})^n = A_n + \sqrt{3}B_n$

and $(2 - \sqrt{3})^n = A_n - \sqrt{3}B_n$

So, $\lim_{n\to\infty}(A_n + \sqrt{3}B_n) \to \infty$

But $\lim_{n\to\infty}(A_n - \sqrt{3}B_n) \to 0$

and $\lim_{n\to\infty}\frac{\sqrt{3}B_n}{A_n} = 1$ and $\frac{\sqrt{3}B_n}{A_n} < 1$

So, $\{A_n + \sqrt{3}B_n\} = \{\sqrt{3}B_n\} \to 1$ as $n \to \infty$

18. (A) - (r), (B) - (r), (C) - (q)

19. (A) - (s), (B) - (p), (C) - (q), (D) - (r)

JEE Advanced

1. (a) 2. (a) 3. (b)

$P(A \cap B) \leq P(B)$

$P(A \cap B)_{\min} = \frac{3}{4} + \frac{2}{3} - 1$

$P(A/B) = \frac{P(A \cap B)}{P(B)}$, max. $\frac{2/3}{2/3}$ and min. $\frac{5/12}{2/3}$

$P(B/A) = \frac{P(B \cap A)}{P(A)}$, max. $\frac{2/3}{3/4}$ and min. $\frac{5/12}{3/4}$

4. (b)

5. (c)

6. (a)

7. (a) : Putting $u = f^{-1}(t)$ and integrating by parts,

$$\int_0^x f(t)\,dt + \int_0^{f(x)} f^{-1}(t)\,dt = \int_0^x f(t)\,dt + \int_0^x uf'(u)\,du$$
$$= xf(x)$$

8. (b): Notice that, $C^2 + 3C + 2I = 0$

where, $C = \begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix}$

So, $C = (C + I)^3 + (-I)$
Hence, $A = C + I$, $B = -I$

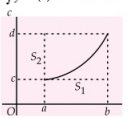
9. (b): Required probability $= 3 \times \sum_{k=1}^{6} \frac{1}{6} \times \frac{(k-1)}{36}$

10. (b)

11. (b)

12. (150)

13. (5): $\int_a^b f(t)\,dt + \int_c^d f^{-1}(t)\,dt = bd - ac$



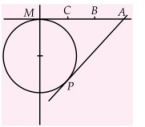
14. (1): Let $P(-1, -1)$, $Q(x_1, y_1)$, $R(x_2, y_2)$
The midpoint of QR lies on the altitude from P on QR.
Using this and simplifying a bit,
$(1 - x_1x_2)\,[(x_1 + x_2)(1 + x_1x_2) + 2x_1x_2] = 0$
As the second factor is positive hence, $1 - x_1x_2 = 0$

15. (7): Let the given series be S, then using binomial theorem, we have
$5S = 1 + (-1 + 5)^n$

i.e., $S = \frac{4^n + 1}{5} = \frac{1}{5}[(\sqrt{u} - 1)^2 + u][(\sqrt{u} + 1)^2 + u]$, where $u = 2^{n-1}$

For $n \geq 5$, hence S can't be a prime number.

16. (1): $AP = AM$, $BQ = BM$, $CR = CM$
$AB \cdot CR + BC \cdot AP$
$= AB \cdot CM + BC \cdot AM$
$= AB \cdot CM + BC \cdot (AC + CM)$
$= (AB + BC) \cdot CM + AC \cdot BC$
$= AC \cdot CM + AC \cdot BC$
$= AC \cdot (CM + BC)$
$= AC \cdot BM = AC \cdot BQ$



17. (a): For each a_i, there are 3 possibilities. It may be in A only, in B only or in both. Hence there are $3^3 = 27$ pairs (A, B) for which $A \cup B = \{a_1, a_2, a_3\}$.

18. (a): The result follows from mean value theorem.

19. (c): $f'(x) = -\frac{x^n}{n!}e^{-x}$

So, in case when n is odd, $f(0) = 1$ is a global maximum value.

20. (a): The reflection of focus in any tangent falls on the directrix line.

21. (c): For $0 < a < 1$, $\log_a x$ is a decreasing function and for $0 < \theta < \pi/4$, $0 < \sin\theta < \cos\theta < 1$.

22. (a): If $a^2 + b^2 > c^2$, then $a^2 + b^2$ and c^2 are real. Hence S_1 is true, but S_2 might not be true.
For example, take $a = 3 + 2i$, $b = 2i$, $c^2 = 12i$

MTG MAGAZINES GO DIGITAL



Now, no need to wait for the shipping to arrive! You can read all the four digital magazines on iPAD, iPhone, iPod touch and Android running devices anywhere anytime.

Flexible subscription offer for each magazine:
Single issue: Rs 30/-
6 months subscription: Rs 180
12 months subscription: Rs 300

Now available on digital magazine stands:

Visit the link to subscribe your Digital magazines
http://www.magzter.com/publishers/MTG-Learning-Media-Pvt-Ltd

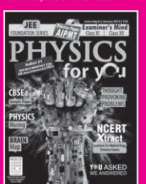
MAGZTER DIGITAL NEWSSTAND TAP • READ • ENJOY
readwhere
Rockstand

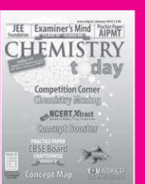
Now, save up to Rs 2,220*

Subscribe to MTG magazines today.

Our 2014 offers are here. Pick the combo best suited for your needs. Fill-in the Subscription Form at the bottom and mail it to us today. If in a rush, log on to www.mtg.in now to subscribe online.

*On cover price of ₹ 30/- each.









For **JEE (Main & Advanced), AIPMT, PMTs, All State Level Engg. & Medical Exams**

About MTG's Magazines

Perfect for students who like to prepare at a steady pace, MTG's magazines – Physics For You, Chemistry Today, Mathematics Today & Biology Today – ensure you practice bit by bit, month by month, to build all-round command over key subjects. Did you know these magazines are the only source for solved test papers of all national and state level engineering and medical college entrance exams?

Trust of over 1 Crore readers. Since 1982.

- Practice steadily, paced month by month, with very-similar & model test papers
- Self-assessment tests for you to evaluate your readiness and confidence for the big exams
- Content put together by a team comprising experts and members from MTG's well-experienced Editorial Board
- Stay up-to-date with important information such as examination dates, trends & changes in syllabi
- All-round skill enhancement – confidence-building exercises, new studying techniques, time management, even advice from past IIT/PMT toppers
- **Bonus:** exposure to competition at a global level, with questions from Intl. Olympiads & Contests

SUBSCRIPTION FORM

Please accept my subscription to:
(Confirm your choice by ticking the appropriate boxes)

Best Offer

PCMB combo
☐ 1 yr: ₹ 900 *(save ₹ 540)* ☐ 2 yrs: ₹ 1,500 *(save ₹ 1,380)* ☐ 3 yrs: ₹ 2,100 *(save ₹ 2,220)*

PCM combo
☐ 1 yr: ₹ 800 *(save ₹ 280)* ☐ 2 yrs: ₹ 1,200 *(save ₹ 960)* ☐ 3 yrs: ₹ 1,700 *(save ₹ 1,540)*

PCB combo
☐ 1 yr: ₹ 800 *(save ₹ 280)* ☐ 2 yrs: ₹ 1,200 *(save ₹ 960)* ☐ 3 yrs: ₹ 1,700 *(save ₹ 1,540)*

Individual magazines
☐ Physics ☐ Chemistry ☐ Mathematics ☐ Biology
☐ 1 yr: ₹ 300 *(save ₹ 60)* ☐ 2 yrs: ₹ 500 *(save ₹ 220)* ☐ 3 yrs: ₹ 675 *(save ₹ 405)*

Name:_____

Complete Postal Address:_____

_____

_____

_____

Pin Code ☐☐☐☐☐☐ Mobile # ☐☐☐☐☐☐☐☐☐☐

Other Phone # 0 ☐☐☐☐☐☐☐☐

Email _____

Enclose Demand Draft favouring **MTG Learning Media (P) Ltd**, payable at New Delhi. You can also pay via Money Orders. Mail this Subscription Form to Subscription Dept., **MTG Learning Media (P) Ltd**, Plot 99, Sector 44, Gurgaon – 122 003 (HR).

Note: Magazines are despatched by Book-Post on 4th of every month (each magazine separately). Should you want us to send you your copies by Courier or Regd. Post instead, additional charges apply (₹ 150, ₹ 250, ₹ 350 for 1-yr, 2-yr, 3-yr subscriptions respectively).

Email *info@mtg.in*. Visit *www.mtg.in* to subscribe online. **Call (0)8800255334/5 for more info.**