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Contents

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editorial

Physics, Philosophy and Language

The correct language leads to the truth. Loaded with assumptions, simple conjugations can mislead a person from the path of truth. The philosophy of one's thoughts often clouds the way of thinking. According to a very respected philosopher, "With all the emphasis on the usefulness of science, the value of humanities will be forgotten!"

Science or philosophy, maths or chemistry or biology is wrongly worded. 'Or' has to be replaced by 'And'. Many great scientists were experts in music and were also artists. There is unity in music and one searches for unity in science.

Recent achievements of our research establishment, not only ISRO, Nuclear Science but also C.S.I.R have shown that we have the cream of great scientists and we have to continue to produce great scientists of the calibre of the top scientists in India. We need more science education and more education in humanities also. Our needs have grown tremendously. We cannot have the luxury of retired scientists. They are badly needed as advisors in universities and other scientific establishments. While different branches of mathematics and science subjects are taught as separate topics in universities, by associating the top scientists as advisors, a total picture will be available to the students of research.

To upgrade the quality of research, top scientists should be associated in teaching and experimental work. The choice is not quality or quantity. The need of the hour is quantity and quality.

Anil Ahlawat
Editor

PHYSICS MUSING

Physics Musing was started in August 2013 issue of Physics For You with the suggestion of Shri Mahabir Singh. The aim of Physics Musing is to augment the chances of bright students preparing for JEE (Main and Advanced) / AIIMS / Other PMTs with additional study material.

In every issue of Physics For You, 10 challenging problems are proposed in various topics of JEE (Main and Advanced) / various PMTs. The detailed solutions of these problems will be published in next issue of Physics For You.

The readers who have solved five or more problems may send their solutions. The names of those who send atleast five correct solutions will be published in the next issue.

We hope that our readers will enrich their problem solving skills through "Physics Musing" and stand in better stead while facing the competitive exams.

By : Akhil Tewari

PROBLEM Set 19

SINGLE OPTION CORRECT TYPE

1. A particle of mass m is subjected to a force $\vec{F} = F_0[\cos(t)\hat{i} + \sin(t)\hat{j}]$. If initially ($t$ = 0) the particle was at rest, the kinetic energy of the particle at time t is given by

(a) $\frac{F_0^2}{m}(1-\cos 2t)$ (b) $\frac{F_0^2}{m}(1-\cos t)$

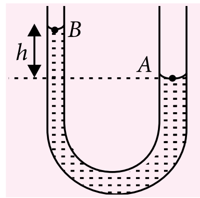
(c) $\frac{F_0^2}{m}(1-\sin t)$ (d) $\frac{F_0^2}{m}t$

2. A certain radioactive sample is obtained by mixing equal number of nuclides of species A and species B. Half life of species A is T and that of B is $2T$. After certain time, it is found that the remaining radioactive nuclei of A and B combined together are 5/32 of the total number of nuclei. The number of half lives of A elapsed is

(a) 4 (b) 3

(c) 2 (d) 1

3. The given figure shows difference in water level h of communicating capillary tubes of different radius. Radius at A and B are 1 mm and 1.5 mm respectively. Surface tension of water = 0.07 N m^{-1} and angle of contact between the glass and the water is 0°. The value of h (in mm) is (Take g = 9.8 m s^{-2})



(a) 3.78

(b) 4.76

(c) 5.00

(d) 5.32

4. Sixty percent of given sample of oxygen gas when raised to a high temperature dissociates into atoms. Ratio of its initial heat capacity (at constant volume) to the final heat capacity (at constant volume) will be

(a) $\frac{6}{7}$ (b) $\frac{25}{26}$ (c) $\frac{10}{7}$ (d) $\frac{25}{27}$

5. An amount Q of heat is added to a monoatomic ideal gas in a process in which the gas performs a work $Q/2$ to its surrounding. The molar heat capacity for the process is

(a) R (b) $2R$

(c) $3R$ (d) $4R$

6. An electric kettle heater has two coils when one coil is switched on, the water in the kettle begins to boil after 15 minutes, when the other coil is switched on, the water boils in 30 minutes.

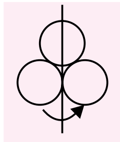
(a) Water boils in 60 minutes if the coils are connected in series.

(b) Water boils in 10 minutes if the coils are connected in parallel.

(c) Water boils in 22.5 minutes if the coils are connected in series.

(d) Water boils in 20 minutes if the coils are connected in parallel.

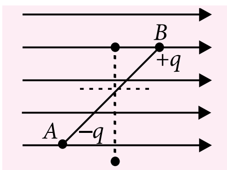
7. Three rings each of mass m and radius r are placed in a plane touching each other as shown. The radius of gyration of the system about the axis as shown in the figure is



(a) $\sqrt{\frac{7}{6}}\,r$ (b) $\sqrt{\frac{5}{3}}\,r$

(c) $\sqrt{\frac{5}{6}}\,r$ (d) $\sqrt{\frac{7}{2}}\,r$

8. A dipole consists of a light rod of length l, having two point charges $+q$ and $-q$ of same mass m joined at ends A and B of the rod. A uniform electric field E is present in the region and perpendicular to the length of the rod. The rod is released from rest. The angular velocity of the rod, when it becomes parallel to the electric field for the first time is

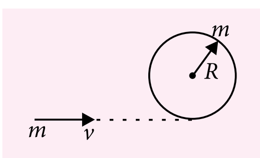


(a) $\sqrt{\frac{qE}{ml}}$

(b) $2\sqrt{\frac{qE}{ml}}$

(c) $\sqrt{\frac{qE}{2ml}}$ (d) $\frac{1}{2}\sqrt{\frac{qE}{ml}}$

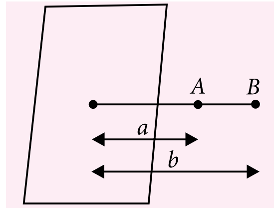
9. A circular hoop of mass m and radius R rests flat on a horizontal frictionless surface. A bullet, also of mass m and moving with a velocity v, strikes the hoop and gets embedded in it. The thickness of the hoop is much smaller than R. The angular velocity with which the system rotates after the bullet strikes the hoop is



(a) $\frac{v}{4R}$ (b) $\frac{v}{3R}$

(c) $\frac{2v}{3R}$ (d) $\frac{3v}{4R}$

10. Two points A and B are at a distance a and b respectively from an infinite metal plate having charge density as σ C m^{-2}. Potential difference between A and B is



(a) $\frac{\sigma}{\varepsilon_0}(b-a)$ (b) $\frac{\sigma}{\varepsilon_0}(a-b)$

(c) $\frac{\sigma}{2\varepsilon_0}(b-a)$ (d) $\frac{\sigma}{2\varepsilon_0}(a-b)$

SOLUTION OF JANUARY 2015 CROSSWORD

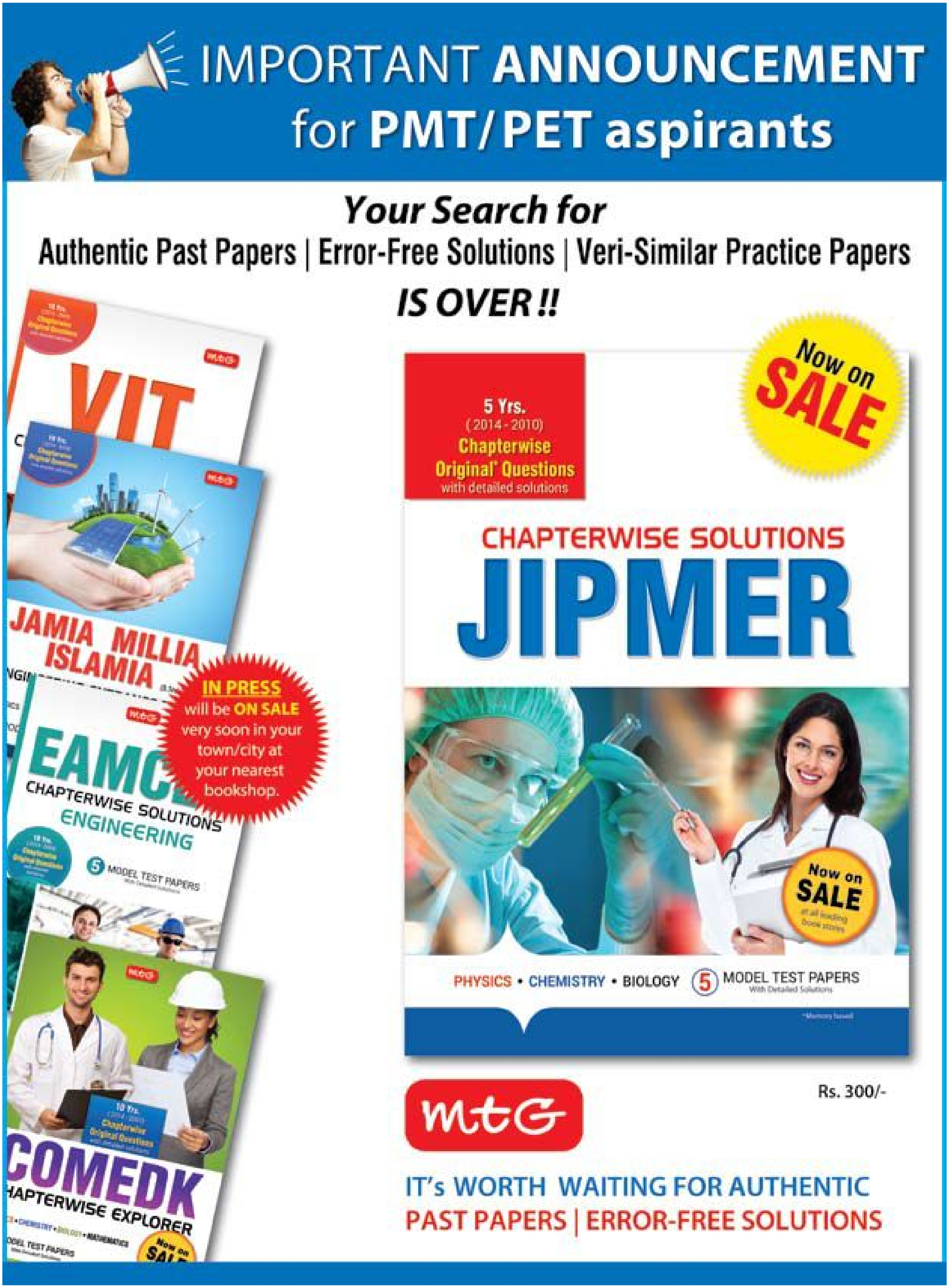
	1 A				2 R						3 T					4 F	E	E	D	B	A	C	K		
5 U	N	I	T	V	E	C	T	O	R		H					E									
	T				S						U		6 M	I	C	R	O			7 F					
	I				I						N					M			8 P	L	A	S	M	A	
	9 N	I	E	L	S	B	O	H	R		D					I				O					
	O				T						E			10 S	H	O	C	K	W	A	V	E			11 P
	D			12 D	I	E	L	E	C	T	R	I	C			N				T			13 S		E
	E				V												14 R	O	T	A	M	E	T	E	R
			15 D	R	I	F	T	V	E	L	O	16 C	I	T	Y					T			E		I
17 O					T							E				18 M	Y	O	P	I	A		R		G
P			19 D		Y							N								O			A		E
A			I					20 E	L	E	C	T	R	O	P	L	A	T	I	N	G		D		E
21 C	R	U	S	T								R											I		
I			P		22 C	E	N	T	R	I	P	E	T	A	L	F	O	R	C	E			A		
T			L									O									23 S		N		
Y			A		24 B		25 P					26 F	I	L	A	M	E	N	T		H				
			C		27 A	U	R	O	R	A		G									U				
			E		R		E					R				28 G	29 R	A	T	I	N	G			
	30 M		31 M	O	N	O	C	H	R	O	M	A	T	I	C		E				T				
	O		E				E					V					M					32 L			
	B		N		33 P		34 S	H	O	35 O	T	I	N	G	S	T	A	R		36 L	O	A	D		
	I		T		H		S			C		T					N					S			
	L				A		I			T		Y			37 A	P	E	R	T	U	R	E			
38 D	I	M	E	N	S	I	O	N		A							N					R			
	T				O		N			V							C								
	Y				R					E		39 M	A	N	O	M	E	T	E	R					

WINNERS (January 2015)

- **Atriz Roy**
- **Shambhavi Chowdhary :** "The crossword provides a quick round-up of all topics and is very helpful to all those preparing for various competitive exams.
- **Prabhat Singh :** "The crossword is a puzzle game of revising basic principles, discoveries, concepts and applications of physics".

Solution Senders (December 2014)

- **Kashish Arora**
- **Saayan Mitra**
- **Mohd. Taha Khan**



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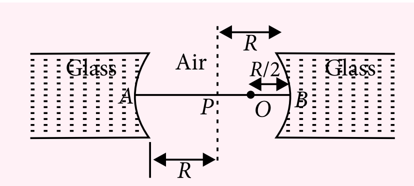
PRACTICE PAPER 2015

JEE Main

Exam on 4th April

1. Two concave refracting surfaces of equal radii of curvature and refractive index 1.5 face each other in air as shown in figure. A point object *O* is placed as shown in figure. What is the separation between the images of *O* formed by each refracting surface?



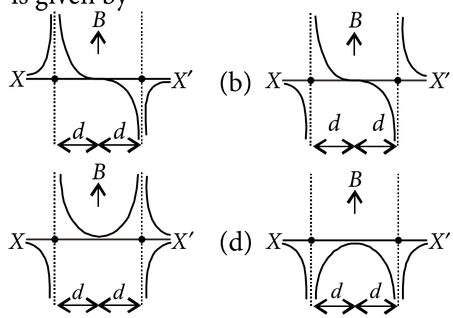
(a) $114R$ (b) $1.0114R$
(c) $0.114R$ (d) $0.4R$

2. If momentum (*P*), area (*A*) and time (*T*) are taken to be fundamental quantities, then energy has the dimensional formula
(a) $[P^1A^{-1}T^1]$ (b) $[P^2A^1T^1]$
(c) $[P^1A^{-1/2}T^1]$ (d) $[P^1A^{1/2}T^{-1}]$

3. A system consists of a uniformly charged sphere of radius *R* and a surrounding medium filled by a charge with the volume density $\rho = \frac{\alpha}{r}$, where α is a positive constant and *r* is the distance from the centre of the sphere. Find the charge of the sphere for which the electric field intensity *E* outside the sphere is independent of *R*.
(a) $\frac{\alpha}{2\epsilon_0}$ (b) $\frac{2}{\alpha\epsilon_0}$
(c) $2\pi\alpha R^2$ (d) None of these

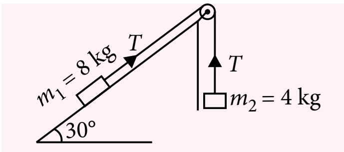
4. The bob of a simple pendulum is a spherical hollow ball filled with water. A plugged hole near the bottom of the oscillating bob gets suddenly unplugged. During observation, till water is coming out, the time period of oscillation would
(a) remain unchanged
(b) increase towards a saturation value
(c) first increase and then decrease to the original value
(d) first decrease and then increase to the original value

5. Two long parallel wires are at a distance $2d$ apart. They carry steady equal currents flowing out of the plane of the paper, as shown. The variation of the magnetic field *B* along the line *XX′* is given by



6. A photon collides with a stationary hydrogen atom in ground state inelastically. Energy of the colliding photon is 10.2 eV. After a time interval of the order of microsecnd another photon collides with same hydrogen atom inelastically with an energy of 15 eV. What will be observed by the detector?
(a) One photon of energy 10.2 eV and an electron of energy 1.4 eV.
(b) Two photons of energy 1.4 eV.
(c) Two photons of energy 10.2 eV.
(d) One photon of energy 10.2 eV and another photon of 1.4 eV.

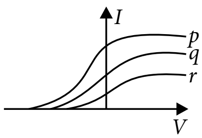
7. Two masses are connected by a string as shown in the figure over a frictionless pulley.

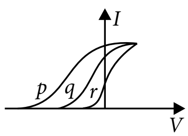


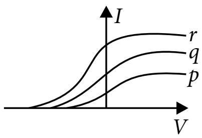
The acceleration of the system is

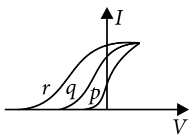
(a) 4 m s^{-2} (b) 2 m s^{-2}
(c) zero (d) 9.8 m s^{-2}

8. Photoelectric effect experiments are performed using three different metal plates p, q and r having work functions $\phi_p = 2.0$ eV, $\phi_q = 2.5$ eV and $\phi_r = 3.0$ eV, respectively. A light beam containing wavelengths of 550 nm, 450 nm and 350 nm with equal intensities illuminates each of the plates. The correct I-V graph for the experiment is

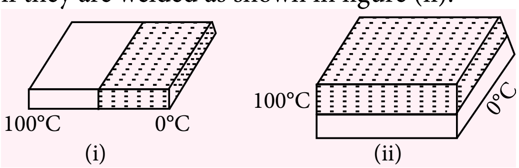
(a) 

(b) 

(c) 

(d) 

9. Two identical rectangular rods of metal of thermal resistance R, are welded end to end as shown in figure (i) and 10 J of heat flows through the rods in 2 min. How long would it take for 30 J of heat to flow through the the rods if they are welded as shown in figure (ii).



(a) 2 min (b) 1.5 min
(c) 1 min (d) 4 min

10. Distance between the centres of two stars is $10a$. The masses of these stars are M and $16M$ and their radii a and $2a$ respectively. A body of mass m is fired straight from the surface of the larger star towards the smaller star. The minimum initial speed for the body to reach the surface of smaller star is

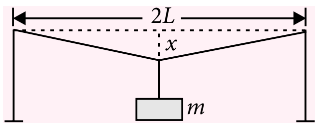
(a) $\frac{2}{3}\sqrt{\frac{Gm}{a}}$ (b) $\frac{3}{2}\sqrt{\frac{5Gm}{a}}$
(c) $\frac{2}{3}\sqrt{\frac{5Gm}{a}}$ (d) $\frac{3}{2}\sqrt{\frac{Gm}{a}}$

11. The magnetic field of a beam emerging from a filter facing a floodlight is given by
$B = 12 \times 10^{-8} \sin(1.20 \times 10^7 z - 3.60 \times 10^{15} t)$T.

What is the average intensity of the beam?

(a) 1.25 W m^{-2} (b) 1.72 W m^{-2}
(c) 0.2 W m^{-2} (d) 0.25 W m^{-2}

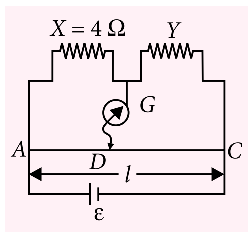
12. A mild-steel wire of length $2L$ and cross-sectional area A is stretched, well within elastic limit, horizontally between two pillars,



A mass m is suspended from the midpoint of the wire. Strain in the wire is

(a) $\frac{x^2}{2L^2}$ (b) $\frac{x}{L}$ (c) $\frac{x^2}{L}$ (d) $\frac{x^2}{2L}$

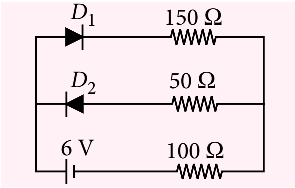
13. Figure shows a meter bridge, wire AC has uniform cross-section. The length of wire AC is 100 cm. X is a standard resistor of 4 Ω and Y is a coil.



When Y is immersed in melting ice, the null point is at 40 cm from point A. When the coil Y is heated to 100°C, a 100 Ω resistor has to be connected in parallel with Y in order to keep the bridge balanced at the same point.
Temperature coefficient of resistance of the coil is

(a) 6.3×10^{-4} K^{-1} (b) 4.3×10^{-4} K^{-1}
(c) 8.3×10^{-4} K^{-1} (d) 2.3×10^{-4} K^{-1}

14. The circuit shown in the figure contains two diodes each with a forward resistance of 50 ohm and with infinite backward resistance. If the battery voltage is 6 V, the current through the 100 ohm resistance (in ampere) is

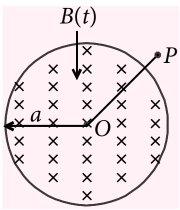


(a) zero (b) 0.02 (c) 0.03 (d) 0.033

15. Two point masses of 0.3 kg and 0.7 kg are fixed at the ends of a rod of length 1.4 m and of negligible mass. The rod is set rotating about an axis perpendicular to its length with a uniform angular speed. The point on the rod through which the axis should pass in order that the work required for rotation of the rod is minimum, is located at a distance of

(a) 0.42 m from mass of 0.3 kg
(b) 0.70 m from mass of 0.7 kg
(c) 0.98 m from mass of 0.3 kg
(d) 0.98 m from mass of 0.7 kg.

16. A uniform but time-varying magnetic field $B(t)$ exists in a circular region of radius a and is directed into the plane of the paper, as shown in the figure.

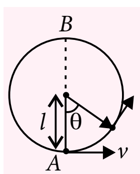


The magnitude of the induced electric field at point P at a distance r from the centre of the circular region

(a) is zero (b) decreases as $\frac{1}{r}$

(c) increases as r (d) decreases as $\frac{1}{r^2}$.

17. A bob of mass m is suspended by a massless string of length l. The horizontal velocity v at position A is just sufficient to make it reach the point B.



The angle θ at which the speed of the bob is half of that at A, satisfies

(a) $\theta = \frac{\pi}{4}$ (b) $\frac{\pi}{4} < \theta < \frac{\pi}{2}$

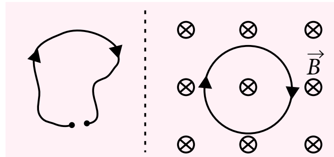
(c) $\frac{\pi}{2} < \theta < \frac{3\pi}{4}$ (d) $\frac{3\pi}{4} < \theta < \pi$

18. An insulated container containing n moles of monoatomic gas of molar mass m is moving with a velocity v_0. If the container is suddenly stopped, find the change in temperature.

(a) $\frac{mv_0^2}{3R}$ (b) $\frac{mv_0^2}{3nR}$

(c) $\frac{mnv_0^2}{R}$ (d) $\frac{mv_0^2}{2R}$

19. A thin flexible wire of length L is connected to two adjacent fixed points and carries a current I in the clockwise direction as shown in figure.

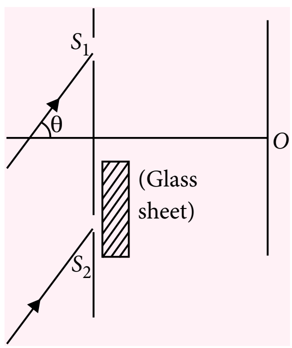


When the system is put in a uniform magnetic field of strength B going into the plane of the paper, the wire takes the shape of a circle. The tension in the wire is

(a) IBL (b) $\frac{IBL}{\pi}$

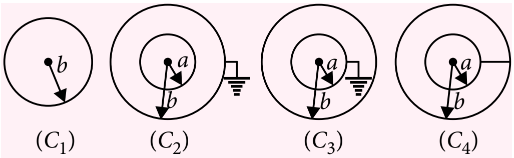
(c) $\frac{IBL}{2\pi}$ (d) $\frac{IBL}{4\pi}$

20. A monochromatic beam of light falls on Young's double slit experiment apparatus at some angle (say θ) as shown in figure. A thin sheet of glass is inserted in front of the lower slit S_2. The central bright fringe (path difference = 0) will be obtained



(a) anywhere depending on angle θ, thickness of plate t and refractive index of glass μ
(b) at O
(c) below O (d) above O

21. Consider the following four arrangement of spherical shells of radius a and b ($a << b$).



Then which of the following holds good for the value of their capacitances.

(a) $C_3 < C_2 > C_1 = C_4$ (b) $C_3 > C_1 = C_4 > C_2$
(c) $C_2 > C_3 > C_1 > C_4$ (d) $C_2 > C_1 = C_4 > C_3$

22. An object of specific gravity ρ is hung from a thin steel wire. The fundamental frequency for transverse standing waves in the wire is 300 Hz. The object is immersed in water so that one half of its volume is submerged. The new fundamental frequency in Hz is

(a) $300\left(\frac{2\rho-1}{2\rho}\right)^{1/2}$ (b) $300\left(\frac{2\rho}{2\rho-1}\right)^{1/2}$

(c) $300\left(\frac{2\rho}{2\rho-1}\right)$ (d) $300\left(\frac{2\rho-1}{2\rho}\right)$.

23. A student uses a simple pendulum of exactly 1 m length to determine g, the acceleration due to gravity. He uses a stop watch with the least count of 1 s for this and records 40 s for 20 oscillations. For this observation, which of the following statements is true?

(a) Error ΔT in measuring T, the time period, is 0.02 seconds.
(b) Error ΔT in measuring T, the time period, is 1 second.
(c) Percentage error in the determination of g is 5%.
(d) Percentage error in the determination of g is 2.5%.

24. The ratio of contributions made by the electric field and magnetic field components to the intensity of an e.m. wave is

(a) $c : 1$ (b) $c^2 : 1$
(c) $1 : 1$ (d) $\sqrt{c} : 1$

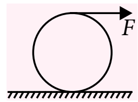
25. The water flows from a tap of diameter 1.25 cm with a rate of 5×10^{-5} m^3 s^{-1}. The density and coefficient of viscosity are 10^3 kg m^3 and 10^{-3} Pa s respectively. The flow of water is

(a) steady with Reynolds number 5100
(b) turbulent with Reynolds number 5100
(c) steady with Reynolds number 3900
(c) turbulent with Reynolds number 3900

26. Two bulbs consume same power when operated at 200 V and 300 V respectively. When the two bulbs are connected in series across a DC source of 500 V, then ratio of

(a) potential difference across them is 3 : 2
(b) potential difference across them is 2 : 3
(c) power consumed by them is 4 : 9
(d) power consumed by them is 2 : 3

27. A force F is applied at the top of a ring of mass M and radius R placed on a rough horizontal surface as shown in figure.



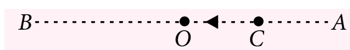
Friction is sufficient to prevent slipping. The friction force acting on the ring is

(a) $\frac{F}{2}$ towards right (b) $\frac{F}{3}$ towards left
(c) $\frac{2F}{3}$ towards right (d) zero

28. In refrigerator one removes heat from a lower temperature and deposits to the surroundings at a higher temperature. In this process, mechanical work has to be done, which is provided by an electric motor. If the motor is of 1 kW power, and heat is transferred from −3°C to 27°C, find the heat taken out of the refrigerator per second assuming its efficiency is 50% of a perfect engine.

(a) 14 J (b) 12 J (c) 19 J (d) 20 J

29. A particle is in linear simple harmonic motion between two points A and B, 10 cm apart, as shown in figure. Take the direction from A to B as the +ve direction and choose the incorrect statement.



$AO = OB = 5$ cm
$BC = 8$ cm

(a) The signs of velocity, acceleration and force on the particle when it is 3 cm away from A going towards B are positive.

(b) The signs of velocity of the particle at *C* going towards *O* is negative.

(c) The signs of velocity, acceleration and force on the particle when it is 4 cm away from *B* going towards *A* are negative.

(d) The signs of acceleration and force on the particle when it is at point *B* are negative.

30. The electron emitted in beta radiation originates from

(a) inner orbits of atoms

(b) free electrons existing in nuclei

(c) decay of a neutron in a nuclei

(d) photon escaping from the nucleus.

SOLUTIONS

1. **(c) :** For image formed by the surface on the right side

$$\mu_1 = 1, \mu_2 = 1.5, u = \frac{-R}{2}, R_1 = -R$$

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1}$$

$$\frac{1.5}{v} + \frac{2}{R} = \frac{1.5-1}{-R} \Rightarrow v = \frac{-3R}{5}$$

The image is at a distance $\left(R - \frac{3R}{5}\right) = 0.4R$ from the centre *P* towards the right hand side.

For the surface on the left hand side

$$u = \frac{-3R}{2}, R_2 = -R, \mu_1 = 1, \mu_2 = 1.5$$

So, $\frac{1.5}{v} + \frac{2}{3R} = \frac{1.5-1}{-R}$ or $v = \frac{-9R}{7}$

The image is at a distance of $\left(\frac{9R}{7} - R\right) = \frac{2R}{7}$ from the centre *P* towards the right hand side.

∴ The distance between the two images is

$$= 0.4R - \frac{2R}{7} = 0.114\, R$$

Therefore answer is (c).

2. **(d):** Let energy, $E \propto P^a A^b T^c$

or $E = kP^a A^b T^c$

or $[ML^2T^{-2}] = [MLT^{-1}]^a[L^2]^b[T]^c = [M^aL^{a+2b}T^{-a+c}]$

whence, $a = 1, b = \frac{1}{2}, c = -1$

Dimensional formula for *E* is $[P^1A^{1/2}T^{-1}]$.

3. **(c) :** Using Gauss theorem for spherical surface of radius *r* outside the sphere with the charge *q*

$$\int_0^r E ds = \frac{1}{\varepsilon_0}\int_0^r \rho(r) dV$$

$$E4\pi r^2 = \frac{1}{\varepsilon_0}\left(q + \int_R^r \frac{\alpha}{r}(4\pi r^2)dr\right)$$

$$E4\pi r^2 = \frac{(q - 2\pi\alpha R^2)}{\varepsilon_0} + \frac{4\pi\alpha r^2}{2\varepsilon_0}$$

The intensity *E* does not depend on *R* if

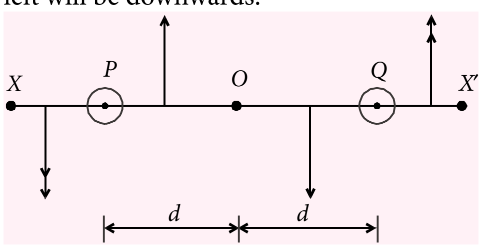
$$\frac{q - 2\pi\alpha R^2}{\varepsilon_0} = 0 \text{ or } q = 2\pi\alpha R^2$$

4. **(c) :** For a pendulum, $T = 2\pi\sqrt{\frac{l}{g}}$ where l is measured upto centre of gravity. The centre of gravity of system is at centre of sphere when hole is plugged. When unplugged, water drains out. Centre of gravity goes on descending. When the bob becomes empty, centre of gravity is restored to centre.

∴ Length of pendulum first increases, then decreases to original value.

∴ *T* would first increase and then decrease to the original value

5. **(b):** The steady currents in the two parallel wires flow out of paper. *O* is a point in the centre between two wires. The wires are at *P* and *Q*. Since the current flows out of the plane of the paper, the magnetic field at points to the right of given wire will be upwards and to the left will be downwards.



(i) At point *O*, net field will be zero. The fields due to wires at *P* and *Q* are equal in magnitude but opposite in directions.

(ii) Region *QX′* : Magnetic field will be upwards. The region lies to right of both the wires.

(iii) Region OQ : Net field will be downwards. Due to nearness, wire at Q dominates.

(iv) Region OP : Net field will be upwards. Due to nearness, wire at P dominates.

(v) Region PX : Net field will be downwards. The region lies to left of both the wires.

Graph (b) satisfies all the five conditions.

6. **(a) :** For a hydrogen atom,

$$E_1 = -13.6 \text{ eV}, E_2 = \frac{13.6}{2^2} = -3.4 \text{ eV}$$

$\therefore \quad E_2 - E_1 = -3.4 + 13.6 = 10.2$ eV

During inelastic collision, photon of energy 10.2 eV is absorbed by the hydrogen atom. This excites the electron from ground state to the second orbit. The electron jumps back to the ground state in less than a microsecond releasing a photon of energy 10.2 eV.

The second photon of 15 eV energy will ionise the atom because ionisation energy is just 13.6 eV. The knocked out electron will retain the balance energy = 15 – 13.6 = 1.4 eV.

7. **(c) :**

$$a = \frac{m_2 g - m_1 g \sin\theta}{m_1 + m_2} = \frac{(4g - 8g\sin 30°)}{m_1 + m_2} = 0$$

8. **(a) :** Energies of incident photons of different wavelengths will be

$$E(550 \text{ nm}) = \frac{hc}{\lambda} = \frac{1240}{550} = 2.25 \text{ eV}$$

$$E(450 \text{ nm}) = \frac{1240}{450} = 2.75 \text{ eV}$$

$$E(350 \text{ nm}) = \frac{1240}{350} = 3.54 \text{ eV}$$

In case of plate p, all radiations will cause photoelectric emission.

In case of metal q, wavelengths 450 nm and 350 nm will cause photoelectric emission.

In case of metal r, only wavelength 350 nm will cause photoelectric emission.

Hence saturation current will be maximum for plate p, intermediate for q and minimum for r.

9. **(b) :** The thermal resistance of each rod is R.

In case (i) the rods are in series.

$\therefore \quad R_S = R + R = 2R$

Rate of flow of heat will be

$$\frac{dQ}{dt} = \frac{\Delta T}{R_S}; \ \frac{10}{2} = \frac{(100-0)}{2R}$$

$\therefore \quad R = 10$

In case (ii) the rods are parallel; so

$$\frac{1}{R_P} = \frac{1}{R} + \frac{1}{R} \Rightarrow R_P = \frac{R}{2} = 5$$

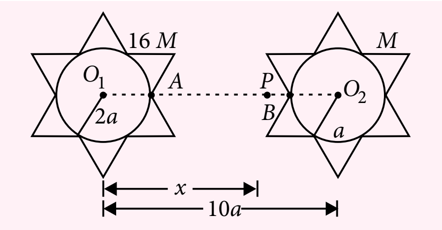
Now, rate of flow of heat in this case will be

$$\frac{dQ}{dt} = \frac{\Delta T}{R_P} \Rightarrow \frac{30}{t} = \left(\frac{100-0}{5}\right)$$

or $\quad t = 1.5$ minute.

10. **(b) :** First we have to find a point where the resultant field due to both is zero. Let the point P be at a distance x from centre of bigger star.

$$\Rightarrow \quad \frac{G(16M)}{x^2} = \frac{GM}{(10a - x)^2} \Rightarrow x = 8a(\text{from } O_1)$$



i.e., once the body reaches P, the gravitational pull of attraction due to M takes the lead to make m move towards it automatically as the gravitational pull of attraction due to $16M$ vanishes i.e., a minimum KE or velocity has to be imparted to m from surface of $16M$ such that it is just able to overcome the gravitational pull of $16M$. By law of conservation of energy

$$\frac{1}{2}mv^2 + \left[-\frac{G(16M)m}{2a} - \frac{GMm}{8a}\right] = 0$$
$$+\left[-\frac{GMm}{2a} - \frac{G(16M)m}{8a}\right]$$

$$\Rightarrow \quad \frac{1}{2}mv^2 - \frac{GMm}{8a}(45) \Rightarrow v = \frac{3}{2}\sqrt{\frac{5GM}{a}}$$

11. **(b) :** Comparing $B = B_0 \sin(kz - \omega t)$ T with

$B = 12 \times 10^{-8} \sin(1.20 \times 10^7 z - 3.60 \times 10^{15} t)$ T,

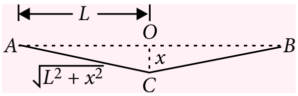
we get $B_0 = 12 \times 10^{-8}$ T

Average intensity of the beam,

$$I_v = \frac{cB_0^2}{2\mu_0}$$

$$= \frac{3\times10^8 \times (12\times10^{-8})^2}{2\times4\pi\times10^{-7}} = 1.72 \text{ W m}^{-2}$$

12. (a) :



Change in length, $\Delta L = AC - AO$

$$= [L^2 + x^2]^{1/2} - L = L\left[1 + \frac{1}{2}\frac{x^2}{L^2}\right] - L = \frac{x^2}{2L}$$

Longitudinal strain $= \frac{\Delta L}{L} = \frac{x^2/2L}{L} = \frac{x^2}{2L^2}$

13. (a) : $\frac{X}{R_0} = \frac{l}{100-l}$

Since null point remains unchanged

$$\frac{X}{R_0} = \frac{40}{60}; R_0 = 6\,\Omega$$

And $6 = \frac{100R_t}{R_t + 100}; R_t = 6.38\,\Omega$

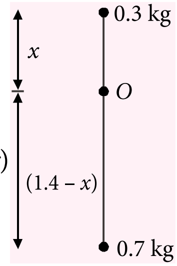
$$\therefore \quad \alpha = \frac{R_t - R_0}{R_0 \Delta T} = 6.3\times10^{-4} \text{ K}^{-1}$$

14 (b): In the given circuit, diode D_1 is forward biased while D_2 is reverse biased. Moreover, D_1 has a forward resistance of 50 Ω.

$$\therefore \quad I = \frac{6}{50+150+100} = \frac{6}{300} = 0.02 \text{ A}$$

Current through 100 Ω resistance = 0.02 A.

15. (c) : Let the axis of rotation pass through O.



$I = mr^2$ for point mass.

$\therefore \quad I = I_1 + I_2$

$= 0.3\,x^2 + 0.7\,(1.4 - x)^2$

$= 0.3x^2 + 0.7\,(1.96 + x^2 - 2.8x)$

$= x^2 + 1.372 - 1.96x$

The work done for rotation of the rod is stored as rotational kinetic energy, $\frac{1}{2} I\omega^2$, of rod.

or $\quad W = \frac{I\omega^2}{2} = \frac{1}{2}(x^2 + 1.372 - 1.96x)\omega^2$

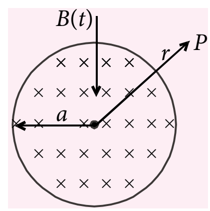
For work done to be minimum, $\frac{dW}{dx} = 0$

$$\therefore \quad \frac{d}{dx}\left[(x^2 + 1.372 - 1.96x)\right]\frac{\omega^2}{2} = 0$$

or $\quad 2x + 0 - 1.96 = 0$

or $\quad 2x = 1.96 \quad$ or $\quad x = 0.98$ m

16. (b): Magnetic field $B(t)$ is directed into the plane of the paper. P lies outside the field.



$$\oint \vec{E}\,.\,d\vec{l} = \frac{d\phi}{dt}$$

or $\quad E(2\pi\, r) = \frac{d}{dt}(\vec{B}\bullet\vec{A})$

or $\quad 2\pi r E = \pi a^2 \left(\frac{dB}{dt}\right)\cos 0°$

or $\quad E = \frac{a^2}{2r}\frac{dB}{dt}$ or $E \propto \frac{1}{r}$

17. (d)

18. (a) : There is n moles of the monoatomic gas in the container.
molar mass of the gas = m,
Total mass of the gas in the container, $M = mn$
Change in KE of the gas when the container is suddenly stopped, *i.e.*,

$$\Delta K = (\text{KE})_{\text{initial}} - (\text{KE})_{\text{final}} = \frac{1}{2}Mv_0^2 - 0$$
$$= \frac{1}{2}Mv_0^2 = \frac{1}{2}mnv_0^2$$

This change in kinetic energy (ΔK) result in a change in internal energy (ΔU) of the gas.

$$\Delta U = nC_V\Delta T = n\left(\frac{3}{2}R\right)\Delta T$$

Here, ΔT is the change in temperature of the gas.

As $\Delta U = \Delta K,\ n\left(\frac{3}{2}R\right)\Delta T = \frac{1}{2}mnv_0^2$

$$\therefore \quad \Delta T = \frac{mv_0^2}{3R}$$

19. (c)

20. (a) : The position of the central bright fringe will depend on angle θ, thickness of plate, distance between the slits and refractive index of the glass. For example, if $d \sin\theta = (\mu - 1)t$, the central bright fringe is at O. If $d \sin\theta > (\mu - 1)t$, the central bright frigne lies above O, and if $d \sin\theta < (\mu - 1)t$, the central bright fringe lies below O. Therefore the answer is (a).

21. (b) : $C_1 = 4\pi\varepsilon_0 b, C_2 = \dfrac{4\pi\varepsilon_0 ab}{b-a}$

$$C_3 = \frac{4\pi\varepsilon_0 b^2}{b-a}, C_4 = 4\pi\varepsilon_0 b$$

Clearly $C_1 = C_4$

also $C_3 > C_1 = C_4 \quad \left(\because \dfrac{b^2}{b-a} > b\right)$

Now $C_2 = \dfrac{4\pi\varepsilon_0 ab}{b-a} \approx 4\pi\varepsilon_0 a$

$$\Rightarrow \quad C_3 > C_1 = C_4 > C_2$$

22. (a) : The steel wire is first stretched by an object of specific gravity ρ in air. Then the object is half submerged in water. The stretching force diminishes due to upthrust of water on the object. Let σ denote specific gravity of water.
Weight of the object = $V\rho g$

Upthrust of water on object = $\dfrac{V}{2}\sigma g$.

$$\therefore \quad \text{Tension } T' = V\rho g - \frac{V\sigma g}{2}$$

or $\quad T' = Vg\left(\dfrac{2\rho - \sigma}{2}\right)$

$$\upsilon = \frac{1}{2l}\sqrt{\frac{T}{\mu}} \quad \text{where } T = V\rho g$$

$$\upsilon' = \frac{1}{2l}\sqrt{\frac{T'}{\mu}} \quad \therefore \quad \frac{\upsilon'}{\upsilon} = \sqrt{\frac{T'}{T}}$$

or $\quad \dfrac{\upsilon'}{\upsilon} = \sqrt{\dfrac{Vg\,(2\rho - 1)}{2} \times \dfrac{1}{Vg\rho}}$

or $\quad \upsilon' = \upsilon\sqrt{\dfrac{2\rho - 1}{2\rho}}$ or $\upsilon' = 300\left[\dfrac{2\rho - 1}{2\rho}\right]^{1/2}$.

23. (c) : Relative error in measurement of time,

$$\frac{\Delta t}{t} = \frac{1\text{ s}}{40\text{ s}} = \frac{1}{40}$$

Time period, $T = \dfrac{40\text{ s}}{20} = 2\text{ s}$

Error in measurement of time period,

$$\Delta T = T \times \frac{\Delta t}{t} = 2\text{ s} \times \frac{1}{40} = 0.05\text{ s}$$

The time period of simple pendulum is

$$T = 2\pi\sqrt{\frac{l}{g}} \quad \text{or} \quad T^2 = \frac{4\pi^2 l}{g} \quad \text{or} \quad g = \frac{4\pi^2 l}{T^2}$$

$$\therefore \quad \frac{\Delta g}{g} = \frac{2\Delta T}{T} = 2 \times \frac{1}{40} = \frac{1}{20}\left(\because \frac{\Delta T}{T} = \frac{\Delta t}{t}\right)$$

Percentage error in determination of g is

$$\frac{\Delta g}{g} \times 100 = \frac{1}{20} \times 100 = 5\%$$

24. (c) : As intensity due to electric field

$$I_E = \frac{1}{2}c\varepsilon_0 E^2,$$

Intensity due to magnetic field $I_B = \dfrac{cB^2}{2\mu_0}$

$$\frac{I_E}{I_B} = \frac{\frac{1}{2}c\varepsilon_0 E^2}{cB^2/2\mu_0} = (\varepsilon_0\mu_0)(E/B)^2 = \left(\frac{1}{c^2}\right)(c^2) = 1$$

$$\left(\text{as } c = \frac{1}{\sqrt{\varepsilon_0\mu_0}} \text{ and } E/B = c\right)$$

25. (b) : Reynolds number,

$$R_e = \frac{v\rho D}{\eta} \text{ and } v = \frac{4Q}{\pi D^2} \left(\text{as } Q = \frac{\pi D^2}{4} v\right),$$

$$R_e = \frac{4Q\rho}{\pi D \eta}$$

Here,

$D = 1.25$ cm $= 1.25 \times 10^{-2}$ m

$\rho = 10^3$ kg m^{-3}, $\eta = 10^{-3}$ Pa s

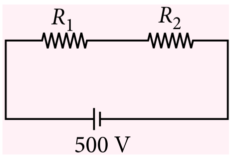
$Q = 5 \times 10^{-5}$ m^3 s^{-1}

$$R_e = \frac{4 \times 5 \times 10^{-5} \times 10^3}{3.14 \times 1.25 \times 10^{-2} \times 10^{-3}} = 5095 \approx 5100.$$

For $R_e > 3000$, the flow is turbulent.

26. (c) : Let resistance of bulbs are R_1 and R_2

$$\Rightarrow \quad \frac{(200)^2}{R_1} = \frac{(300)^2}{R_2}$$



$$\frac{R_2}{R_1} = \frac{9}{4}$$

$$V_1 = IR_1 = \left(\frac{500}{R_1 + R_2}\right) \times R_1$$

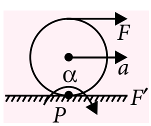
$$V_2 = \frac{500}{R_1 + R_2} \times R_2$$

$$\frac{V_1}{V_2} = \frac{R_1}{R_2} = \frac{4}{9}$$

$$\frac{P_1}{P_2} = \left(\frac{V_1}{V_2}\right)^2 \times \frac{R_2}{R_1} = \frac{4}{9}$$

27. (d) : Let F' be the friction on the ring towards right, a its linear acceleration and α the angular acceleration about center of mass. Point of contact P is momentarily as rest *i.e.*, ring will rotate about P.

$$\alpha = \frac{\tau_P}{I_P} = \frac{F(2R)}{2MR^2} = \frac{F}{MR}$$



Now $\quad F + F' = Ma = MR\alpha = F$

or $\quad F' = 0$

28. (c) : Efficiency of a perfect engine working between – 3°C and 27°C (*i.e.*, T_2 = 270 K and T_1 = 300 K)

$$\eta_{\text{engine}} = 1 - \frac{T_2}{T_1} = 1 - \frac{270 \text{ K}}{300 \text{ K}} = 0.1$$

Since efficiency of the refrigerator ($\eta_{\text{ref.}}$) is 50% of η_{engine}

$\therefore \quad \eta_{\text{ref.}} = 0.5\, \eta_{\text{engine}} = 0.05$

If Q_1 is the heat transferred per second at higher temperature by doing work W, then

$$\eta_{\text{ref.}} = \frac{W}{Q_1} \text{ or } Q_1 = \frac{W}{\eta_{\text{ref.}}} = \frac{1 \text{ kJ}}{0.05} = 20 \text{ kJ}$$

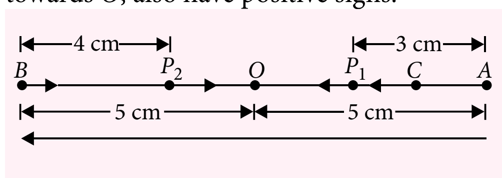
(as W = 1 kW × 1s = 1 kJ)

Since $\eta_{\text{ref.}}$ is 0.05, heat removed from the refrigerator per second, *i.e.*,

$Q_2 = Q_1 - \eta_{\text{ref.}}\, Q_1 = Q_1(1 - \eta_{\text{ref.}})$

$= 20$ kJ $(1 - 0.05) = 19$ kJ

29. (b) : (a) Particle at P_1 is 3 cm away from A and is going towards B. As such sign of v is positive. Force and acceleration, which are directed towards O, also have positive signs.



(b) Particle at C is going from A to B and as such sign of v is positive.

(c) Particle at P_2 is 4 cm away from B and is going towards O. As such sign of v is negative. Force and acceleration which are directed towards O also have negative signs.

(d) Particle at B is going towards O and as such signs of acceleration and force are negative.

30. (c) : In β-emission, a neutron of nucleus decays into a photon and a β-particle.

$${}^1_0 n \rightarrow {}^1_1\text{H} + {}^{\;\,0}_{-1}e$$

"Every great and deep difficulty bears in itself its own solution. It forces us to change our thinking in order to find it."

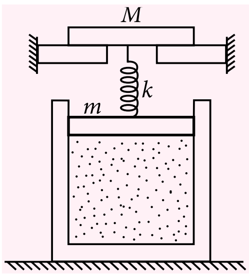
– Niels Bohr

Thought Provoking Problems

Thermodynamics

By : Prof. Rajinder Singh Randhawa*

1. 0.01 mole of an ideal diatomic gas is enclosed in an adiabatic cylinder of cross-sectional area $A = 10^{-4}\ m^2$. In the arrangement shown in figure, a block of mass $M = 0.8$ kg is placed on a horizontal support, and another block of mass $m = 1$ kg is suspended from a spring of stiffness constant $k = 16\ N\ m^{-1}$. Initially, the spring is relaxed and the volume of the gas is $V = 1.4 \times 10^{-4}\ m^3$.

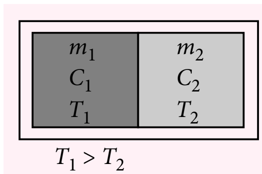


(a) Find the initial pressure of the gas.

(b) If the block m is gently pushed down and released, it oscillates harmonically, find its angular frequency of oscillation.

(c) When the gas in the cylinder is heated up, the piston starts moving up and spring gets compressed so that the block M is just lifted up. Determine the heat supplied. (Take atmospheric pressure, $P_0 = 10^5\ N\ m^{-2}$ and $g = 10\ m\ s^{-2}$).

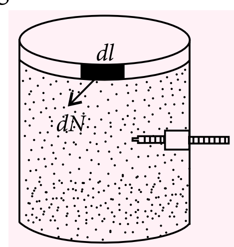
2. An insulated container has a thin membrane partition. Each of the two chambers has two different liquids as shown in figure. The membrane is punctured and liquids are allowed to mix. Find the entropy change associated with this free mixing of liquids.



3. Two blocks with heat capacities C_1 and C_2 are connected by a rod of length l, cross-sectional area A and heat conductivity K. Initial temperature difference between the two blocks is T_0. Assuming the entire system to be isolated from surroundings, heat capacity of the rod to be negligible, determine the temperature difference between the blocks as a function of time.

4. A cylindrical tube of radius r and length l, fitted with a cork is shown in figure.

The coefficient of friction between the cork and the tube is μ. The tube contains an ideal gas at temperature T, and atmospheric pressure P_0. The tube is slowly heated, the cork pipe out when temperature is doubled.



What is normal force per unit length exerted by the cork on the periphery of tube? Assume uniform temperature throughout gas at any instant.

*Randhawa Institute of Physics, S.C.O. 208, First Fl., Sector-36D & S.C.O. 38, Second Fl., Sector-20C, Chandigarh, Ph. 09814527699

5. An ideal gas of molar mass M is filled in a horizontal cylinder closed at one end. The cylinder rotates with a constant angular velocity ω about a vertical axis passing through the open end of the cylinder. The pressure at the axis passing through the open end of the cylinder is P_0 and temperature T. Find the air pressure as a function of the distance r from the rotation axis, assuming isothermal condition.

6. A double-plane window consists of two glass sheets each of area 1 m^2 and thickness 0.01 m separated by a 0.05 m thick stagnant air space. In the steady state, the room glass interface and the glass outdoor interface are at constant temperature of 27°C and 0°C respectively.

(Given, $K_{glass} = 0.8 \text{ W m}^{-1} \text{ K}^{-1}$

$K_{air} = 0.08 \text{ W m}^{-1} \text{ K}^{-1}$)

(a) Calculate the rate of heat flow through the window pane.

(b) Find the temperature of other interfaces.

SOLUTIONS

1. (a)

$$P = P_0 + \frac{mg}{A} = 10^5 + \frac{(1)(10)}{10^{-4}} = 2\times10^5 \text{ N m}^{-2}$$

(b) For an adiabatic process, $\frac{dP}{P} = -\gamma\frac{dV}{V}$

or $dP = -\gamma P\frac{dV}{V}$

Let the block be displaced by small distance x.

$\therefore$ The resultant restoring force on the piston,

$$m\frac{d^2x}{dt^2} = -\left[\gamma PA\frac{dV}{V} + kx\right] = -\left[\frac{\gamma PA^2 x}{V} + kx\right]$$

$$\therefore \frac{d^2x}{dt^2} = -\left[\frac{\gamma PA^2}{mV} + \frac{k}{m}\right]x$$

which gives,

$$\omega = \sqrt{\frac{\gamma PA^2}{mV} + \frac{k}{m}}$$

$$= \sqrt{\frac{(1.4)(2\times10^5)(10^{-4})^2}{1\times1.4\times10^{-4}} + \frac{16}{1}} = 6 \text{ rad s}^{-1}$$

(c) Compression in the spring to lift the block is given by

$$x' = \frac{Mg}{k} = \frac{(0.8)\times10}{16} = 0.5 \text{ m}$$

Work done by the gas is

$$W = mgx' + \frac{1}{2}kx'^2 + P_0Ax'$$

$$W = 1\times10\times(0.5) + \frac{1}{2}\times16\times(0.5)^2 + 10^5\times10^{-4}\times(0.5) = 12 \text{ J}$$

$\therefore$ Change in internal energy,

$$\Delta U = \frac{5}{2}nR(T'-T) = \frac{5}{2}(P'V'-PV)$$

As, $P' = P_0 + \frac{mg}{A} + \frac{kx'}{A}$

$$= 10^5 + \frac{1\times10}{10^{-4}} + \frac{16\times0.5}{10^{-4}} = 2.8\times10^5 \text{ N m}^{-2}$$

$V' = V + Ax' = (1.4\times10^{-4}) + (10^{-4})\times(0.5)$
$= 1.9\times10^{-4} \text{ m}^3$

$$\Delta U = \frac{5}{2}[(2.8\times10^5)(1.9\times10^{-4}) - (2\times10^5)(1.4\times10^{-4})] = 63 \text{ J}$$

Using first law of thermodynamics,

$Q = W + \Delta U = 12 + 63 = 75 \text{ J}$

2. Since $Q_{net} = 0$

$\therefore$ $m_1c_1(T_r - T_1) + m_2c_2(T_r - T_2) = 0$

$\therefore$ Resultant temperature,

$$T_r = \frac{m_1c_1T_1 + m_2c_2T_2}{m_1c_1 + m_2c_2}$$

Change in entropy of liquid 1,

$$\Delta S_1 = \int_{T_1}^{T_r}\frac{dQ}{T} = m_1c_1\ln\frac{T_r}{T_1}$$

Since $T_r < T_1$, this entropy change is negative.
The entropy change in liquid 2,

$$\Delta S_2 = m_2c_2\ln\frac{T_r}{T_2}$$

Since $T_r > T_2$, this entropy change is positive.

$$\Delta S_{total} = \Delta S_1 + \Delta S_2 = m_1c_1\ln\frac{T_r}{T_1} + m_2c_2\ln\frac{T_r}{T_2}$$

The first term is negative and second is positive, but total entropy change is positive.

Special case : $m_1 = m_2$, $c_1 = c_2$

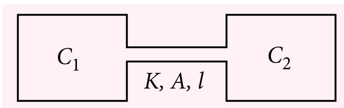
$$\therefore T_r = \frac{T_1 + T_2}{2}$$

$$\Delta S_{total} = mc\ln\frac{T_r^2}{T_1T_2} = mc\ln\left[\frac{\left(\frac{T_1+T_2}{2}\right)^2}{T_1T_2}\right]$$

Since $\frac{T_1 + T_2}{2}$ is the arithmetic mean of two temperatures T_1 and T_2, $\sqrt{T_1 T_2}$ is geometric mean, and $AM > GM$.

$\therefore$ $\Delta S_{total} > 0$ which shows that total entropy change will be positive in this irreversible process of mixing.

3. Let T be the temperature difference between two blocks at time t.



Heat transferred per second,

$$\frac{dQ}{dt} = \frac{KAT}{l} \quad \text{...(i)}$$

Also, $dT = dT_1 + dT_2$ ...(ii)

Heat lost by one block is equal to the heat gained by the other, $C_1 dT_1 = C_2 dT_2$ ...(iii)

From equations, (ii) and (iii), we get

$$dT = \left(\frac{C_1 + C_2}{C_2}\right) dT_1 \quad \text{...(iv)}$$

If a block loses heat, $dQ = -C_1 dT_1$,

From equation (i), $\frac{dQ}{dt} = -C_1 \frac{dT_1}{dt} = \frac{KAT}{l}$ ...(v)

From equations (iv) and (v), we get

$$\frac{-C_1 C_2}{C_1 + C_2} \frac{dT}{dt} = \frac{KA}{l} T$$

$$\frac{dT}{T} = \frac{-KA(C_1 + C_2)}{C_1 C_2} dt$$

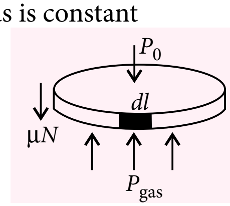
$$\int_{T_0}^{T} \frac{dT}{T} = \frac{-KA(C_1 + C_2)}{C_1 C_2} \int_0^t dt$$

$$\ln \frac{T}{T_0} = \frac{-KA(C_1 + C_2)}{C_1 C_2} t$$

$$T = T_0 \exp\left(\frac{-KA(C_1 + C_2)t}{C_1 C_2}\right)$$

4. Since the volume of the gas is constant

$$\frac{P_i}{T_i} = \frac{P_f}{T_f}$$



$$P_f = P_i\left(\frac{T_f}{T_i}\right) = 2P_i = 2P_0$$

$$(\because \quad T_f = 2T_i)$$

At equilibrium, $P_0 \times A + \mu N = 2P_0 A$

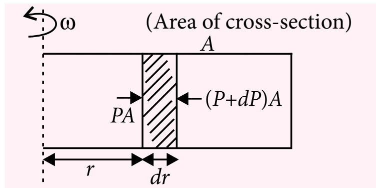
or $N = \frac{P_0 A}{\mu}$

N is the total normal force exerted by the tube on the cork,

hence contact force per unit length is

$$\frac{dN}{dl} = \frac{N}{2\pi r} = \frac{P_0 A}{2\pi \mu r}$$

5. We consider a differential layer of thickness dr, at a distance r from axis.



$$(P + dP)A - PA = (dm)r\omega^2$$

or $dP\,A = (\rho A dr)r\omega^2$ or $dP = \rho\omega^2 r dr$ ...(i)

Since, $\rho = \frac{PM}{RT}$ ...(ii)

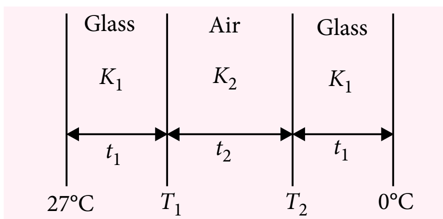
(from ideal gas equation)

Putting (ii) in equation (i), we get

$$dP = \frac{PM}{RT}\omega^2 r\,dr \text{ or } \int_{P_0}^{P} \frac{dP}{P} = \frac{M\omega^2}{RT}\int_0^r rdr$$

$$\ln \frac{P}{P_0} = \frac{M\omega^2}{RT}\frac{r^2}{2} \text{ or } P = P_0\, e^{M\omega^2 r^2/2RT}$$

6.



(a) Total thermal resistance is

$$R = \frac{t_1}{K_1 A_1} + \frac{t_2}{K_2 A_2} + \frac{t_1}{K_1 A_1}$$

$$R = 2 \times \frac{0.01}{0.8 \times 1} + \frac{0.05}{0.08 \times 1} = 0.65\ \text{W}^{-1}\text{K}$$

$\therefore$ Heat current, $H = \frac{\Delta T}{R} = \frac{27 - 0}{0.65} = 41.5$ W

(b) $T_1 = 27 - \frac{Ht_1}{K_1 A_1} = 27 - 0.52 = 26.48\ °\text{C}$

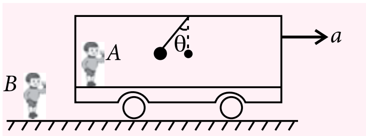
and $T_2 = 0 + \frac{Ht_1}{K_1 A_1} = 0.52\ °\text{C}$

CORE CONCEPT on

Pseudo Force

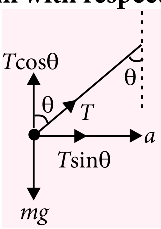
Pseudo Force

Before we describe the term, let us consider the simple situation of a bus accelerating horizontally from the roof of which a pendulum is hanging. Two observers A and B are observing the situation, A is inside the bus while B is on ground. Let us see how both of them describe the motion.



The bob gets inclined at an angle θ with respect to vertical at equilibrium (steady state and not $F_{net} = 0$). B begins by describing that since the bob is not accelerated with respect to the bus hence, it too has the same acceleration as that of bus.

Free body diagram with respect to *B*



Vertically it is not accelerated, hence,

$$F_{vertical} = 0$$

$\Rightarrow$ $T\cos\theta - mg = 0$ ... (i)

Horizontally it is accelerated, so

$$F_{horizontal} = ma$$

$\Rightarrow$ $T\sin\theta = ma$ ... (ii)

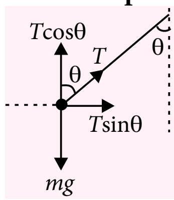
From (i) and (ii), we get

$$\tan\theta = \frac{a}{g}$$

Now, let us see, how A describes the situation.

A says, that he can see the bob is at equilibrium, *i.e.* not accelerated. Hence the net force in all directions should be zero.

Free body diagram with respect to *A*



Along the vertical,

$T\cos\theta - mg = 0$ ... (iii)

(Exactly identical to equation (i) that B wrote.)

But the problem starts when he starts considering the horizontal direction.

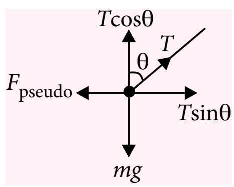
He finds that there is a net force $T\sin\theta$ but no acceleration in this direction, which is quite contradict according to him, since he knows that for any direction,

$F_{net} = ma$ is valid from Newton's second law of motion.

So where is he wrong?

The problem is with his frame of reference, which is an accelerated frame, *i.e.*, a non-inertial frame and Newton's laws are not valid in such frames in their original form.

We can justify the equilibrium if there is a force in opposite direction to balance $T\sin\theta$. This force is said to be pseudo force. It is a ficticious force also known as inertial force.



Clearly, if

$$T\sin\theta - F_{pseudo} = 0,$$

then equilibrium can be established.

Equation (ii) suggest that

$F_{pseudo} = ma$ ... (iv)

Contributed By: Bishwajit Barnwal, Aakash Institute, Kolkata

Hence, $T\sin\theta - ma = 0$ ... (v)

From (iii) and (v) we again have, $\tan\theta = \frac{a}{g}$, same as obtained for B.

So, from next time, when you say that pseudo force is acting on an object, ask yourself, on which frame are you situated, is it an accelerated or a non-accelerated frame, since pseudo forces aren't applied from non accelerated frames.

Two important things about pseudo force

- The magnitude of pseudo force is F_{pseudo} = mass of the object under consideration × acceleration of the frame from which the object is being observed.
- The direction of the pseudo force is opposite to the acceleration of the frame.

Hence, we will always have two ways to solve such question in which one object moves while there are other objects which are over these objects. One by fixing our frame on the accelerated object and the other by standing on ground.

So, how do we know, which frame to chose? The answer is obvious. If one object A appears to move with respect to another object B which itself is in motion, then it would be better to analyse the motion of A with respect to B since its motion with respect to ground would be a lot complicated.

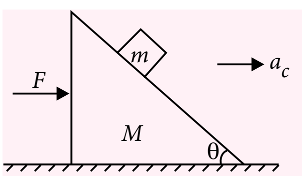
Let us try to understand these by taking few examples.

Example 1 : A block of mass m is placed over a smooth wedge of mass M which is placed on smooth horizontal ground. Find the magnitude of horizontal force to be exerted on wedge such that

(i) block can stay in equilibrium with respect to wedge

(ii) block leaves contact with wedge.

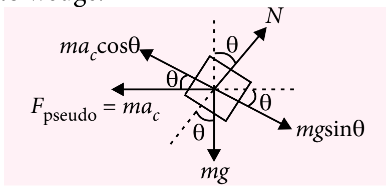
Sol.: (i)



Since the block does not move relative to wedge, they can together be taken as the system. Hence,

$$F = (m + M)a_c \quad \text{... (i)}$$

Now consider free body diagram of block with respect to wedge.



$\therefore \quad ma_c \cos\theta = mg\sin\theta$

$a_c = g\tan\theta$

$\therefore \quad F = (m + M)g\tan\theta$

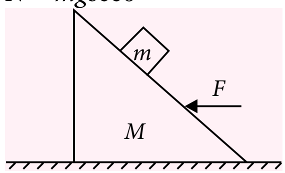
(Students should remember this as a standard result)

Clearly, the normal reaction in this case is

$$N = \sqrt{(mg)^2 + (ma_c)^2}$$

$$N = mg\sec\theta$$

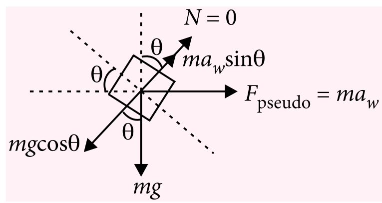
(ii)



Since the block is about to leave contact, normal reaction between the blocks will be zero. Hence wedge moves without block over it. Hence acceleration of wedge,

$$a_w = \frac{F}{M}$$

Hence, free body diagram with respect to M

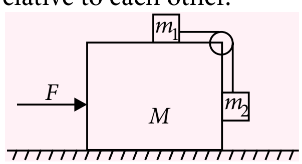


To make $N = 0$,

$ma_w \sin\theta \geq mg\cos\theta$

$\Rightarrow \quad F \geq Mg\cot\theta$

Example 2 : Find the value of F for which the blocks do not slip relative to each other.



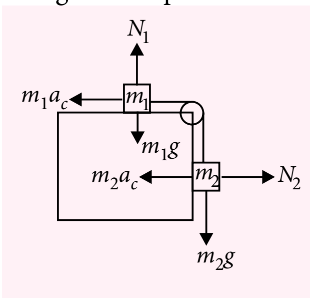
All surfaces are smooth.

$m_1 = 2m$, $m_2 = 5m$, $M = m$

Sol.: Since the blocks do not slip relative to each other, their common acceleration towards right is

$$a_c = \frac{F}{m_1 + m_2 + M} = \frac{F}{8m}$$

Now, considering with respect to M.



The situation in which blocks are connected end-to-end with string can always be compared to tug-of-war.

Since no slipping takes place,

$$m_1 a_c = m_2 g$$

$$\Rightarrow \quad 2m\frac{F}{8m} = 5mg$$

$$\Rightarrow \quad F = 20mg$$

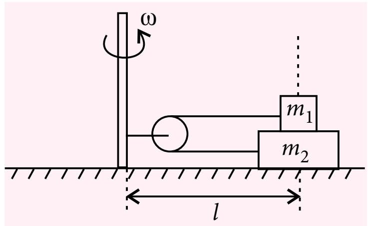
Note : A rotatory frame is also a non-inertial frame, hence we need to consider a pseudo force known as centrifugal force (directed away from the centre), whose magnitude will be $\frac{mv^2}{r}$ or $m\omega^2 r$ where

r = distance of the object from axis of rotation,

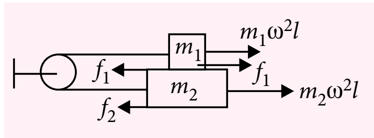
v = linear speed along the tangent to circular path,

ω = angular speed with respect to the axis of rotation which is equal to $\frac{v}{r}$

Example 3 : A horizontal turn tube is rotating with constant angular velocity ω at the centre of which a vertical pole is fixed which in turn is connected to two blocks m_1 and m_2 through a pulley as shown. Find maximum ω such that blocks do not slip relative to each other. Given, friction co-efficient is μ everywhere. $(m_2 > m_1)$



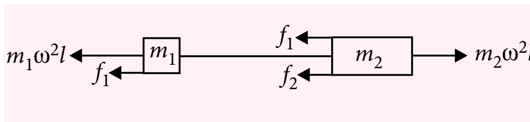
Soln.: When we consider with respect to rotating frame, we would exactly be able to understand, why the blocks try to slip relative to each other.



Since, $m_2 > m_1$, hence $m_2\omega^2 l > m_1\omega^2 l$

Therefore, m_2 has a tendency to move towards right, due to which m_1 has a tendency to move towards left. Hence the direction of friction can be determined.

Again comparing this with tug-of-war,



$\therefore$ For no slipping,

$$m_1\omega^2 l + 2f_1 + f_2 \ge m_2\omega^2 l$$

$$\therefore \quad (m_2 - m_1)\omega^2 l \le 2f_{1(\text{lim})} + f_{2(\text{lim})}$$

$$= 2\mu m_1 g + \mu(m_1 + m_2)g$$

$$\Rightarrow \quad (m_2 - m_1)\omega^2 l \le 3\mu m_1 g + \mu m_2 g$$

$$\Rightarrow \quad \omega \le \sqrt{\frac{(3m_1 + m_2)\mu g}{(m_2 - m_1)l}}$$

Solution Senders of Physics Musing

SET-18

1. Sadhna Tyagi (New Delhi)
2. Akash Jha (Bihar)
3. Gaurav Kumar (U.P.)

SET-17

1. R. Anantha Krishnan (Kerala)
2. Kashish Arora (Ambala Cantt)
3. Rushikesh Joshi (Maharashtra)

YOU ASKED WE ANSWERED

Do you have a question that you just can't get answered?

Use the vast expertise of our mtg team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough.

The best questions and their solutions will be printed in this column each month.

Q1. Why does the flame burn upright even when the thread associated with the lamp is inverted?

– Sudarshan. G.S. (Mysore, Karnataka)

Ans. When a lamp burns, it heats up the surrounding air. Since hot air is lighter than cold air, it rises above, making the flame burn upright and not upside down. That is why the flame burn upright even when the thread associated with the lamp is inverted.

Q2. Isothermal process is not possible in practical but theoretically it is. How?

– Krishna Yadav (Punjab)

Ans. In isothermal process, temperature of the system is constant and also equal to that of surrounding in contact with the system. Heat supplied to a system is possible if there is some temperature difference between the system and surrounding. Therefore during an isothermal expansion, it could be considered that the surroundings are at a slightly elevated temperature above the system. So the necessary condition that the system shall receive heat would be met. Since the temperature difference would be small, the rate of heat transfer would be very slow. So from a practical point of view, the isothermal process is very slow. Actually this is almost impossible as a practical process. Hence, isothermal process is not possible practically.

Q3. Human eyes resembles a photographic camera. Explain.

– Pankaj Mishra, Chandauli (U.P.)

Ans. A camera and the human eye have much in common. Cameras are made in ways that are similar to the ways in which an eye functions. A camera and an eye are both capable of viewing different colours. They are also both capable of seeing near and far, judging the size of different things. When light passes through the lens and cornea, it is focused in the back portion of the eye. This light beam is shone on the patch of photoreceptors which make up the retina. The retina changes the physical light energy into electrical pulses. This energy is then transmitted through the back of your eyes and into the brain. The brain then deciphers the electrical impulses it has been sent and produces pictures of what we are seeing. The cornea of the eye is the frontal, transparent surface of the eye, it is very similar to the lens of a camera. The purpose of spherical curvature is to allow the cornea and the lens view the left and right, not just straight ahead. Another similarity they possess is with the eye's iris and the camera's aperture. The size of a camera's aperture refers to the amount of light it let into the camera to be reflected onto the sensor. The iris performs a similar job, letting different amount of light in depending on the size of the pupil. Both the eye and the camera are able to focus on single objects while blurring the rest, regardless of if this object is in the foreground or the background. Both can focus on large images. The retina of the eye and the film of a camera perform similar functions. While the eyes retina collects reflected light from the surrounding environment, the film or digital sensors in a camera do the same thing. Both cameras and our eyes receive images upside down. However these faults are corrected by the brain (eye) and when the picture is digitally formatted or processed (camera).

Q4. Between stress and strain, which is cause and which is result? And why?

– Abhijit, Burdwan, (W.B.)

Ans. When a deforming force is applied on a body, it changes the configuration of the body by changing the normal positions of the molecules or atoms of the body. As a result an internal restoring force comes into play which tends to bring the body back to its initial configuration. This internal restoring force acting per unit area of a deformed body is called stress. When a deforming force is applied on a body, there is a change occur in the configuration of the body. The body is said to be strained or deformed. The ratio of change in configuration to the original configuration is called strain.

Stress causes a strain in a body, so we can say that stress is the cause and strain is the result.

Foundation Series

JEE

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UNIT-7 Magnetic Effects of Current, Magnetism, EMI & AC and Electromagnetic Waves

MAGNETIC FIELD

It is the region or space around a magnet or current carrying conductor or a moving charge in which its effect can be felt. It is a vector quantity and its dimensional formula is $[ML^0T^{-2}A^{-1}]$. SI unit of magnetic field is tesla (T) or Wb m^{-2}.

BIOT SAVART'S LAW

According to this law, the magnetic field at a point P due to a current element of length $d\vec{l}$, carrying current I, at a distance r from the element is

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,(d\vec{l}\times\vec{r})}{r^3} \quad \text{or} \quad d\vec{B} = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2}$$

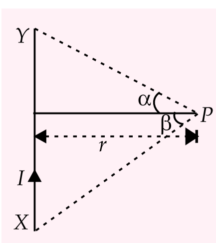
where θ is the angle between $d\vec{l}$ and $\vec{r}$ and μ_0 is the permeability of free space.

$\mu_0 = 4\pi \times 10^{-7}$ Wb A^{-1} m^{-1} or henry m^{-1}

The dimensional formula of μ_0 is $[MLT^{-2}A^{-2}]$.

The direction of $d\vec{B}$ is that of $d\vec{l}\times\vec{r}$, which is perpendicular to the plane containing $d\vec{l}$ and $\vec{r}$, and is directed as given by right hand screw rule.

MAGNETIC FIELD DUE TO A STRAIGHT CURRENT CARRYING WIRE



The magnetic field B at a point P due to a straight wire of finite length carrying current I at a perpendicular distance r is

$$B = \frac{\mu_0 I}{4\pi r}[\sin\alpha + \sin\beta].$$

- If the wire is of infinite length and the point P lies near the centre of the straight wire, then

$$\alpha = \beta = 90^\circ$$

$$B = \frac{\mu_0 2I}{4\pi r} = \frac{\mu_0 I}{2\pi r}$$

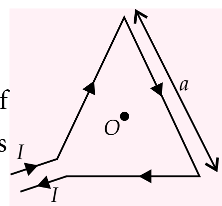
- If the wire is of infinite length and the point P lies near one end, then

$$\alpha = \frac{\pi}{2},\ \beta = 0^\circ$$

$$B = \frac{\mu_0 I}{4\pi r}$$

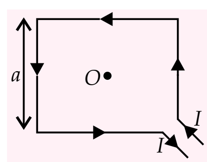
- The magnetic field at the centre O (*i.e.* centroid) of an equilateral triangle of side a, carrying current I is

$$B = \frac{9\mu_0 I}{2\pi a}$$



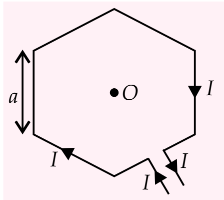
- The magnetic field at the centre O of a square coil of side a, carrying current I is

$$B = 2\sqrt{2}\,\frac{\mu_0 I}{\pi a}$$



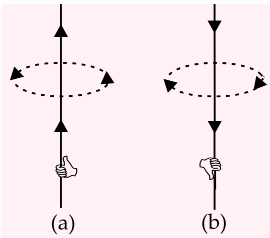
- The magnetic field at the centre O of a regular hexagonal coil of side a, carrying current I is

$$B = \frac{\mu_0 I\sqrt{3}}{\pi a}$$



Right Hand Thumb Rule

The direction of magnetic field due to a straight current carrying wire is given by right hand thumb rule. According to this rule, if you grasp the wire in your right hand with your extended thumb pointing in the direction of the current. Your fingers curling around the wire give the direction of the magnetic field lines.



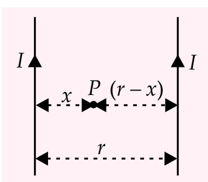
KEY POINT

- Whenever the current in a wire reverses direction, the force exerted on the wire by a given magnetic field also reverses its direction.

MAGNETIC FIELD DUE TO TWO LONG PARALLEL AND STRAIGHT CURRENT CARRYING WIRES

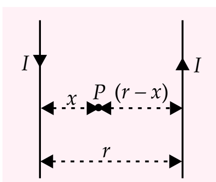
If the point P is in between the wires and currents in both the wires are in the same direction

$$B = \frac{\mu_0 I}{2\pi}\left(\frac{1}{x} - \frac{1}{r-x}\right)$$



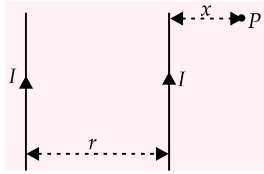
If the point P is in between the wires and currents in both the wires are in opposite directions

$$B = \frac{\mu_0 I}{2\pi}\left(\frac{1}{x} + \frac{1}{r-x}\right)$$



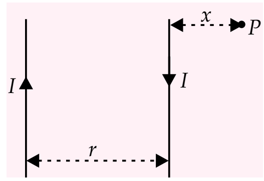
If the point P is outside the wires and current in both the wires are in the same direction

$$B = \frac{\mu_0 I}{2\pi}\left(\frac{1}{x} + \frac{1}{r+x}\right)$$



If the point P is outside the wires and current in both the wires are in opposite directions

$$B = \frac{\mu_0 I}{2\pi}\left(\frac{1}{x} - \frac{1}{r+x}\right)$$



MAGNETIC FIELD AT THE CENTRE OF A CURRENT CARRYING CIRCULAR COIL

The magnetic field at the centre of a circular coil of radius a carrying current I is

$$B = \frac{\mu_0}{4\pi}\frac{2\pi I}{a} = \frac{\mu_0 I}{2a}$$

If the circular coil consists of N turns, then

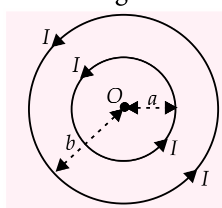
$$B = \frac{\mu_0}{4\pi}\frac{2\pi NI}{a} = \frac{\mu_0 NI}{2a}$$

The direction of magnetic field at the centre of a circular coil carrying current is given by right hand rule. According to which, the direction of magnetic field at the centre of the circular coil is perpendicular to the plane of coil downwards for the clockwise current, and perpendicular to the plane of coil outwards for the anticlockwise current.

The magnetic field at the common centre O of two concentric coils of radii a and b having turns N_1 and N_2 in which the same current I is flowing

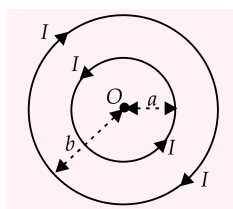
- when current flows in the same direction

$$B = \frac{\mu_0 I}{2}\left[\frac{N_1}{a} + \frac{N_2}{b}\right]$$



- when current flows in the opposite direction

$$B = \frac{\mu_0 I}{2}\left[\frac{N_1}{a} - \frac{N_2}{b}\right]$$



MAGNETIC FIELD AT A POINT ON THE AXIS OF A CIRCULAR CURRENT CARRYING COIL

The magnetic field at a point on the axis of the circular current carrying coil is

$$B=\frac{\mu_0}{4\pi}\frac{2\pi NIa^2}{(a^2+x^2)^{3/2}}$$

where a is the radius of coil, x is the distance of the point on the axis from the centre of the coil, N is the number of turns in the coil.

If the point lies at the centre of the coil, *i.e.* $x = 0$, then

$$B=\frac{\mu_0}{4\pi}\frac{2\pi NIa^2}{(a^2)^{3/2}}=\frac{\mu_0}{4\pi}\cdot\frac{2\pi NI}{a}=\frac{\mu_0 NI}{2a}$$

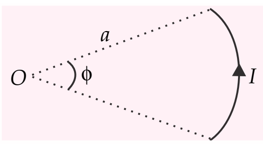
If $x >> a$, then

$$B=\frac{\mu_0}{4\pi}\frac{2\pi NIa^2}{x^3}=\frac{\mu_0}{4\pi}\frac{2NIA}{x^3}=\frac{\mu_0}{4\pi}\frac{2M}{x^3}\quad(\because NIA=M)$$

where M is the magnetic dipole moment of current loop and A is cross sectional area of loop.

MAGNETIC FIELD AT THE CENTRE DUE TO CURRENT CARRYING CIRCULAR ARC

The magnetic field at the centre O of the circular arc of radius a carrying current I is $B=\frac{\mu_0 I\phi}{4\pi a}$.



SELF CHECK

1. A current I flows in an infinitely long wire with cross-section in the form of a semicircular ring of radius R. The magnitude of the magnetic induction along its axis is
   (a) $\frac{\mu_0 I}{\pi^2 R}$ (b) $\frac{\mu_0 I}{2\pi^2 R}$
   (c) $\frac{\mu_0 I}{2\pi R}$ (d) $\frac{\mu_0 I}{4\pi R}$
   (AIEEE 2011)

AMPERE'S CIRCUITAL LAW

It states that the line integral of magnetic field around any closed path in vacuum is equal to μ_0 times the total current passing the closed path.

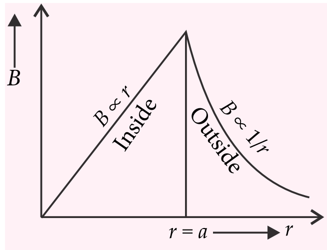
$$\oint \vec{B}\cdot d\vec{l}=\mu_0 I$$

Applications of Ampere's Circuital Law

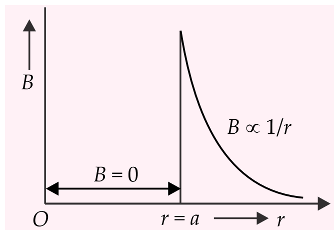
Magnetic field due to an infinitely long straight solid cylindrical wire of radius a, carrying current I

- Magnetic field at a point outside the wire *i.e.* $(r > a)$ is $B=\frac{\mu_0 I}{2\pi r}$
- Magnetic field at a point inside the wire *i.e.* $(r < a)$ is $B=\frac{\mu_0 Ir}{2\pi a^2}$
- Magnetic field at a point on the surface of a wire *i.e.* $(r = a)$ is $B=\frac{\mu_0 I}{2\pi a}$

The variation of magnetic field B and the distance r from axis is as shown in the figure.

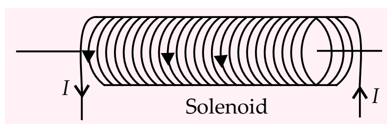


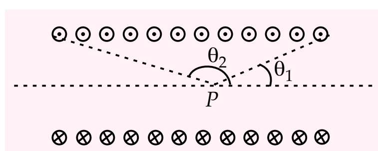
If the cylindrical wire is hollow *i.e.* it is in the form of pipe, then the magnetic field inside the wire is zero. The variation of magnetic field and distance r from the axis is as shown in the figure.



Solenoid

A solenoid consists of a long insulating wire closely wound in the form of a helix. Its length is very large as compared to its diameter.





The magnetic field at a point P on the axis of the solenoid having n turns per unit length is

$$B = \frac{\mu_0 nI}{2}(\cos\theta_1 - \cos\theta_2)$$

If the solenoid is of infinite length and point is in the middle of a solenoid, then $\theta_2 = 180°$, $\theta_1 = 0°$

$$\therefore \quad B = \mu_0 nI$$

If the solenoid is of infinite length and point is near one end, then $\theta_1 = 0°$ and $\theta_2 = \frac{\pi}{2}$ $\therefore$ $B = \frac{\mu_0 nI}{2}$

KEY POINT

- Ampere's circuital law can be derived from the Biot-Savart law.
- Its relationship to the Biot-Savart law is similar to the relationship between Gauss's law and Coulomb's law.

FORCE BETWEEN TWO PARALLEL CURRENT CARRYING CONDUCTORS

Two parallel conductors carrying currents in the same direction attract each other while those carrying currents in the opposite direction repel each other. When two parallel conductors separated by a distance r carry currents I_1 and I_2, the magnetic field of one will exert a force on the other. The force per unit length on either conductor is $f = \frac{\mu_0}{4\pi}\frac{2I_1 I_2}{r}$

The force of attraction or repulsion acting on each conductor of length l due to currents in two parallel conductor is $F = \frac{\mu_0}{4\pi}\frac{2I_1 I_2}{r}$.

The force acting between currents in parallel wires is the basis for the definition of the ampere.

Definition of Ampere : One ampere is that constant current which, if maintained in two straight, parallel conductors of infinite lengths, of negligible circular cross-section, and placed 1 m apart in vacuum, would produce on each of these conductors a force of magnitude 2×10^{-7} N, per meter of wire length.

FORCE ON A CHARGED PARTICLE IN A UNIFORM ELECTRIC FIELD

When a charged particle of charge q moving or at rest is subjected to a uniform electric field $\vec{E}$, the force acting on it is $\vec{F} = q\vec{E}$

The direction of $\vec{F}$ is same as that of $\vec{E}$, if q is positive and $-\vec{E}$ if q is negative.

FORCE ON A CHARGED PARTICLE IN A UNIFORM MAGNETIC FIELD

When a charged particle of charge q, moving with velocity $\vec{v}$ is subjected to a uniform magnetic field $\vec{B}$, the force acting on it is $\vec{F} = q(\vec{v} \times \vec{B})$ or $F = qvB\sin\theta$ where θ is the angle between $\vec{v}$ and $\vec{B}$.

The direction of this force is perpendicular to the plane containing $\vec{v}$ and $\vec{B}$. $\vec{F} = 0$ if $\vec{v} = 0$, *i.e.* a charge at rest does not experience any magnetic force. $\vec{F} = 0$ if $\theta = 0°$ or $180°$ *i.e.*, the magnetic force vanishes if $\vec{v}$ is either parallel or antiparallel to the direction of $\vec{B}$. Force will be maximum if $\theta = 90°$, *i.e.*, if $\vec{v}$ is perpendicular to $\vec{B}$, the magnetic force has a maximum value and is given by $F_{max} = qvB$.

MOTION OF A CHARGED PARTICLE IN A UNIFORM MAGNETIC FIELD

When a charged particle of charge q and mass m moves with velocity $\vec{v}$ in a uniform magnetic field $\vec{B}$, the force acting on it is $F = qvB\sin\theta$. The following two case arise :

Case I : When the charged particle is moving perpendicular to the field *i.e.* $\theta = 90°$.

In this case path is circular. Radius of circular path is $R = \frac{mv}{Bq} = \frac{\sqrt{2mK}}{qB}$

where K is the kinetic energy of the charged particle.

Time period of revolution is $T = \frac{2\pi R}{v} = \frac{2\pi m}{qB}$.

The frequency is $\upsilon = \frac{1}{T} = \frac{qB}{2\pi m}$.

The angular frequency is $\omega = 2\pi\upsilon = \frac{qB}{m}$.

Case II : When the charged particle is moving at an angle θ to the field (other than 0°, 90° or 180°).

In this case, path is helical. Due to component of $\vec{v}$ perpendicular to $\vec{B}$, *i.e.*, $v_\perp = v\sin\theta$, the particle describes a circular path of radius R, such that

$$\frac{mv_\perp^2}{R} = qv_\perp B \quad \text{or} \quad R = \frac{mv_\perp}{qB} = \frac{mv\sin\theta}{qB}$$

Time period of revolution is $T = \frac{2\pi R}{v\sin\theta} = \frac{2\pi m}{qB}$.

The frequency is $\upsilon = \frac{qB}{2\pi m}$.

The angular frequency is $\omega = 2\pi\upsilon = \frac{qB}{m}$

The pitch of the helical path is

$$p = v_{\parallel} \times T = v\cos\theta \times T = \frac{2\pi mv}{qB}\cos\theta = \frac{2\pi R}{\tan\theta}$$

FORCE ON A CHARGED PARTICLE IN COMBINED UNIFORM ELECTRIC AND MAGNETIC FIELDS

When a charged particle of charge q moving with velocity $\vec{v}$ is subjected to an electric field $\vec{E}$ and magnetic field $\vec{B}$, the total force acting on the particle is $\vec{F} = \vec{F}_e + \vec{F}_m = q\vec{E} + q(\vec{v} \times \vec{B}) = q(\vec{E} + \vec{v} \times \vec{B})$

This force is known as Lorentz force and is named after the Dutch physicist Hendrik Anton Lorentz.

Cyclotron

It is a heavy particle accelerator, invented in 1929 by E.O. Lawrence for accelerating charged particles such as protons, deutrons, or alpha particles to high velocities. Cyclotron frequency, $\upsilon = \frac{Bq}{2\pi m}$

where m and q are mass and charge of the particle and B is the strength of magnetic field.

FORCE ON A CURRENT CARRYING CONDUCTOR IN A UNIFORM MAGNETIC FIELD

The force experienced by a straight conductor of length l carrying current I when placed in a uniform magnetic field $\vec{B}$ is $\vec{F} = I(\vec{l} \times \vec{B})$ or $F = IlB\sin\theta$

where θ is the angle between $\vec{l}$ and $\vec{B}$.

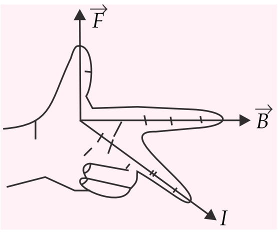
If $\theta = 0°$, then $F = 0$ (minimum).

If $\theta = 90°$, then $F = BIl$ (maximum).

The direction of this force is given by Fleming's left hand rule.

Fleming's Left Hand Rule

Stretch the fore-finger, central finger and thumb of left hand mutually perpendicular. Then if the fore-finger is along the direction of field $(\vec{B})$, the central finger in the direction of current I, the thumb gives the direction of force as shown in the figure.



TORQUE ON A CURRENT CARRYING COIL PLACED IN A UNIFORM MAGNETIC FIELD

When a current carrying coil is placed in a uniform magnetic field, the net force on it is always zero but different parts of the coil experience forces in different directions. Due to it, the coil may experience a torque or couple.

When a coil of area A having N turns, carrying current I is placed in uniform magnetic field B, it will experience torque which is given by

$$\tau = NIAB\sin\theta = MB\sin\theta$$

where θ is the angle between the direction of magnetic field and normal to the plane of the coil.

If the plane of the coil is perpendicular to the direction of magnetic field *i.e.* $\theta = 0°$, then

$$\tau = 0 \text{ (minimum)}$$

If the plane of the coil is parallel to the direction of magnetic field *i.e.* $\theta = 90°$, then

$$\tau = NIAB \text{ (maximum)}$$

If α is the angle between plane of the coil and the magnetic field, then torque on the coil is

$$\tau = NIAB\cos\alpha = MB\cos\alpha$$

Potential energy of the coil is $U = -\vec{M}\cdot\vec{B}$

Work done in rotating the coil through an angle θ from the field direction is $W = MB(1 - \cos\theta)$

where $\vec{M} = NI\vec{A}$ is magnetic dipole moment of the loop.

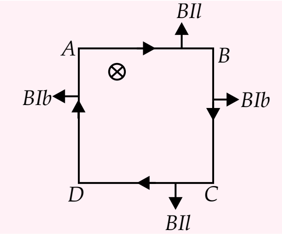
Torque on Current Loop, Magnetic Dipole

A rectangular loop carrying a steady current I and placed in a uniform magnetic field experiences a torque. It does not experience a net force.

The torque, $\tau = BNIA\sin\theta$ or $\vec{\tau} = \vec{M} \times \vec{B}$

Case I : When plane of the loop is perpendicular to magnetic field.

Length of $AB = DC = l$ and that of $BC = AD = b$. Forces experienced by all the sides are shown in the figure.

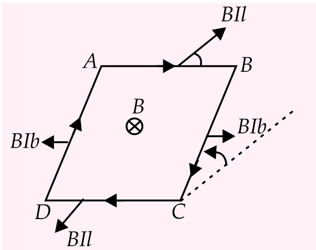


∴ Force on AB and DC are equal and opposite to the each other and that on BC and AD too

$\Rightarrow \quad \Sigma F = 0$

Since the line of action of the forces on AB and DC is same, therefore torque is zero.

Case II : When the plane of the loop is inclined to the magnetic field. In this case again $\Sigma F = 0$.



∴ Lines of action of the forces on AB and DC are different, therefore they form a couple and produce torque.

MOVING COIL GALVANOMETER

It is used for the detection and measurement of small currents.

Principle of a moving coil galvanometer: When a current carrying coil is placed in a magnetic field, it experiences a torque.

In moving coil galvanometer the current I passing through the galvanometer is directly proportional to its deflection θ.

$$I \propto \theta \quad \text{or,} \quad I = G\theta.$$

where $G = \frac{k}{NAB}$ = galvanometer constant,

A = area of a coil, N = number of turns in the coil, B = strength of magnetic field, k = torsional constant of the spring *i.e.* restoring torque per unit twist.

Current sensitivity : It is defined as the deflection produced in the galvanometer, when unit current flows through it.

$$I_s = \frac{\theta}{I} = \frac{NAB}{k}.$$

The unit of current sensitivity is rad A^{-1} or div A^{-1}.

Voltage sensitivity : It is defined as the deflection produced in the galvanometer when a unit voltage is applied across the two terminals of the galvanometer.

$$V_s = \frac{\theta}{V} = \frac{\theta}{IR} = \frac{NAB}{kR}.$$

The unit of voltage sensitivity is rad V^{-1} or div V^{-1}.

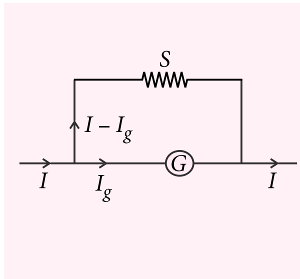
$$V_s = \frac{1}{R} I_s.$$

AMMETER

It is an instrument used to measure current in an electrical circuit.

Conversion of Galvanometer into Ammeter

A galvanometer can be converted into an ammeter of given range by connecting a suitable low resistance S called shunt in parallel to the given galvanometer, whose value is given by



$$S = \left(\frac{I_g}{I - I_g}\right) G$$

where I_g is the current for full scale deflection of galvanometer, I is the current to be measured by the galvanometer and G is the resistance of galvanometer.

Ammeter is a low resistance instrument and it is always connected in series to the circuit. An ideal ammeter has zero resistance.

In order to increase the range of an ammeter n times, the value of shunt resistance to be connected in parallel is $S = G/(n - 1)$.

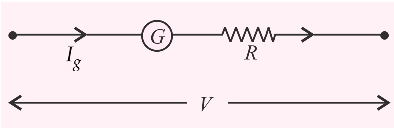
VOLTMETER

It is an instrument used to measure potential difference across any element in an electrical circuit.

Conversion of Galvanometer into Voltmeter

A galvanometer can be converted into voltmeter of given range by connecting a suitable resistance R in series with the galvanometer, whose value is given by $R = \frac{V}{I_g} - G$

where V is the voltage to be measured, I_g is the current for full scale deflection of galvanometer and G is the resistance of galvanometer.



Voltmeter is a high resistance instrument and it is always connected in parallel with the circuit element across which potential difference is to be measured. An ideal voltmeter has infinite resistance.

In order to increase the range of voltmeter n times the value of resistance to be connected in series with galvanometer is $R = (n - 1)G$.

THE BAR MAGNET

When a bar magnet is freely suspended, it points in the north-south direction. The tip which points to the geographic north is called the north pole and the tip which points to the geographic south is called the south pole of the magnet.

MAGNETIC FIELD LINES

The magnetic field lines are a visual and intuitive realisation of the magnetic field. Their properties are as follows

- The magnetic field lines of a magnet (or a solenoid) form continuous closed loops. This is unlike the electric dipole where these field lines begin from a positive charge and end on the negative charge or escape to infinity.
- The tangent to the field line at a given point represents the direction of the net magnetic field $\vec{B}$ at that point.
- The larger the number of field lines crossing per unit area, the stronger is the magnitude of the magnetic field $\vec{B}$.
- The magnetic field lines do not intersect, for if they did, the direction of the magnetic field would not be unique at the point of intersection.

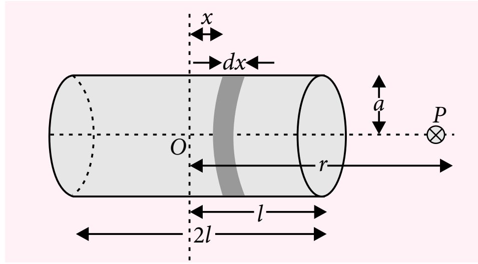
MAGNETIC DIPOLE

A magnetic dipole consists of a unlike poles of equal strength and separated by a small distance.

Magnetic Dipole Moment

The magnetic dipole moment of a dipole is given by $\vec{M} = m(2\vec{l})$. It is a vector directed from south pole to north pole. Its SI unit is A m^2.

BAR MAGNET AS AN EQUIVALENT SOLENOID



Let us consider a solenoid consisting of n turns per unit length, having total length of $2l$ and radius a as shown in figure.

Let I be the current through the solenoid. To find the magnetic field at point P on the axial line at a distance r from the center O of the solenoid, we consider a small circular element of thickness dx of the solenoid at a distance x from the center of the solenoid. Let number of turns in the element dx be ndx. The magnetic field at point P due to the circular element will be

$$dB = \frac{\mu_0 ndxIa^2}{2[(r-x)^2 + a^2]^{3/2}}$$

The magnitude of the net magnetic field B at point P due to the entire solenoid,

$$B = \frac{\mu_0 nIa^2}{2}\int_{-l}^{+l}\frac{dx}{[(r-x)^2 + a^2]^{3/2}} \quad \text{...(i)}$$

If point P lies very far from the center of the solenoid ($r >> a$ and $r >> x$) then we can approximate

$$[(r - x)^2 + a^2]^{3/2} \approx r^3. \quad \text{...(ii)}$$

From equation (i) and (ii), we get

$$B = \frac{\mu_0 nIa^2}{2r^3}\int_{-l}^{+l} dx = \frac{\mu_0 nIa^2}{2r^3} \times 2l \quad \text{...(iii)}$$

That the magnetic moment of the solenoid is

Magnetic moment, M = total number of turns × current × area of cross-section

$$\therefore \quad M = (n \times 2l) \times (I) \times (\pi a^2). \quad \text{(iv)}$$

From equation (iii) and (iv), we get

$$B = \frac{\mu_0}{4\pi} \times \frac{2M}{r^3}$$

MAGNETIC FIELD AT A POINT DUE TO MAGNETIC DIPOLE (BAR MAGNET)

The magnetic field due to a bar magnet at any point on the axial line (end on position) is

$$B_{\text{axial}} = \frac{\mu_0}{4\pi}\frac{2Mr}{(r^2 - l^2)^2}$$

where r = distance between the centre of the magnet and the given point on the axial line, $2l$ = magnetic length of the magnet and M = magnetic moment of the magnet.

For short magnet $l^2 << r^2$, $B_{\text{axial}} = \frac{\mu_0 2M}{4\pi r^3}$

The direction of B_{axial} is along SN.

The magnetic field due to a bar magnet at any point on the equatorial line (broad-side on position) of the bar magnet is $B_{\text{equatorial}} = \frac{\mu_0 M}{4\pi(r^2 + l^2)^{3/2}}$

For short magnet, $B_{\text{equatorial}} = \frac{\mu_0 M}{4\pi r^3}$

The direction of $B_{\text{equatorial}}$ is parallel to NS.

Work Done in Rotating the Magnetic Dipole in a Uniform Magnetic Field

Work done in rotating the magnetic dipole from θ_1 to θ_2 with respect to uniform magnetic field is

$$W = \int_{\theta_1}^{\theta_2} MB\sin\theta\, d\theta = -MB(\cos\theta_2 - \cos\theta_1)$$

$$= MB(\cos\theta_1 - \cos\theta_2)$$

If the dipole is rotated from field direction *i.e.* $\theta_1 = 0°$ to position θ *i.e.* $\theta_2 = \theta$

$$W = MB(1 - \cos\theta).$$

Potential Energy of a Magnetic Dipole

Potential energy of a magnetic dipole in a uniform magnetic field is $U = -\vec{M}\cdot\vec{B} = -MB\cos\theta$

The potential energy of the dipole will be minimum when $\theta = 0°$, *i.e.*, the dipole is parallel to the field, and maximum when $\theta = 180°$, *i.e.*, the dipole is antiparallel to the field.

Current Loop as a Magnetic Dipole

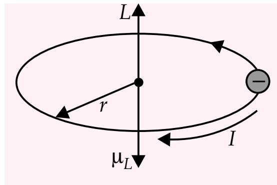
A current loop behaves as a magnetic dipole whose magnetic dipole moment is $M = IA$ where A is the area enclosed by loop and I is the current flowing in the loop. If there are N turns in a loop, then $M = NIA$.

Magnetic Dipole Moment of a Revolving Electron

An electron revolving around the central nucleus in an atom has a magnetic moment and it is given by

$$\vec{\mu}_L = -\frac{e}{2m_e}\vec{L}$$

The negative sign shows that $\vec{\mu}$ is in the opposite direction to $\vec{L}$.



In magnitude $\mu_L = \frac{e}{2m_e}L$ where L is the magnitude of the angular momentum of the revolving electron.

GAUSS'S LAW FOR MAGNETISM

Gauss's law for magnetism states that the net magnetic flux through any closed surface is zero.

$$\phi = \sum_{\substack{\text{all area} \\ \text{elements } \Delta S}} \vec{B}\cdot\Delta\vec{S} = 0$$

This law establishes that isolated magnetic poles do not exist.

EARTH'S MAGNETISM

The strength of earth magnetic field varies from place to place on the earth's surface. Its value is of the order of 10^{-5} T.

Geographical Meridian and Magnetic Meridian

The vertical plane passing through the geographical north pole and south pole at given place is known as the geographical meridian of that place. And a vertical plane passing through the axis of a freely suspended or pivoted magnet is known as magnetic meridian.

Elements of the Earth's Magnetic Field

Three quantities are needed to specify the magnetic field of the earth on its surface – the horizontal component, the magnetic declination and the magnetic dip. These are known as elements of the earth's magnetic field or magnetic elements.

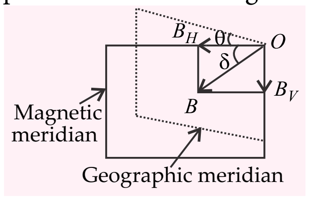
Magnetic declination : Magnetic declination at a place is defined as the angle between the geographic meridian and magnetic meridian.

Magnetic dip or inclination : Magnetic dip at a place is defined as the angle made by the earth's magnetic field with the horizontal in the magnetic meridian. It is denoted by δ.

Horizontal component : It is component of earth's magnetic field along the horizontal direction in the magnetic meridian. It is denoted by B_H.

If B is intensity of earth's total magnetic field, then the horizontal component of earth's magnetic field is given by $B_H = B\cos\delta$

Also, the vertical component of earth's magnetic field, $B_V = B\sin\delta$



$$\therefore\ B = \sqrt{B_H^2 + B_V^2}$$

and $\tan\delta = \dfrac{B_V}{B_H}$

The earth always has a vertical component except at equator. The earth always has a horizontal component except at the poles. In a vertical plane at an angle θ to magnetic meridian.

$B'_H = B_H\cos\theta$ and $B'_V = B_V$

$$\therefore\ \tan\delta' = \frac{B'_V}{B'_H} = \frac{B_V}{B_H\cos\theta} = \frac{\tan\delta}{\cos\theta}$$

If at a given place δ_1 and δ_2 are angles of dip in two arbitrary vertical planes which are perpendicular to each other, the true angle of dip δ is given by

$\cot^2\delta = \cot^2\delta_1 + \cot^2\delta_2$

Angle of dip δ at a plane is related to its magnetic latitude λ through the relation $\tan\delta = 2\tan\lambda$

KEY POINT

- The magnetic declination is greater at higher latitudes and smaller near the equator.

Magnetic Intensity

When a magnetic material is placed in a magnetic field, it becomes magnetised. The capability of the magnetic field to magnetise a material is expressed by means of a magnetic vector $\vec{H}$, called the magnetic intensity of the field. The relation between magnetic induction B and magnetising field H is $B = \mu H$

where μ is the permeability of medium. It is a vector quantity and its SI unit is A m^{-1}.

Intensity of Magnetisation

It is defined as the magnetic moment per unit volume.

$$I = \frac{\text{magnetic moment}}{\text{volume}} = \frac{M}{V}$$

If A = uniform area of cross-section of the magnetised specimen (a rectangular bar)

$2l$ = magnetic length of the specimen

m = strength of each pole of the specimen,

then, $I = \dfrac{m \times 2l}{A \times 2l} = \dfrac{m}{A}$

The intensity of magnetisation is a vector quantity and its SI unit is A m^{-1}. Its dimensional formula is $[M^0L^{-1}T^0A]$.

Magnetic Susceptibility

It is defined as the ratio of the intensity of magnetisation I to the magnetising field.

$$\chi_m = \frac{I}{H}$$

It is a scalar quantity with no units and dimensions. Physically, it represents the ease with which a magnetic material can be magnetised. *i.e.*, large value of χ_m implies that the material is more susceptible to the field and hence can be easily magnetised.

Magnetic Permeability

It is defined as the ratio of magnetic induction to the magnetising field H. $\mu = \dfrac{B}{H}$

It is a scalar quantity having unit H m^{-1} and its dimensional formula is $[MLT^{-2}A^{-2}]$.

It measures the degree to which a magnetic material can be penetrated or permeated by the magnetising field.

Relative permeability : It is defined as ratio of permeability of a medium to that of free space

$$\mu_r = \frac{\mu}{\mu_0}$$

It has no units and dimensions.

Relationship between magnetic permeability and suspectibility

$$\mu_r = 1 + \chi_m$$

CLASSIFICATION OF MAGNETIC MATERIALS

On the basis of magnetic properties, different materials have been classified into three categories :

Diamagnetic substances : Diamagnetic substances are those in which the individual atoms/molecules/ions do not posses any net magnetic moment on their own. When such substances are placed in an external magnetic field, they get feebly magnetised in a direction opposite to the magnetic field. *e.g.* antimony, bismuth, copper, lead, gold, silver, water, alcohol, mercury, air, hydrogen, nitrogen and all inert gases like helium, neon, argon etc.

The relative permeability of diamagnetic substances is less than one. The magnetic susceptibility of diamagnetic substances is small and negative. The magnetic susceptibility of diamagnetic substances is independent of temperature.

Paramagnetic substances : Paramagnetic substances are those in which each individual atom/molecule/ion has a net non zero magnetic moment of its own. When such substances are placed in an external magnetic field, they get feebly magnetised in the direction of the magnetic field.

The relative permeability of paramagnetic substances is just greater than one. The magnetic susceptibility of paramagnetic substances is small and positive. The magnetic susceptibility of paramagnetic substance is dependent on the temperature and it varies with temperature according to the given equation $\chi_m = \frac{C}{T}$

This is known as Curie law. The constant C is known as Curie's constant.

Ferromagnetic substances : Ferromagnetic substances are those in which each individual atom/molecule/ion has a non zero magnetic moment, as in a paramagnetic substance. When such substances are placed in an external magnetic field, they get strongly magnetised in the direction of the field. *e.g.* iron, cobalt, nickel, gadolinium and a number of their alloys.

The relative magnetic permeability of ferromagnetic substances is very large. The magnetic susceptibility of ferromagnetic substance is positive and very large. At high temperature, ferromagnet becomes paramagnet. The temperature of transition from ferromagnetic to paramagnetic is known as Curie temperature (T_C). The susceptibility above the Curie temperature *i.e.* in the paramagnetic phase is given by $\chi_m = \frac{C}{T - T_C} \; (T > T_C)$

This is known as Curie Weiss law.

HYSTERESIS

Hysteresis is the phenomenon of lagging of magnetic induction B or intensity of magnetisation I behind the magnetising field H, when a specimen is taken through a cycle of magnetisation. From the hysteresis loop of material, we can study about retentivity, coercivity etc. of the material. The study of these characteristics enables us to select suitable materials for different purposes.

- **Retentivity :** The value of magnetic induction left in the specimen when the magnetizing force is reduced to zero is called retentivity or remanence or residual magnetism of the material.
- **Coercivity :** The value of magnetizing force which is applied to reduce the residual magnetism or retentivity to zero is known as coercivity.
- **Soft magnetic materials :** They have low retentivity, low coercivity and small hysteresis loss. These are used for making electromagnets, cores of transformers, motors and generators. Soft iron, mu-metal and stalloy are examples of these materials.
- **Hard magnetic materials :** They have high retentivity, high coercivity and large hysteresis-loss. These are used in making permanent magnets of various kinds of electric meters and loudspeakers. Steel, alnico, alcomax and ticonal are examples of these materials.

KEY POINT

- Diamagnetism is universal, it is present in all materials, but it is weak and hard to detect if the substance is para- or ferromagnetic.

SELF CHECK

2. The coercivity of a small magnet where the ferromagnet gets demagnetized is 3×10^3 A m^{-1}. The current required to be passed in a solenoid of length 10 cm and number of turns 100, so that the magnet gets demagnetized when inside the solenoid, is
   (a) 6 A (b) 30 mA
   (c) 60 mA (d) 3 A
   (JEE Main 2014)

3. Two short bar magnets of length 1 cm each have magnetic moments 1.20 A m^2 and 1.00 A m^2 respectively. They are placed on a horizontal table parallel to each other with their N poles pointing towards the South. They have a common magnetic equator and are separated by a distance of 20.0 cm. The value of the resultant horizontal magnetic induction at the mid-point *O* of the line joining their centres is close to (Horizontal component of earth's magnetic induction is 3.6×10^{-5} Wb m^{-2})
(a) 5.80×10^{-4} Wb m^{-2}
(b) 3.6×10^{-5} Wb m^{-2}
(c) 2.56×10^{-4} Wb m^{-2}
(d) 3.50×10^{-4} Wb m^{-2}

(JEE Main 2013)

MAGNETIC FLUX

The total number of magnetic field lines crossing through the surface is known as the magnetic flux of the any surface held in magnetic field $\vec{B}$.

If a loop enclosing an area *A* is placed in a magnetic field $\vec{B}$, then the magnetic flux through the loop is

$$\phi_B = \int \vec{B} \cdot d\vec{A}$$

FARADAY'S LAW OF INDUCTION

According to Faraday's first law of electromagnetic induction, whenever the magnetic flux linked with a circuit changes, an emf is induced in the circuit. The SI unit of induced emf is volt.

According to Faraday's second law of electromagnetic induction, the magnitude of the induced emf in a circuit is equal to the time rate of magnetic flux through the circuit.

$$i.e.\ \ \varepsilon = -\frac{d\phi}{dt}$$

In the case of a closely wound coil of *N* turns, change of flux associated with each turn is the same.

$$\therefore \quad \varepsilon = -N\frac{d\phi}{dt}$$

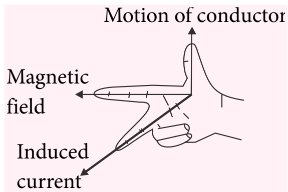
LENZ'S LAW AND ENERGY CONSERVATION

According to Lenz's law, the induced current has a direction such that the magnetic field due to the current opposes the change in the magnetic flux that induces the current. In other words, the polarity of the induced emf/current is such that it always opposes the change in magnetic flux which produces the induced emf/current.

Lenz's law is in accordance with the principle of conservation of energy.

Fleming's Right Hand Rule

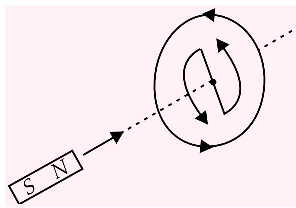
Fleming's right hand rule also gives us the direction of induced emf or current, in a conductor moving in a magnetic field.



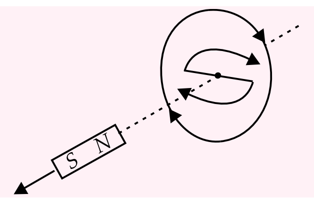
According to this rule, if we stretch the fore finger, central finger and thumb of our right hand in mutually perpendicular directions such that fore finger points along the direction of the field and thumb is along the direction of motion of the conductor, then the central finger would give us the direction of induced current or emf. The direction of induced current or emf given by Lenz's law and Fleming's right hand rule is the same.

Applications of Lenz's Law

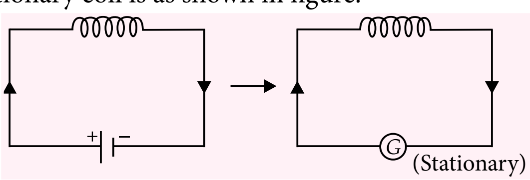
When a north pole of a bar magnet is moved towards a coil, the current induced in the coil will be in anticlockwise direction as shown in the figure.



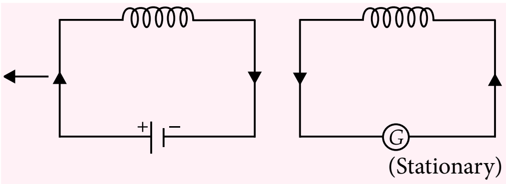
When a north pole of bar magnet is moved away from the coil, the current induced in the coil will be in clockwise direction as shown in the figure.



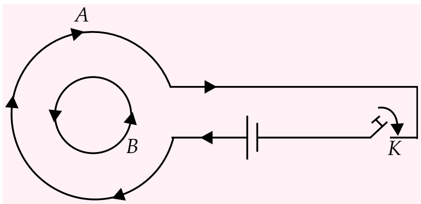
When a current carrying coil is moved towards a stationary coil, the direction of current induced in stationary coil is as shown in figure.



When a current carrying coil is moved away from a stationary coil, the direction of current induced in stationary coil is as shown in figure.

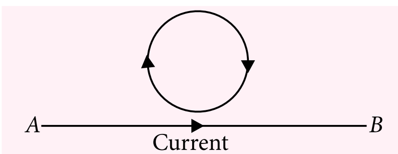


When two coils A and B are arranged as shown in figure, then on pressing K, current in A increases in clockwise direction. Therefore, induced current in B will be in anticlockwise direction.



However, when key K is released, current in A decreases in clockwise direction. Therefore, induced current in B will be in clockwise direction.

When current in a straight conductor AB is increased, induced current in loop will be in clockwise direction as shown in the figure. If current in AB is decreasing, the induced current in the loop will be in anticlockwise direction.



MOTIONAL EMF

If a conductor is moving with velocity $\vec{v}$ in a magnetic field, electrons inside it experience a force $\vec{F} = e(\vec{v} \times \vec{B})$ and accumulate at the end of the conductor. Very soon, an electric field is established. Eventually component of magnetic force along the conductor length is balanced by the electric field force and the drifting of electrons stops and an emf is established.

Now, $\varepsilon = -\int \vec{E} \cdot d\vec{l} = \int (\vec{v} \times \vec{B}) \cdot d\vec{l}$

This is general expression for induced emf in a conducting wire. If $\vec{v}, \vec{B}$ and $\vec{l}$ are mutually perpendicular to each other then, $\varepsilon = Bvl$.

Motional emf in Loop

If a conducting rod moves on two parallel conducting rails then an emf is induced whose magnitude is $|\varepsilon| = Blv$ and direction is given by Fleming's right hand rule. Induced current $|I| = \frac{|\varepsilon|}{R} = \frac{Blv}{R}$

ROTATIONAL EMF

If a conducting rod of length l rotates about an axis passing through one of its ends (that end may be fixed) with an angular velocity ω in a plane perpendicular to the magnetic field $\vec{B}$. Then an induced emf is set up between ends of rod whose magnitude is given by $|\varepsilon| = \frac{1}{2} Bl^2\omega$

EDDY CURRENTS

The current induced in bulk pieces of conductors when the magnetic flux linked with the conductor changes are known as Eddy currents.

INDUCTANCE

An electric current can be induced in a coil by flux change produced by another coil in its vicinity or flux change produced by the same coil. The flux through a coil is proportional to the current *i.e.* $\phi \propto I$

For a closely wound coil of N turns, $N\phi \propto I$

or $\phi = \frac{LI}{N}$

where L is the constant of proportionality called inductance. SI unit of inductance is T m^2 A^{-1} or henry (H) and its dimensional formula is $[ML^2T^{-2}A^{-2}]$

Self Inductance

Whenever the current passing through a coil or circuit changes, the magnetic flux linked with it will also change. As a result of this, an emf is induced in the coil or the circuit which opposes the change that causes it. This phenomenon is known as self induction and the emf induced is known as self induced emf or back emf.

When a current I flows through a coil and ϕ is the magnetic flux linked with the coil, then

$\phi \propto I$ or $\phi = LI$

where L is coefficient of self induction or self inductance of the coil.

The self induced emf is $\varepsilon = -\frac{d\phi}{dt} = -L\frac{dI}{dt}$

Self inductance of a solenoid is $L = \mu_0 n^2 lA$

where l is length of the coil solenoid, n is number of turns per unit length of a solenoid and A is area of cross section of the solenoid.

Self inductance of a circular coil is $L = \frac{\mu_0 N^2 \pi R}{2}$

where R is the radius of a coil and N is the number of turns.

Mutual Inductance

Whenever the current passing through a coil or circuit changes, the magnetic flux linked with a neighbouring coil or circuit will also change. Hence an emf will be induced in the neighbouring coil or circuit. This phenomenon is known as mutual induction. The coil or circuit in which the current changes is known as primary, while the other in which emf is set up is known as secondary.

Let I_P be the current flowing through primary coil at any instant. If ϕ_S is the flux linked with secondary coil then $\phi_S \propto I_P$ or $\phi_S = MI_P$

where M is coefficient of mutual inductance of two coils.

The emf induced in the secondary coil is given by

$$\varepsilon_S = -M \frac{dI_P}{dt}$$

Combination of Inductances

When two coils having self inductance L_1 and L_2 are connected in series keeping them far apart (so that mutual inductance between them is negligible), the equivalent inductance of the combination is given by

$$L = L_1 + L_2$$

When two coils are connected in parallel, the equivalent inductance is given by

$$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}$$

Energy Stored by an Inductor

A current-carrying inductor stores energy in the magnetic field associated with it and energy stored for current I and an inductance L are related as $\frac{1}{2} LI^2$.

SELF CHECK

4. A circular loop of radius 0.3 cm lies parallel to a much bigger circular loop of radius 20 cm. The centre of the small loop is on the axis of the bigger loop. The distance between their centres is 15 cm. If a current of 2.0 A flows through the smaller loop, then the flux linked with bigger loop is

(a) 6.6×10^{-9} weber (b) 9.1×10^{-11} weber
(c) 6×10^{-11} weber (d) 3.3×10^{-11} weber

(JEE Main 2013)

AC GENERATOR

It is a device used to obtain a supply of alternating emf by converting rotational mechanical energy into electrical energy. It is based on the phenomenon of electromagnetic induction. *i.e.* when a coil is rotated in uniform magnetic field, an induced emf is produced in it.

The instantaneous value of the e.m.f. produced is given by $\varepsilon = NBA\omega\sin\omega t$, where N is number of turns of the coil, A is the area of coil and ω is angular frequency of rotation of the coil in a magnetic field strength B.

ALTERNATING CURRENT

It is the current which varies continuously in magnitude and periodically in direction. It can be represented by $I = I_0 \sin\omega t$ or $I = I_0\cos\omega t$.

where I_0 is peak value of current and is known as amplitude of ac and I is the instantaneous value of ac.

$\omega = \frac{2\pi}{T} = 2\pi\upsilon$ where T is period of ac and υ is frequency of ac.

MEAN OR AVERAGE VALUE OF AC

Average value of the alternating current over a half cycle is the value of direct current which will send the same amount of charge in a circuit in a time of half cycle as is sent by the given ac in the same circuit in the same time.

$$I_m \text{ or } \bar{I} \text{ or } I_{av} = \frac{\int_0^T I_0 \sin\omega t \, dt}{\int_0^T dt} = 0$$

$$V_m \text{ or } \bar{V} \text{ or } V_{\text{av}} = \frac{\int_0^T V_0 \sin\omega t\, dt}{\int_0^T dt} = 0$$

Average value of alternating current for first half cycle is

$$I_{\text{av}} = \frac{\int_0^{T/2} I_0 \sin\omega t\, dt}{\int_0^{T/2} dt} = \frac{2I_0}{\pi} = 0.637 I_0$$

Similarly, for alternating voltage, the average value over first half cycle is

$$V_{\text{av}} = \frac{\int_0^{T/2} V_0 \sin\omega t dt}{\int_0^{T/2} dt} = \frac{2V_0}{\pi} = 0.637 V_0$$

Average value of alternating current for second cycle is

$$I_{\text{av}} = \frac{\int_{T/2}^{T} I_0 \sin\omega t dt}{\int_{T/2}^{T} dt} = -\frac{2I_0}{\pi} = -0.637\, I_0$$

Similarly, for alternating voltage, the average value over second half cycle is

$$V_{\text{av}} = \frac{\int_{T/2}^{T} V_0 \sin\omega t dt}{\int_{T/2}^{T} dt} = -\frac{2V_0}{\pi} = -0.637\, V_0$$

ROOT MEAN SQUARE VALUE OF ALTERNATING CURRENT

The value of that direct current which produces heat at the same rate as the alternating current in a given resistor is known as the rms value of alternating current. rms value is also known as virtual value or effective value. All ac instruments measure virtual value.

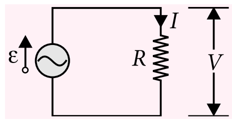
$$I_{\text{rms}} = \frac{I_0}{\sqrt{2}} = 0.707\, I_0 \text{ or } V_{\text{rms}} = \frac{V_0}{\sqrt{2}} = 0.707\, V_0$$

$$\text{Form factor} = \frac{I_{\text{rms}}}{I_{\text{av}}} = \frac{0.707 I_0}{0.637 I_0} = 1.11$$

KEY POINT

- The average value of alternating current during the first and second half cycles are equal but opposite in sign so that the average over one cycle is zero.
- An ammeter connected in a circuit always measures rms value of alternating current.

AC VOLTAGE APPLIED TO A RESISTOR



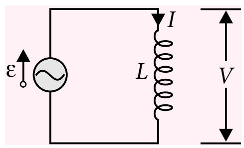
The varying potential difference

$V = V_0 \sin\omega t$

then $I = \frac{V}{R} = \frac{V_0 \sin\omega t}{R} = I_0 \sin\omega t$

Here the alternating voltage is in phase with current, when ac flows through the resistor.

AC VOLTAGE APPLIED TO AN INDUCTOR



Potential difference across the inductor,

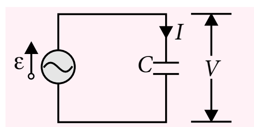
$V = V_0 \sin\omega t$

or $I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$ where $I_0 = \frac{V_0}{\omega L}$

The alternating current lags behind the alternating voltage by a phase angle of $\frac{\pi}{2}$ when ac flows through an inductor.

Inductive reactance : It is the opposition offered by the inductor to the flow of alternating current through it. $X_L = \omega L = 2\pi\upsilon L$

AC VOLTAGE APPLIED TO A CAPACITOR



Potential difference across capacitor,

$V = V_0 \sin\omega t$

or $I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$ where $I_0 = (\omega C) V_0$

The alternating current leads the voltage by a phase angle of $\frac{\pi}{2}$, when ac flows through a capacitor.

Capacitive reactance : It is the opposition offered by the capacitor to the flow of alternating current through it.

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\upsilon C}$$

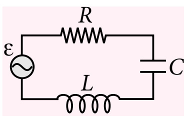
KEY POINT

- The inductive reactance is zero for dc and has a finite value for ac.
- The capacitive reactance is infinite for dc and has a finite value for ac.

AC VOLTAGE APPLIED TO A SERIES *LCR* CIRCUIT

Let $V = V_0\sin\omega t$, then $I = I_0\sin(\omega t - \phi)$

where $I_0 = \frac{V_0}{Z}$



Here Z is the impedence of series LCR circuit,

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

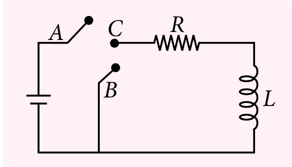
The alternating current lags behind the voltage by a phase angle ϕ. $\tan\phi = \frac{X_L - X_C}{R}$

When $X_L > X_C$, tan ϕ is positive. Therefore, ϕ is positive. Hence current lags behind the voltage by a phase angle ϕ. The ac circuit is inductance dominated circuit.

When $X_L < X_C$, tan ϕ is negative. Therefore, ϕ is negative. Hence current leads the voltage by a phase angle ϕ. The ac circuit is capacitance dominated circuit.

SELF CHECK

5. In the circuit shown here, the point C is kept connected to point A till the current flowing through the circuit becomes constant. Afterward, suddenly, point C is disconnected from point A and connected to point B at time $t = 0$. Ratio of the voltage across resistance and the inductor at $t = L/R$ will be equal to



(a) $\frac{1-e}{e}$ (b) $\frac{e}{1-e}$

(c) 1 (d) –1

(JEE Main 2014)

6. In a series LCR circuit $R = 200\ \Omega$ and the voltage and the frequency of the main supply is 220 V and 50 Hz respectively. On taking out the capacitance from the circuit the current lags behind the voltage by 30°. On taking out the inductor from the circuit the current leads the voltage by 30°. The power dissipated in the LCR circuit is

(a) 242 W (b) 305 W

(c) 210 W (d) 0 W

(AIEEE 2010)

RESONANCE

The phenomenon of resonance is shown by system which have a tendency to oscillate at a particular frequency (natural frequency). If such a system is driven by an external source whose frequency is equal to natural frequency of the system, the amplitude of oscillation becomes large and resonance is said to occur.

Resonant Series *LCR* Circuit

When the frequency of ac supply is such that the inductive reactance and capacitive reactance become equal ($X_L = X_C$), the impedance of the series LCR circuit is equal to the ohmic resistance in the circuit. As such, the current in the circuit becomes maximum. Such a series LCR circuit is known as resonant series LCR circuit and the frequency of the ac supply is known as resonant frequency (υ_r). The resonant frequency is

$$\upsilon_r = \frac{1}{2\pi\sqrt{LC}} \text{ or } \omega_r = \frac{1}{\sqrt{LC}}$$

The series resonance circuit is known as acceptor circuit.

KEY POINT

- Resonance phenomenon is exhibited by a circuit only if both L and C are present in the circuit. Then only voltages across L and C cancel each other.
- We cannot have resonance in a RL or RC circuit.

Quality Factor

It is a measure of sharpness of resonance. It is defined as the ratio of reactance of either the inductance or capacitance at the resonant angular frequency to the total resistance of the circuit.

$$Q = \frac{X_L}{R} = \frac{\omega_r L}{R}$$

$$Q = \frac{X_C}{R} = \frac{1}{\omega_r CR}$$

$$\therefore \quad Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Quality factor is also expressed in terms of bandwidth.

$$Q = \frac{\text{resonant frequency}}{\text{bandwidth}}$$

POWER IN AN AC CIRCUIT

The instantaneous rate at which energy is dissipated in the resistor in an LCR circuit is

$$I = I_0 \sin(\omega t - \phi)$$

or power, $P = I^2R = I_0^2 \sin^2(\omega t - \phi)$ ...(i)

The average rate at which energy is dissipated

$$P_{av} = \frac{I_0^2 R}{2} = \left(\frac{I_0}{\sqrt{2}}\right)^2 R \quad \text{...(ii)}$$

$$\therefore \quad I_{rms} = \frac{I_0}{\sqrt{2}} \quad \text{...(iii)}$$

From equation (ii), $P_{av} = I^2_{av}R$

Also rms value for voltage and emf for ac circuit is

$$V = \frac{V_0}{\sqrt{2}} \text{ and } \varepsilon = \frac{\varepsilon_0}{\sqrt{2}}$$

$$I = \frac{\varepsilon}{Z} = \frac{\varepsilon}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$P_{av} = \frac{\varepsilon}{Z} \times IR = \varepsilon I \frac{R}{Z}$$

As $\quad \cos\phi = \frac{V_0}{\varepsilon} = \frac{IR}{IZ} = \frac{R}{Z}$

$\therefore \quad P_{av} = \varepsilon I \cos\phi$

where $\cos\phi$ is the power factor.

Average power of purely resistive circuit

$$P_{av} = \varepsilon I \cos\phi = \varepsilon I \cos 0° = \varepsilon I$$

Since phase angle $\phi = 0°$ for a purely resistive circuit.

Average power of a purely inductive circuit

$$P_{av} = \varepsilon I \cos\phi = \varepsilon I \cos 90° = 0$$

Since phase angle $\phi = 90°$ for a purely inductive circuit.

Average power of purely capacitive circuit

$$P_{av} = \varepsilon I \cos\phi = \varepsilon I \cos 90° = 0$$

Since $\phi = 90°$ for a purely capacitive circuit.

Average power of LCR series circuit

$$P_{av} = \varepsilon I \cos\phi.$$

Average power of a series LCR circuit at resonance

$$P_{av} = \varepsilon I \cos\phi = \varepsilon I \cos 0° = \varepsilon I$$

Since at resonance, the series LCR circuit behaves as a purely resistive circuit.

KEY POINT

- There are no power losses associated with pure capacitances and pure inductances in an ac circuit.

SELF CHECK

7. In an a.c. circuit the voltage applied is $E = E_0 \sin \omega t$. The resulting current in the circuit is $I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$. The power consumption in the circuit is given by

(a) $P = \sqrt{2}E_0 I_0$ (b) $P = \frac{E_0 I_0}{\sqrt{2}}$

(c) $P = \text{zero}$ (d) $P = \frac{E_0 I_0}{2}$

(AIEEE 2007)

Wattless Current

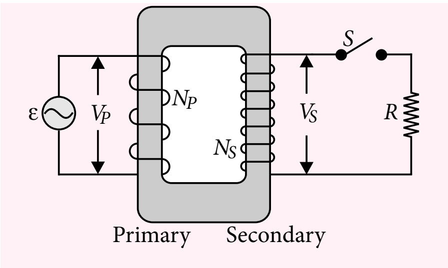
It is the current which consumes no power for its maintenance in the circuit. Current flowing in a purely inductive or purely capacitive circuit is an example of wattless current.

***LC* oscillations**

The frequency of LC oscillations is $\upsilon = \frac{1}{2\pi\sqrt{LC}}$

TRANSFORMERS

It is a device used for converting a low alternating voltage to a high alternating voltage and viceversa. It works on the principle of mutual induction.



For ideal transformer, $\frac{V_S}{V_P} = \frac{I_P}{I_S} = \frac{N_S}{N_P} = k$

where k is called transformation ratio.

For a step-up transformer,

$k > 1$. *i.e.* $V_S > V_P$, $I_S < I_P$ and $N_S > N_P$.

For a step-down transformer,

$k < 1$. *i.e.* $V_S < V_P$, $I_S > I_P$ and $N_S < N_P$.

Efficiency of a transformer,

$$\eta = \frac{\text{output power}}{\text{input power}} = \frac{V_S I_S}{V_P I_P}.$$

ELECTROMAGNETIC WAVES

Displacement current

It is the current which comes into play in the region in which the electric field and the electric flux are changing with time.

Displacement current is given by $I_0 = \varepsilon_0 \frac{d\phi_E}{dt}$

where ε_0 is the permittivity of free space and $\frac{d\phi_E}{dt}$ is the rate of change of electric flux .

MAXWELL'S EQUATIONS

Four fundamental equations of electromagnetism, called maxwell's equations, which are as follows.

Gauss's law for electrostatics	$\oint \vec{E} \cdot d\vec{A} = Q/\varepsilon_0$	Relates net electric flux to net enclosed electric charge.
Gauss's law for magnetism	$\oint \vec{B} \cdot d\vec{A} = 0$	Relates net magnetic flux to net enclosed magnetic charge.
Faraday's law	$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$	Relates induced electric field to changing magnetic flux.
Ampere-Maxwell law	$\oint \vec{B} \cdot d\vec{l} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$	Relates induced magnetic field to changing electric flux and current.

ELECTROMAGNETIC WAVES

Waves in which, there is a sinusoidal variation of electric and magnetic field vectors, such that they are perpendicular to each other and also the direction of propagation of wave are known as electromagnetic waves.

For a plane progressive electromagnetic wave propagating along the $+z$ direction, the electric and magnetic fields can be written as

$$E = E_0 \sin(kz - \omega t),\ B = B_0 \sin(kz - \omega t)$$

The amplitudes of electric and magnetic fields in free space, in electromagnetic waves are related by

$$E_0 = cB_0 \quad \text{or} \quad B_0 = \frac{E_0}{c}$$

The speed of electromagnetic wave in free space is

$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$ where μ_0 and ε_0 are the permeability and permittivity of free space.

The speed of electromagnetic wave in a medium is

$$v = \frac{1}{\sqrt{\mu\varepsilon}}$$

where μ and ε are permeability and permittivity of the medium.

Properties of Electromagnetic Waves

Electromagnetic waves do not carry any charge. These waves are not deflected by electric and magnetic fields. They travel with the speed of light in vacuum. The frequency of electromagnetic wave does not change when it goes from one medium to another but its wavelength changes. These waves are transverse in nature, hence they can be polarised.

Production of Electromagnetic Waves

Maxwell showed that an accelerating charge produces electromagnetic waves. An electric charge oscillating harmonically with frequency υ produces electromagnetic waves of the same frequency υ.

Energy Density of Electromagnetic Waves

Electromagnetic waves carry energy as they travel through space and this energy is equally shared by electric field and magnetic field of electromagnetic wave.

The energy density of the electric field is

$$u_E = \frac{1}{2}\varepsilon_0 E^2$$

The energy density of magnetic field is

$$u_B = \frac{1}{2}\frac{B^2}{\mu_0}$$

Average energy density of the electric field is

$$< u_E > = \frac{1}{4}\varepsilon_0 E_0^2$$

Average energy density of the magnetic field is

$$< u_B > = \frac{1}{4}\frac{B_0^2}{\mu_0} = \frac{1}{4}\varepsilon_0 E_0^2$$

Average energy density of electromagnetic wave is

$$< u > = \frac{1}{2}\varepsilon_0 E_0^2$$

Intensity of Electromagnetic Wave

It is defined as energy crossing per unit area per unit time perpendicular to the direction of propagation of electromagnetic wave. The intensity of electromagnetic wave is

$$I = < u > c = \frac{1}{2}\varepsilon_0 E_0^2 c$$

KEY POINT

- An electric dipole is a basic source of electromagnetic waves.

SELF CHECK

8. During the propagation of electromagnetic waves in a medium
   (a) Both electric and magnetic energy densities are zero.
   (b) Electric energy density is double of the magnetic energy density.
   (c) Electric energy density is half of the magnetic energy density.
   (d) Electric energy density is equal to the magnetic energy density.
   (JEE Main 2014)

9. The magnetic field in a travelling electromagnetic wave has a peak value of 20 nT. The peak value of electric field strength is
   (a) 12 V m^{-1} (b) 3 V m^{-1}
   (c) 6 V m^{-1} (d) 9 V m^{-1}
   (JEE Main 2014)

ELECTROMAGNETIC SPECTRUM

It is the orderly distribution of electromagnetic waves according to their frequency or wavelength.

Different types of electromagnetic waves in the decreasing order of wavelength are as follows.

Radio waves

They are produced by the accelerated motion of charges in conducting wires. They are generally in the frequency range from 500 kHz to about 1000 MHz. Radio waves are used is television, radio cellular phones etc.

Microwaves

Microwaves are produced by special vacuum tubes. Frequency of microwave is in the order of GHz. They are suitable for the radar system used in aircraft navigation. Microwave ovens are domestic application of these waves.

Infrared waves

Infrared waves are produced by hot bodies and molecules. Infrared waves are sometimes referred to as heat waves. Electronic devices emit infrared and are widely used in the remote switches of household electronic systems such as TV sets, video recorders etc. Frequency of infrared waves are in the range of $3 \times 10^{11} - 4 \times 10^{14}$ Hz.

Visible rays

It is the part of the spectrum that is detected by the human eye. Its frequency range is from about 4×10^{14} Hz to about 7×10^{14} Hz. Our eyes are sensitive to this range.

Ultraviolet rays

Ultraviolet radiation is produced by special lamps and very hot bodies. It covers wavelengths ranging from about 4×10^{-7} m (400 nm) down to 6×10^{-10} m (0.6 nm).

X-rays

One common way to generate X-rays is to bombard a metal target by high energy electrons. It covers wavelengths from about 10^{-8} m down to 10^{-13} m. X-rays are used as a diagnostic tool in medicine and as a treatment for certain forms of cancer.

Gamma rays

They lie in the upper frequency range of the electromagnetic spectrum and have wavelengths of from about 10^{-10} m to less than 10^{-14} m. This high frequency radiation is produced in nuclear reactions and also emitted by radioactive nuclei. They are used in medicine to destroy cancer cells.

KEY POINT

- Infrared rays are emitted by all hot bodies and cannot be seen by naked eyes
- The basic difference between various types of electromagnetic waves lies in their wavelengths or frequencies since all of them travel through vacuum with the same speed.
- The waves differ considerably in their mode of interaction with matter.

SELF CHECK

10. Match List-I (Electromagnetic wave type) with List-II (Its association/application) and select the correct option from the choices given.

List-I	List-II
(P) Infrared waves	(i) To treat muscular strain
(Q) Radio waves	(ii) For broadcasting
(R) *X*-rays	(iii) To detect fracture of bones
(S) Ultraviolet rays	(iv) Absorbed by the ozone layer of the atmosphere

	P	Q	R	S
(a)	(i)	(ii)	(iii)	(iv)
(b)	(iv)	(iii)	(ii)	(i)
(c)	(i)	(ii)	(iv)	(iii)
(d)	(iii)	(ii)	(i)	(iv)

(JEE Main 2014)

ANSWER KEYS (SELF CHECK)

1. (a) **2.** (d) **3.** (c) **4.** (b) **5.** (d)
6. (a) **7.** (c) **8.** (d) **9.** (c) **10.** (a)

SCIENCE BEHIND PYROELECTRIC THERMOMETER

Human body, like any object, radiates infrared radiation, and the amount emitted depends on the temperature of the body. Infrared radiation can be detected by sensors. An ear thermometer, like the pyroelectric thermometer determines the body's temperature by measuring the amount of infrared radiation that emanates from the eardrum and surrounding tissue. Ear is one of the best places to measure body temperature because it is close to the hypothalamus, an area at the bottom of the brain that controls body temperature. The ear is also not cooled or warmed by eating, drinking, or breathing. When the probe of the thermometer is inserted into the ear canal, infrared radiation travels down the barrel of the probe and strikes the sensor. The absorption of infrared radiation warms the sensor, and, as a result, its electrical conductivity changes. The change in electrical conductivity is measured by an electronic circuit. The output from the circuit is sent to a microprocessor, which calculates the body temperature and displays the result on a digital screen.

BRAIN MAP

THERMAL PROPERTIES OF MATTER, THERMODYNAMICS & KINETIC THEORY

Heat and temperature are not the same thing. Temperature of a body is the degree of hotness or coldness whereas heat is a form of energy that flows between two bodies by virtue of temperature difference between

Thermal Properties of Matter

Heat and Temperature

- Heat is a form of energy which produces in us the sensation of warmth.
- Temperature is a relative measure of hotness or coldness of a body.
- $$\frac{T_C+0}{100}=\frac{T_F-32}{180}=\frac{T_K-273.15}{100}=\frac{T_R-0}{80}$$
  where T_C, T_F, T_K and T_R are the temperatures on Celcius, Fahrenheit, Kelvin and Reaumur scales respectively.

Thermal Expansion

- **Coefficient of linear expansion (α) :** Increase in length per unit length per unit rise in temperature.
  $$\alpha=\frac{BL}{L\Delta T}$$
- **Coefficient of area expansion (β) :** Increase in area per unit area per unit rise in temperature.
  $$\beta=\frac{BA}{A\Delta T}$$
- **Coefficient of volume expansion (γ) :** Increase in volume per unit volume per unit rise in temperature.
  $$\gamma=\frac{BV}{V\Delta T}$$
- **Relation between α, β and γ:** $6\alpha=3\beta=2\gamma$

Important Definitions

- **Specific heat capacity (c) :** Amount of heat required to raise the temperature of unit mass of the substance through one degree.
  $$c\ ;\ \frac{Q}{m\Delta T}$$
- **Molar heat capacity (C) :** Amount of heat required to raise the temperature of one mole of a substance through one degree.
  $$C\ ;\ \frac{Q}{n\Delta T}$$
- **Water equivalent (w) :** Quantity of water which would be raised through 1°C by the amount of heat required to raise the temperature of the body through 1°C.
  $$w=mc$$
- **Latent heat (L) :** Amount of heat required to change the state of a unit mass of a substance without rise in temperature.
  $$L\ ;\ \frac{Q}{m}$$
  Latent heat of fusion (L_f) corresponds to solid to liquid phase change whereas latent heat of vapourisation (L_v) corresponds to liquid to gas phase change.

Heat Transfer

- Heat Q that flows across the opposite faces of a rod in time t,
  $$Q\ ;\ KA\frac{dT}{dx}t$$
  where A is area of cross-section, $\frac{dT}{dx}$ is temperature gradient and K is coefficient of thermal conductivity.

Newton's Law of Cooling

- Rate of cooling of a body is proportional to the excess temperature of the body over the surroundings
  i.e. $\frac{dQ}{dt}\ ;\ -k(T_2-T_1)$

Thermodynamics

First Law of Thermodynamics

- $dQ=dU+dW$
  where dQ is amount of heat which is taken as positive when heat is supplied to a system and negative when heat is drawn from the system
  dU is change in internal energy, taken as positive when temperature increases and negative when temperature decreases,
  dW is amount of work done, taken as positive when work is done by the system and negative when work is done on the system.

Thermodynamic Processes

- **Isothermal :** Temperature constant.
  $(dQ = dW)$
- **Adiabatic :** No heat flow between the system and surroundings.
  $(dU = -dW)$
- **Isobaric :** Pressure constant.
- **Isochoric :** Volume constant.
  $(dQ = dU)$

Adiabatic and Isothermal Processes

- For an adibatic process,
  PV^{ε} = constant, $TV^{(\gamma-1)}$ = constant,
  $P^{(1-\gamma)}T^{\gamma}$ = constant
  $$W=\frac{nR(T_1+T_2)}{(\gamma-1)}$$
- For an isothermal process,
  $$W\ ;\ nRT\ln\frac{V_2}{V_1}$$
- Slope of adiabatic curve
  $=\gamma$(slope of isothermal curve)

Heat Engine and Refrigerator

- Net work done (W) = Heat absorbed by the source (Q_1) – Heat rejected to the sink (Q_2)
- Efficiency of a heat engine,
  $$\phi=\frac{W}{Q_1}=1-\frac{Q_2}{Q_1}=1-\frac{T_2}{T_1}$$
- Coefficient of performance of a refrigerator,
  $$\upsilon=\frac{Q_2}{Q_1-Q_2}=\frac{T_2}{T_1-T_2}$$

Entropy

- Entropy (S) is the measure of disorder in a system.
  $$\Delta S=\frac{BQ}{T}.$$
- For irreversible processes, $\Delta S>0$
- For reversible processes, $\Delta S=0$
- Process where $\Delta S<0$ is not possible.

Kinetic Theory

Ideal Gas Laws

- **Boyle's law :** At constant T,
  $P\propto 1/V$, PV = constant
- **Charles' law :** At constant P,
  $V\propto T$, V/T = constant
- **Gay-Lussac's law :** At constant V,
  $P\propto T$, P/T = constant
- **Ideal gas equation :**
  $$PV=nRT=k_BNT.$$

Pressure and Kinetic Energy

- Pressure exerted by an ideal gas
  $$P\ ;\ \frac{1}{3}mnv_{\text{rms}}^2=\frac{1}{3}\frac{M}{V}v_{\text{rms}}^2=\frac{1}{3}\rho v_{\text{rms}}^2$$
- Average kinetic energy per molecule of the gas
  $$E\ ;\ \frac{1}{2}mv_{\text{rms}}^2=\frac{3}{2}k_BT$$
- Relation between P and E: $PV=\frac{2}{3}E$

Speed of a Gas Molecule

- Most probable speed,
  $$v_{\text{mp}}\ ;\ \sqrt{\frac{2k_BT}{m}}=\sqrt{\frac{2RT}{M}}$$
- Average speed,
  $$v_{\text{av}}\ ;\ \sqrt{\frac{8k_BT}{\pi m}}=\sqrt{\frac{8RT}{\pi M}}$$
- Root mean square speed,
  $$v_{\text{rms}}\ ;\ \sqrt{\frac{3k_BT}{m}}=\sqrt{\frac{3RT}{M}}$$

Relation between C_P, C_V, γ, f and R

- $C_V\ ;\ \frac{f}{2}R$
- $C_P\ ;\ \frac{f}{2}R+R$
- $\varepsilon\ ;\ \frac{C_P}{C_V}$
- $\varepsilon=1+\frac{2}{f}$
- $C_P+C_V=R$

where f is the number of degrees of freedom.

Values of C_P, C_V and γ

- For monoatomic gases
  $$C_V\ ;\ \frac{3}{2}R,\ C_P=\frac{5}{2}R,\ \gamma=\frac{5}{3}$$
- For diatomic gases :
  $$C_V\ ;\ \frac{5}{2}R,\ C_P=\frac{7}{2}R,\ \gamma=\frac{7}{5}$$
- For triatomic gases (linear molecule)
  $$C_V\ ;\ \frac{7}{2}R,\ C_P=\frac{9}{2}R,\ \gamma=\frac{9}{7}$$
- For triatomic gases (non-linear molecule)
  $$C_V\ ;\ 3R,\ C_P=4R,\ \gamma=\frac{4}{3}$$

Mean Free Path

- Average distance travelled by a gas molecule between two successive collisions
  $$\varphi=\frac{k_BT}{\sqrt{2}\,\pi d^2P}=\frac{1}{\sqrt{2}\,\pi d^2 n}$$

Blackbody Radiation

- **Wein's displacement law :** $\lambda_m T$ = constant
- **Stefan-Boltzmann law :** For a perfect radiator at temperature T, the energy emitted per unit time, $H=A\sigma T^4$
  For a perfect radiator at temperature T, with surroundings at T_S, $H=\sigma A(T^4-T_S^4)$
  For a body with emissivity e, $H=e\sigma A(T^4-T_S^4)$

Exam Café

QUESTIONS FOR PRACTICE

1. A circular current loop of magnetic moment M is in an arbitrary orientation in an external magnetic field $\vec{B}$. The work done to rotate loop by 30° about an axis perpendicular to its plane is

(a) MB (b) $\sqrt{3}\frac{MB}{2}$

(c) $\frac{MB}{2}$ (d) zero

2. The magnetic field normal to the plane of a wire of n turns and radius r which carries a current I is measured on the axis of the coil at a small distance h from the centre of the coil. This is smaller than the magnetic field at the centre by the fraction

(a) $\frac{2r^2}{3h^2}$ (b) $\frac{3r^2}{2h^2}$

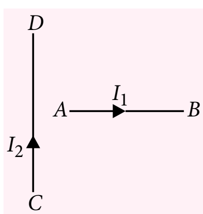
(c) $\frac{2h^2}{3r^2}$ (d) $\frac{3h^2}{2r^2}$

3. A charged particle moving in a uniform magnetic field penetrates a layer of lead and thereby loses one half of its kinetic energy. The radius of curvature of its path

(a) increases by $\sqrt{2}$ (b) reduces by $\frac{1}{\sqrt{2}}$

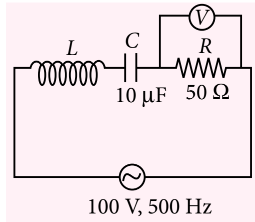
(c) remains the same (d) becomes half.

4. A current I_1 carrying wire AB is placed near an another long wire CD carrying current I_2. If free to move, wire AB will have



(a) rotational motion only
(b) translational motion only
(c) rotational as well as translational motion
(d) neither rotational nor translational motion.

5. In the circuit shown in figure, voltmeter reads 100 V. Then L is



(a) 0.1 H
(b) 0.2 H
(c) 0.02 H
(d) 0.01 H

6. If a magnetic material is having magnetic susceptibility $(\chi) = -1$, then the relative magnetic permeability (μ_r) and type of magnetic material is

(a) 0, diamagnetic (b) 2, ferromagnetic
(c) 1, paramagnetic (d) –1, diamagnetic

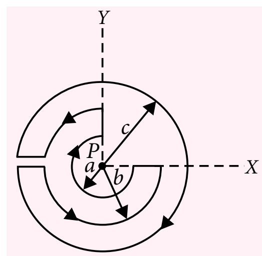
7. From the figure, for $c = 2a$ and $a < b < c$, the magnetic field at centre P will be zero when

(a) $a = b$

(b) $a = \frac{3}{5}b$

(c) $a = \frac{5}{3}b$

(d) $a = \frac{1}{3}b$



8. A particle of mass m and charge q moving with velocity $\vec{v}$ enters a region of uniform magnetic field of induction $\vec{B}$. Which of the following statements is incorrect?

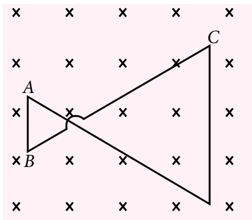
(a) Its path in the region of the field is always circular .

(b) Its path in the region of the field is circular if $\vec{v}\cdot\vec{B} = 0$.

(c) Its path in the region of the field is a straight line if $\vec{v}\times\vec{B} = 0$.

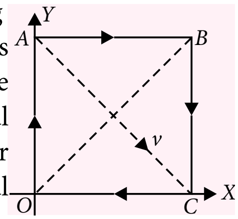
(d) Distance travelled by the particle in time T does not depend on the angle between $\vec{v}$ and $\vec{B}$.

9. A conducting wire frame is placed in magnetic field, which is directed into the paper. The magnetic field is increasing at constant rate. The directions of induced currents in wires AB and CD are



(a) A to B and C to D
(b) B to A and C to D
(c) A to B and D to C
(d) B to A and D to C

10. $OABC$ is a current carrying square loop. An electron is projected from the centre of loop along its diagonal AC as shown. Unit vector in the direction of initial acceleration will be



(a) $\hat{k}$ (b) $-\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$
(c) $-\hat{k}$ (d) $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$

11. A metal rod of resistance 20 Ω is fixed along a diameter of a conducting ring of radius 0.1 m and lies on X-Y plane. There is a magnetic field $\vec{B} = (50\text{ T})\hat{k}$. The ring rotates with an angular velocity ω = 20 rad s^{-1} about its axis. An external resistance of 10 Ω is connected across the centre of the ring and rim. The current through external resistance is

(a) $\frac{1}{4}$ A (b) $\frac{1}{2}$ A (c) $\frac{1}{3}$ A (d) 0

12. A coil having N turns is wound tightly in the form of a spiral with inner and outer radii a and b respectively. When a current I is passed through the coil, the magnetic field at the centre is

(a) $\frac{\mu_0 NI}{b}$ (b) $\frac{2\mu_0 NI}{a}$
(c) $\frac{\mu_0 NI}{2(b-a)}\ln\frac{b}{a}$ (d) $\frac{\mu_0 IN}{(b-a)}\ln\frac{b}{a}$

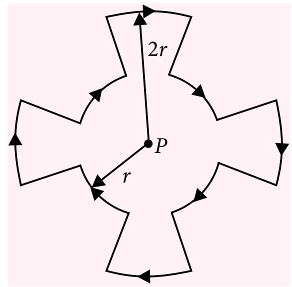
13. An electromagnetic radiation has an energy of 13.2 keV. Then the radiation belongs to the region of

(a) visible light (b) ultraviolet
(c) infrared (d) X-ray

14. A square loop of side a is rotating about its diagonal with angular velocity ω in a perpendicular magnetic field $\vec{B}$. It has 10 turns. The emf induced is

(a) $Ba^2 \sin \omega t$ (b) $Ba^2 \cos \omega t$
(c) $5\sqrt{2}\,Ba^2$ (d) $10\,Ba^2\,\omega \sin \omega t$

15. A current I flows around a closed path in the horizontal plane of the circle as shown in the figure. The path consists of eight arcs with alternating radii r and $2r$.



Each segment of arc subtends equal angle at the common centre P. The magnetic field produced by current path at point P is

(a) $\frac{3}{8}\frac{\mu_0 I}{r}$, perpendicular to the plane of the paper and directed inward.
(b) $\frac{3}{8}\frac{\mu_0 I}{r}$, perpendicular to the plane of the paper and directed outward.
(c) $\frac{1}{8}\frac{\mu_0 I}{r}$, perpendicular to the plane of the paper and directed inward.
(d) $\frac{1}{8}\frac{\mu_0 I}{r}$, perpendicular to the plane of the paper and directed outward.

16. A moving coil galvanometer of resistance 100 Ω is used as an ammeter using a resistance 0.1 Ω. The maximum deflection current in the galvanometer is 100 μA. Find the minimum current in the circuit, so that the ammeter shows maximum deflection.

(a) 100.1 mA (b) 1.001 mA
(c) 10.01 mA (d) 1.01 mA

17. Two concentric coils each of radius equal to 2π cm are placed at right angle to each other. 3 A and 4 A are the currents flowing in each

coil respectively. The magnetic induction (in Wb m^{-2}) at the centre of the coils will be

(a) 12×10^{-5} (b) 10^{-5}
(c) 5×10^{-5} (d) 7×10^{-5}

18. One proton beam enters a magnetic field of 10^{-4} Wb m^{-2} normally. If specific charge is 10^{11}C kg^{-1} and velocity of proton is 10^9 m s^{-1}, then the radius of circle described will be

(a) 0.1 m (b) 10 m (c) 100 m (d) 1 m

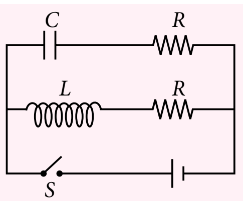
19. A physicist works in a laboratory where the magnetic field is 2 T. She wears a necklace enclosing area 0.01 m^2 in such a way that the plane of the necklace is normal to the field and is having a resistance $R = 0.01\ \Omega$. Due to power failure, the field decays to 1 T in time 10^{-3} s, then the total heat produced in her necklace is

(a) 10 J (b) 20 J (c) 30 J (d) 40 J

20. Suppose C be the capacitance of a capacitor discharging through a resistor R. Suppose t_1 is the time taken for the energy stored in the capacitor to reduce to half its initial value and t_2 is taken for the charge to reduce to one-fourth its initial value. Then the ratio $\frac{t_1}{t_2}$ will be

(a) 2 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

21. In the circuit shown in the figure, the switch S is closed at time $t = 0$.



$\left(\text{Given, } R = \sqrt{\frac{L}{C}}\right)$

The current through the capacitor and inductor will be equal at time t equals

(a) RC (b) $RC\ln 2$
(c) $\frac{1}{RC\ln 2}$ (d) LR

22. The primary of a step-down transformer used for ringing door bell has 2000 turns of fine wire and the secondary has 100 turns. This transformer when connected to a 110 V ac source will deliver at its secondary a potential difference of

(a) 220 V (b) 11 V (c) 55 V (d) 5.5 V

23. A motor is operating at 200 V and draws a current of 5 A at its full speed. If the resistance of armature of motor is 8.5 Ω, the efficiency of motor is

(a) 78.75% (b) 87.75%
(c) 89.75% (d) 98.75%

24. A dip circle is so set that its needle moves freely in the magnetic meridian. In this position, the angle of dip is 40°. Now the dip circle is rotated so that the plane in which the needle moves makes an angle of 30° with the magnetic meridian. In this position, the needle will dip by an angle

(a) 40° (b) 30°
(c) more than 40° (d) less than 40°

25. A horizontal straight wire 10 m long extending east and west is falling at right angles to the horizontal component of earth's magnetic field 0.30×10^{-4} Wb m^{-2}. If the induced emf is 1.5×10^{-3} V, the velocity of wire is

(a) 5×10^4 m s^{-1} (b) 5×10^2 m s^{-1}
(c) 50 m s^{-1} (d) 5 m s^{-1}

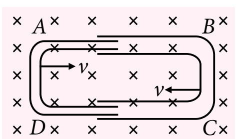
26. Two coils, a primary of 400 turns and a secondary of 20 turns are wound over an iron core of length 20π m and cross-section of 2 cm radius. If $\mu_r = 800$, then coefficient of mutual induction is approximately

(a) 1.6×10^7 H (b) 1.6×10^{-4} H
(c) 1.6×10^3 H (d) 1. 6 H

27. A point source of electromagnetic radiation has an average power output of 800 W. The maximum value of electric field at a distance 4.0 m from the source is

(a) 64.7 V m^{-2} (b) 57.8 V m^{-2}
(c) 56.72 V m^{-2} (d) 54.77 V m^{-2}

28. Oneconducting U tube can slide inside another as shown in figure maintaining electrical contacts between the tubes.



The magnetic field B is perpendicular to the plane of the figure. If each tube moves towards the other at a constant speed v, then the emf induced in the circuit in terms of B, l and v, where l is the width of each tube, will be

(a) Blv (b) $-Blv$ (c) $2Blv$ (d) zero

29. A moving coil galvanometer has 150 equal divisions. Its current sensitivity is 10 divisions m A^{-1} and voltage sensitivity is 2 divisions mV^{-1}. In order that each division reads 1 V, the resistance (in ohm) needed to be connected in series with the coil will be

(a) 10^3 (b) 10^5
(c) 99995 (d) 9995

30. An ideal coil of 10 H is connected in series with a resistance of 5 Ω and a battery of 5 V. Two seconds after the connection is made, the current flowing (in ampere) in the circuit is

(a) $(1 - e^{-1})$ (b) $(1 - e)$
(c) e (d) e^{-1}

SOLUTIONS

1. **(d):** No work is done to rotate the loop about an axis perpendicular to its plane as $\vec{M}$ is directed along the axis. Work is done only when the plane of the loop rotates.

2. **(d):** As $B_c = \frac{\mu_0}{4\pi}\frac{2\pi nI}{r}$ and $B_h = \frac{\mu_0}{4\pi}\frac{2\pi nIr^2}{(r^2+h^2)^{3/2}}$

so $\frac{B_h}{B_c} = \left(1+\frac{h^2}{r^2}\right)^{-3/2}$

Fractional decrease in the magnetic field will be

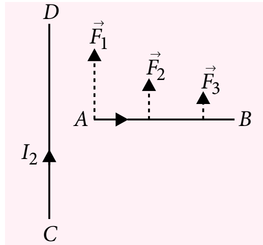
$$= \frac{B_c - B_h}{B_c} = \left(1 - \frac{B_h}{B_c}\right)$$

$$= \left[1 - \left(1+\frac{h^2}{r^2}\right)^{-3/2}\right] = 1 - \left(1 - \frac{3h^2}{2r^2}\right) = \frac{3h^2}{2r^2}$$

3. **(b):** As $r = \frac{\sqrt{2mE}}{Bq}$ and $r' = \frac{\sqrt{2m(E/2)}}{Bq}$

so, $r' = \frac{r}{\sqrt{2}}$

4. **(c) :** Since the magnetic field, due to current through wire CD at various locations on wire AB is not uniform, therefore, the wire AB, carrying current I_1 is subjected to variable magnetic field. Due to which, neither force nor the torque on the wire AB will be zero. As as result of which the wire AB will have both translational and rotational motion.



5. **(d):** As emf of source is 100 V, therefore, voltmeter can read 100 V only at resonance.

As $\upsilon_r = \frac{1}{2\pi\sqrt{LC}}$

$\therefore\ 500 = \frac{1}{2\pi\sqrt{L\times10^{-5}}}$ or $L = 0.01$ H

6. **(a) :** Here, $\chi = -1$

then $\mu_r = 1 + \chi$

or $\mu_r = 1 + (-1) = 1 - 1 = 0$

Since, magnetic susceptibility of given material is less than one, given magnetic material is a diamagnetic material.

7. **(c) :** Magnetic field at the centre P,

$$B = \frac{\mu_0 I}{2c}\otimes + \frac{\mu_0 I}{2b}\left(\frac{3}{4}\right)\odot + \frac{\mu_0 I}{2a}\left(\frac{3}{4}\right)\otimes$$

According to question,

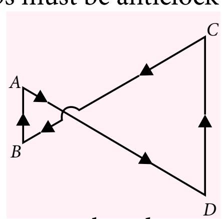
$$0 = \frac{\mu_0 I}{2}\left[\frac{1}{c} - \frac{3}{4b} + \frac{3}{4a}\right]$$

$$\Rightarrow \frac{3}{4}\left[\frac{1}{b} - \frac{1}{a}\right] = \frac{1}{c}$$

If $c = 2a, \frac{3}{4}\left[\frac{1}{b} - \frac{1}{a}\right] = \frac{1}{2a} \Rightarrow a = \frac{5}{3}b$

8. **(a)**

9. **(d):** As the magnetic field directed into the paper is increasing at a constant rate, therefore, induced current should produce a magnetic field directed out of the paper. Thus current in both the loops must be anticlockwise

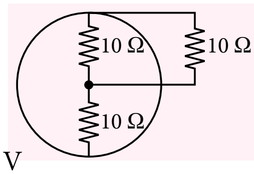


As area of loop on right side is more, therefore, induced emf on right side of loop will be more compared to the emf induced on the left-side of the loop. $\left[\because \quad \varepsilon = -\frac{d\phi}{dt} = -A\frac{dB}{dt}\right]$

Hence the net current induced in the complete loop will be along *DCBAD*.

10. (b)

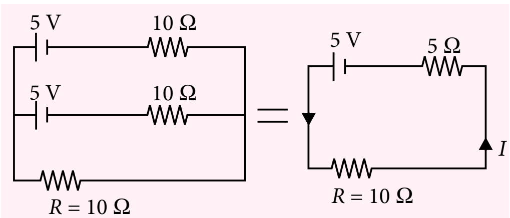
11. (c) : Potential difference between centre of the ring and the rim is



$$V = \frac{1}{2}B\omega r^2$$

$$= \frac{1}{2}(50)(20)(0.1)^2 = 5\text{ V}$$

Now the circuit can be drawn as follows

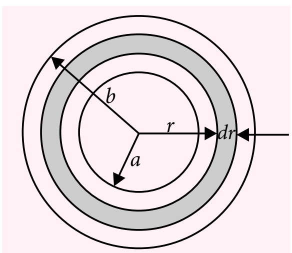


Current through external resistance,

$$I = \frac{5}{10+5} = \frac{1}{3}\text{ A}$$

12. (c) : Consider an element of thickness *dr* at distance *r* from the centre. The number of turns in this element will be

$$dN = \frac{N}{b-a}dr$$



Magnetic field due to this element at the centre of the coil will be

$$dB = \frac{\mu_0 (dN)I}{2r} = \frac{\mu_0 I}{2}\cdot\frac{N}{b-a}\cdot\frac{dr}{r}$$

Total magnetic field at the centre,

$$B = \int dB = \frac{\mu_0 NI}{2(b-a)}\int_{r=a}^{r=b}\frac{1}{r}dr = \frac{\mu_0 NI}{2(b-a)}\ln\left(\frac{b}{a}\right)$$

13. (d): Given: $E = 13.2$ keV

$$\lambda\,(\text{in Å}) = \frac{hc}{E(\text{eV})} = \frac{12400}{13.2\times10^3} = 0.939\text{ Å} \approx 1\text{ Å}.$$

X-rays covers wavelengths ranging from about 10^{-8} m (10 nm) to 10^{-13} m (10^{-4} nm).

An electromagnetic radiation of energy 13.2 keV belongs to X-ray region of electromagnetic spectrum.

14. (d): As, $\phi = NBA\cos\theta = 10\,Ba^2\cos\omega t$

$$\varepsilon = -\frac{d\phi}{dt} = -\frac{d}{dt}(10Ba^2\cos\omega t)$$

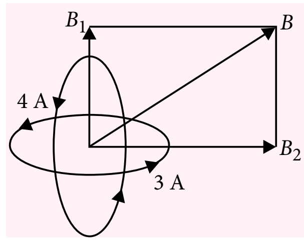
$$= 10\,B\,a^2\omega\sin\omega t.$$

15. (a)

16. (a): As, $I = \frac{G+S}{S}I_g = \frac{100+0.1}{0.1}\times100$

$= 100100\ \mu\text{A} = 100.1$ mA

17. (c) :



$$B_1 = \frac{\mu_0 I_1}{2r} = \frac{4\pi\times10^{-7}\times3}{2\times2\pi\times10^{-2}} = 3\times10^{-5}\text{ Wb m}^{-2}$$

$$B_2 = \frac{\mu_0 I_2}{2r} = \frac{4\pi\times10^{-7}\times4}{2\times2\pi\times10^{-2}} = 4\times10^{-5}\text{ Wb m}^{-2}$$

$$B = \sqrt{B_1^2 + B_2^2} = (\sqrt{3^2+4^2})\times10^{-5}$$

$$= 5\times10^{-5}\text{ Wb m}^{-2}$$

18. (c) : As, $r = \frac{mv}{eB} = \frac{v}{\left(\frac{e}{m}\right)B}$

$$\therefore \quad r = \frac{10^9}{10^{11}\times10^{-4}} = 100\text{ m}$$

19. (a): As $\varepsilon = -\frac{d\phi}{dt} = -A\frac{dB}{dt}$

$$= -0.01\times\frac{(1-2)}{10^{-3}} = 10\text{ V}$$

Heat produced $= \frac{\varepsilon^2 t}{R} = \frac{(10)^2\times10^{-3}}{0.01} = 10$ J

20. (d): As $U = U_0 e^{-\frac{2t}{RC}}$

$$\therefore \quad \frac{U_0}{2} = U_0 e^{\frac{-2t_1}{RC}}$$

$$\Rightarrow \frac{2t_1}{RC} = \log_e 2 \quad \text{...(i)}$$

Also, $q = q_0 e^{-\frac{t}{RC}}$

$$\therefore \frac{q_0}{4} = q_0 e^{-\frac{t_2}{RC}}$$

$$\Rightarrow \frac{t_2}{RC} = \log_e 4 = 2\log_e 2 \quad \text{...(ii)}$$

From equation (i) and (ii)

$$\frac{2t_1}{t_2} = \frac{\log_e 2}{2\log_e 2} \text{ or } \frac{t_1}{t_2} = \frac{1}{4}$$

21. (b): Growth of current in RC circuit,

$I_C = I_0 e^{-t/RC}$

Growth of current in LR circuit,

$I_L = I_0(1 - e^{-Rt/L}) = I_0(1 - e^{-t/RC})$

$$\left[\because R = \sqrt{\frac{L}{C}} \text{ or } \frac{R}{L} = \frac{1}{RC}\right]$$

But $I_C = I_L$

$\therefore \; I_0 e^{-t/RC} = I_0(1 - e^{-t/RC})$

or $2e^{-t/RC} = 1 \Rightarrow e^{t/RC} = 2$

or $\frac{t}{RC} = \ln 2 \qquad \therefore \; t = RC\ln 2$

22. (d): As, $V_S = \frac{N_S}{N_P} \times V_P$

$$= \frac{100}{2000} \times 110 = 5.5 \text{ V}$$

23. (a): As, $I = \frac{V - \varepsilon}{R}$

$\therefore \; \varepsilon = V - IR = 200 - (5 \times 8.5) = 157.5$ V

So, $\eta = \frac{\varepsilon}{V} = \frac{157.5}{200} \times 100\% = 78.75\%$

24. (c) : As, $\tan\delta' = \frac{\tan\delta}{\cos\theta} = \frac{\tan 40°}{\cos 30°} \quad \therefore \; \delta' > \delta$

i.e. δ' is more than 40°

25. (d): From $\varepsilon = Blv$

$$v = \frac{\varepsilon}{Bl} = \frac{1.5\times10^{-3}}{0.3\times10^{-4}\times10} = 5 \text{ m s}^{-1}$$

26. (b): As, $M = \frac{\mu_0\mu_r N_1 N_2 A}{l}$

$$= \frac{4\pi\times10^{-7}\times800\times400\times20\times\pi(2\times10^{-2})^2}{20\,\pi}$$

$m = 1.6 \times 10^{-4}$ H

27. (d) : Intensity of em wave is $I = \frac{P_{av}}{4\pi r^2} = \frac{E_0^2}{2\mu_0 c}$

or $\quad E_0 = \sqrt{\frac{\mu_0 c P_{av}}{2\pi r^2}}$

$$= \sqrt{\frac{(4\pi\times10^{-7})\times(3\times10^8)\times800}{2\pi\times(4)^2}}$$

$= 54.77$ V m^{-2}

28. (c) : Let v_r be the relative velocity of one tube with respect to the other.

Then, $\varepsilon = B \times$ width of the tube $\times v_r$

$= B \times l \times [v - (-v)] = 2Blv$

29. (d): Resistance of the galvanometer,

$$G = \frac{\text{current sensitivity}}{\text{voltage sensitivity}} = \frac{10}{2} = 5\,\Omega$$

Number of divisions on the galvanometer scale,

$n = 150$

Current required for full scale deflection,

$$I_g = \frac{n}{\text{current sensitivity}} = \frac{150}{10} = 15 \text{ mA}$$

$= 15 \times 10^{-3}$ A

Required range of voltmeter = 150 × 1 = 150 V

Required series resistance,

$$R = \frac{V}{I_g} - G = \frac{150}{15\times10^{-3}} - 5 = 9995\,\Omega$$

30. (a) : Here, $L = 10$ H, $R = 5\,\Omega$, $t = 2$ s,

$$\varepsilon = 5 \text{ V}, I_0 = \frac{\varepsilon}{R} = \frac{5}{5} = 1 \text{ A}$$

As $I = I_0(1 - e^{-Rt/L}) = 1(1 - e^{(-5\times2)/10})$

$I = (1 - e^{-1})$ A

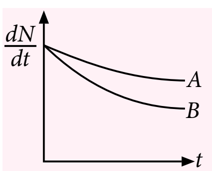
CBSE BOARD PRACTICE PAPER 2015

GENERAL INSTRUCTIONS

(i) All questions are compulsory.
(ii) There are 26 questions in total. Questions 1 to 5 carry one mark each, questions 6 to 10 carry two marks each, questions 11 to 22 carry three marks each, 23 is a VBQ carries 4 marks and questions 24 to 26 carry five marks each.
(iii) There is no overall choice. However, an internal choice has been provided in one question of two marks, one question of three marks and all three questions of five marks. You have to attempt only one of the choices in such questions.
(iv) Use of calculators is not permitted. However, you may use log tables if necessary.

1. An electron does not suffer any deflection while passing through a region of magnetic field. What is the direction of the magnetic field?

2. Which sample, *A* or *B* shown in figure has shorter mean-life?



3. There are two sources of light, each emitting with a power of 100 W. One emits X-rays of wavelength 1 nm and the other visible light at 500 nm. Find the ratio of number of photons of X-rays to the photons of visible light of the given wavelength?

4. How can the rms voltage of an ac circuit be non zero when its average value is zero? Explain.

5. Why are alloys used for making standard resistance coils?

6. Although the band structures of diamond and silicon are quite similar, the band gaps between the valence and conduction bands are different, being 5.33 eV for diamond and 1.14 eV for silicon. How can this difference in band gaps lead to a simple explanation of why silicon has a metallic appearance but diamond is transparent?

7. Why do stable nuclei never have more protons than neutrons?

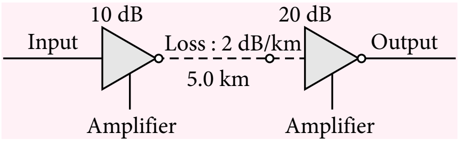
OR

The decay constant, for a given radioactive sample, is 0.3465 day^{-1}. What percentage of this sample will get decayed in a period of 4 days?

8. Consider two conducting spheres of radii R_1 and R_2 with $R_1 > R_2$. If the two are at the same potential, the larger sphere has more charge than the smaller sphere. State whether the charge density of the smaller sphere is more or less than that of the larger one.

9. A current *I* carrying loop consists of 3 identical quarter circles of radius *R*, lying in the positive quadrants of the *XY*-, *YZ*- and *ZX*-planes with their centres at the origin, joined together. Find the direction and magnitude of $\vec{B}$ at the origin.

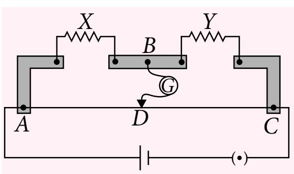
10. Figure shows a communication system. What is the output power when input signal is of 1.01 mW? [gain in dB = $10 \log_{10} (P_o/P_i)$]



11. A beam of light converges at a point P. Now a lens is placed in the path of the convergent beam at 12 cm from P. At what point does the beam converge if the lens is (a) a convex lens of focal length 20 cm, and (b) a concave lens of focal length 16 cm?

12. Find the electric and magnetic fields produced by the radiation coming from a 100 W bulb at a distance of 3 m. Assume that the efficiency of the bulb is 2.5% and it is a point source.

13. The figure shows experimental set up of a meter bridge. When the two unknown resistances X and Y are inserted, the null point D is obtained 40 cm from the end A. When a resistance of 10 Ω is connected in series with X, the null point shifts by 10 cm towards right side. Find the position of the null point when the 10 Ω resistance is connected in series with resistance Y instead of X. Determine the values of the resistances X and Y.



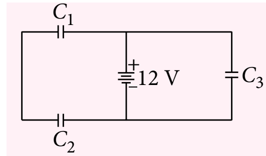
14. The ground state energy of hydrogen atom is –13.6 eV.

(i) What is the kinetic energy of an electron in the 2nd excited state?

(ii) What is the potential energy of an electron in the 3rd excited state?

(iii) If the electron jumps to the ground state from the 3rd excited state, calculate the wavelength of the photon emitted.

15 Three identical capacitors C_1, C_2 and C_3 of capacitance 6 μF each are connected to a 12 V battery as shown in the figure.



Find

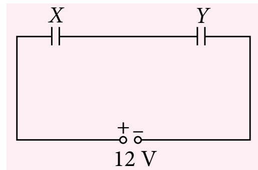
(i) charge on each capacitor.

(ii) equivalent capacitance of the network.

(iii) energy stored in the network of capacitors.

OR

Two parallel plate capacitors, X and Y, have the same area of plates and same separation between them. X has air between the plates while Y contains a dielectric medium of $\varepsilon_r = 4$.



(i) Find capacitance of each capacitor if equivalent capacitance of the combination is 4 μF.

(ii) Find the potential difference between the plates of X and Y.

(iii) What is the ratio of electrostatic energy stored in X and Y ?

16. Draw and explain the graph showing the variation of intensity in the interference pattern in Young's double slit experiment.

17. Prove that the current density of a metallic conductor is directly proportional to the drift speed of electrons.

18. What is the de Broglie wavelength of a nitrogen molecule in air at 300 K? Assume that the molecule is moving with the root mean square speed of molecules at this temperature. (Atomic mass of nitrogen = 14.0075 u)

19. A series LCR circuit with L = 0.12 H, C = 480 nF, R = 23 Ω is connected to a 230 V variable frequency supply.

(a) What is the source frequency for which current amplitude is maximum? Obtain the maximum value.

(b) What is the source frequency for which average power absorbed by the circuit is maximum? Obtain the value of this maximum power.

20. With the help of a schematic diagram, briefly describe an arrangement for transmission and reception of a message signal over a distance of several thousand kilometers.

21. A small bulb is placed at the bottom of a tank containing water to a depth of 80 cm. What is the area of the surface of water through which light from the bulb can emerge out? Refractive index of water is 1.33. (Consider the bulb to be a point source)

22. A hollow charged conductor has a tiny hole cut into its surface. Show that the electric field in the hole is $\left(\frac{\sigma}{2\epsilon_0}\right)\hat{n}$ where $\hat{n}$ is the unit vector in the outward normal direction, and σ is the surface charge density near the hole.

23. Ankush purchased an induction stove and explained to his mother Neelam that due to shortage and heavy cost of LPG, she must utilize other sources that are available to produce heat energy. Ankush also discussed with his younger brother and sister that the oil companies are trying their best to meet out the demand for LPG and that a good citizen must use other sources wherever feasible. The purchased induction stove have a value of 7 H inductor and the flow of current changes from 10 A to 7 A in a time of 9×10^{-2} seconds.

(a) What qualities do you find in Ankush?

(b) What is the advantage of using induction stove over LPG?

(c) Find the emf generated in the induction stove.

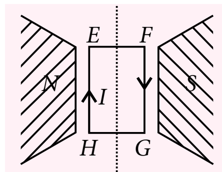
24. Explain with example, the formation of energy bands in solids. For an extrinsic semiconductor, mark the donor and acceptor energy levels in the energy-band diagram.

OR

For a *npn* transistor in the common emitter configuration, draw a labelled circuit diagram of an arrangement for measuring the collector current as a function of collector emitter voltage for at least two different values of base current. Draw the shape of the curves obtained. Define the terms: (i) output resistance and (ii) current amplification factor.

25. (a) Two straight long parallel conductors carry currents I_1 and I_2 in the same direction. Deduce the expression for the force per unit length between them.

(b) A rectangular current carrying loop *EFGH* is kept in a uniform magnetic field as shown in the figure.



(i) What is the direction of the magnetic moment of the current loop?

(ii) When is the torque acting on the loop maximum and zero?

OR

Distinguish the magnetic properties of dia-, para- and ferro-magnetic substances in terms of (i) susceptibility, (ii) magnetic permeability and (iii) coercivity. Give one example of each of these materials.
Draw the field lines due to an external magnetic field near (i) diamagnetic, (ii) paramagnetic substance.

26. What is meant by diffraction? Draw a graph to show the relative intensity distribution for a single slit diffraction pattern. Obtain an expression for the diffraction of the first minimum and first maximum in the diffraction pattern.

OR

State Huygen's principle. Using the geometrical construction of secondary wavelets, explain the refraction of a plane wavefront incident on a plane surface. Hence verify Snell's law of refraction.

SOLUTIONS

1. Since $\vec{F}_m = -e(\vec{v} \times \vec{B}) = e(\vec{B} \times \vec{v})$, and $\vec{F}_m = 0$ (as electron does not suffer deflection),
$\vec{B} \times \vec{v} = 0$, *i.e.*, $\vec{B}$ is parallel to $\vec{v}$, *i.e.*, the direction of motion of the electron.

2. From the graph, it follows that $\lambda_B > \lambda_A$ (as curve B falls off rapidly than curve A).
So, $\lambda_B / \lambda_A > 1$ or $\lambda_A / \lambda_B < 1$.
Since mean-life, $\tau = \frac{1}{\lambda}$,
$$\Rightarrow \frac{\tau_B}{\tau_A} = \frac{\lambda_A}{\lambda_B} < 1 \text{ or } \tau_B < \tau_A$$

3. If n_1 and n_2 denote the numbers of photons of X-rays and visible light of wavelengths λ_1 and λ_2, respectively, then for the same amount of power (*i.e.*, energy incident per second),
$$n_1\left(\frac{hc}{\lambda_1}\right) = n_2\left(\frac{hc}{\lambda_2}\right)$$
(as energy of a photon, $E = \frac{hc}{\lambda}$)
or $$\frac{n_1}{n_2} = \frac{\lambda_1}{\lambda_2} = \frac{1\,\text{nm}}{500\text{ nm}} = \frac{1}{500}$$

4. The average voltage in an ac circuit is zero because it oscillates symmetrically between positive and negative values. To calculate the rms voltage, however, one first squares the voltage. This gives values that are always greater than or equal to zero. Therefore, the rms voltage will be non-zero unless the voltage in the circuit is zero at all times.

5. The alloys (*i.e.*, manganin or constantan) are used for making the standard resistance coils as they have low value of temperature coefficient of resistance and high resistivity.

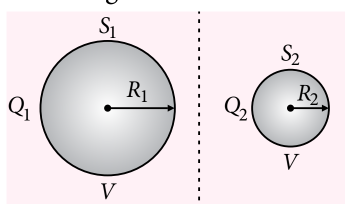
6. The energies of visible-light photons range from about 1.77 eV to 3.10 eV. Because the band gap energy for silicon is smaller than the energy of visible light photon, electrons from the valence band can be photoexcited into conduction band in sufficient quantities to make the silicon metallic and give it a lustrous appearance. On the other hand, the band gap in diamond is much larger than the photon energies, so the visible photons pass through without interacting with the electrons in the diamond crystal. Thus, diamond is transparent.

7. Protons being positively charged, repel one another due to electrostatic force. For those nuclei which have more than 10 protons or so, this repulsion becomes very large. To overcome this repulsion and acquire stability, an excess of neutrons, which produce only attractive forces (due to strong interaction), are required in the nucleus. In all the heavier stable nuclei, the number of neutrons is more than the number of protons.

OR

Here $\lambda = 0.3465$ day^{-1}, $t = 4$ days
$$T_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{0.3465} = 2 \text{ days}$$
$$\therefore \quad n = \frac{t}{T_{1/2}} = \frac{4}{2} = 2$$
Hence sample left undecayed after a period of 4 days,
$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^2 = \frac{1}{4} = 25\%$$

8. Let S_1 and S_2 be the two conducting spheres as shown in the figure



We are given that $R_1 > R_2$ and $Q_1 > Q_2$.
Since S_1 and S_2 are at the same potential,
$$\frac{Q_1}{R_1} = \frac{Q_2}{R_2} \text{ or } \frac{Q_2}{Q_1} = \frac{R_2}{R_1} \quad ...(i)$$
If σ_1 and σ_2 represent the charge densities of the spheres S_1 and S_2, respectively, then
$$\frac{\sigma_2}{\sigma_1} = \frac{Q_2 / 4\pi R_2^2}{Q_1 / 4\pi R_1^2} = \left(\frac{Q_2}{Q_1}\right)\left(\frac{R_1}{R_2}\right)^2 \quad ...(ii)$$
From eqns. (i) and (ii),
$$\frac{\sigma_2}{\sigma_1} = \left(\frac{R_2}{R_1}\right)\left(\frac{R_1}{R_2}\right)^2 = \frac{R_1}{R_2} > 1$$

So, $\sigma_2 > \sigma_1$, i.e., charge density of the smaller spheres is more.

9. Magnetic field due to quarter circle of radius R, carrying current I and lying in the positive quadrant of XY-plane, i.e.,

$$\vec{B}_1 = \frac{1}{4}\left(\frac{\mu_0 I}{2R}\right)\hat{k}$$

Similarly, due to quarter circles lying in YZ-plane and ZX-plane.

$$\vec{B}_2 = \frac{1}{4}\left(\frac{\mu_0 I}{2R}\right)\hat{i}\ ,\ \vec{B}_3 = \frac{1}{4}\left(\frac{\mu_0 I}{2R}\right)\hat{j}$$

Net magnetic field at the origin,

$$\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 = \frac{1}{4}\left(\frac{\mu_0 I}{2R}\right)\left(\hat{i}+\hat{j}+\hat{k}\right)$$

10. Total gain of both the amplifiers

= 10 dB + 20 dB = 30 dB

Loss suffered in transmission path

= (2 dB/km) (5.0 km) = 10 dB

Overall gain of the signal = net gain of the amplifier

= 30 dB – 10 dB = 20 dB

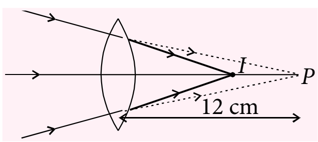
As gain in dB = $10\log_{10}(P_o / P_i)$

$\therefore\ \log_{10}(P_o / P_i) = 2$

or $(P_o / P_i) = 10^2 = 100$

or $P_o = 10^2 P_i = 10^2(1.01\text{ mW}) = 101\text{ mW}$

11. (a) The convex lens is placed in the path of convergent beam,



so the incident rays form a virtual object for the convex lens

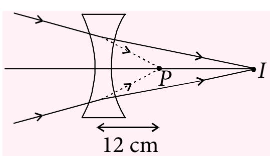
Using lens formula

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f},\ \frac{1}{v}-\frac{1}{+12}=\frac{1}{+20}$$

$$\frac{1}{v}=\frac{1}{20}+\frac{1}{12}=\frac{3+5}{60},\ v=\frac{60}{8}=+7.5\text{ cm}$$

The image is formed by further converging beams at I at a distance 7.5 cm from lens.

(b) A concave lens is placed in the path of convergent beam,



the concave lens further diverge the light.

Using lens formula $\frac{1}{v}-\frac{1}{u}=\frac{1}{f},\ \frac{1}{v}-\frac{1}{+12}=\frac{1}{-16}$

$$\frac{1}{v}=-\frac{1}{16}+\frac{1}{12}=\frac{-3+4}{48}=\frac{1}{48},\ v=+48\text{ cm}$$

The image is formed by diverged rays at a distance 48 cm from concave lens.

12. Since the efficiency of the bulb is 2.5%, effective power of the bulb, $P=\left(\frac{2.5}{100}\right)100\text{ W} = 2.5\text{ W}$

Intensity at a distance r from the bulb,

$$I=\frac{P}{4\pi r^2}=\frac{2.5}{4\times 3.14\times 3^2}=0.022\text{ W m}^{-2}$$

($4\pi r^2$ is the area of a sphere of radius r, centred on the source)

Also, as $I=\frac{1}{2}c\in_0 E_0^2$,

$$E_0=\sqrt{\frac{2I}{c\in_0}}=\sqrt{\frac{2\times 0.022}{\left(3\times 10^8\right)\left(8.85\times 10^{-12}\right)}}$$

or, $E_0 = 4.07\text{ V m}^{-1}$

Clearly, $B_0=\frac{E_0}{c}=\frac{4.07}{3\times 10^8}=1.36\times 10^{-8}\text{ T}$

13. When X and Y are connected in left and right gaps of meter bridge respectively, then

$$\frac{X}{Y}=\frac{l}{100-l}=\frac{40}{100-40}=\frac{40}{60}=\frac{2}{3}$$

or $X=\frac{2}{3}Y$ ...(i)

When 10 Ω is connected in series with X in left gap, then its effective resistance becomes $X' = (X + 10)\ \Omega$ and the balance point shifts by 10 cm towards right side, So, new balancing length becomes $l' = l + 10 = 40 + 10 = 50$ cm

Hence $\frac{X'}{Y}=\frac{l'}{100-l'}$

or $\frac{X+10}{Y}=\frac{50}{100-50}=\frac{50}{50}=1$

or $X + 10 = Y$

or $\frac{2}{3}Y+10=Y$ (Using (i))

or $10=Y-\frac{2}{3}Y=\frac{1}{3}Y$

or $Y = 30\ \Omega$ ...(ii)

and $X=\frac{2}{3}Y=\frac{2}{3}\times 30$

or $X = 20\ \Omega$ ...(iii)

When 10 Ω resistance is connected in series with Y in right gap, then

$$\frac{X}{Y+10} = \frac{l_1}{100-l_1} \text{ or } \frac{20}{30+10} = \frac{l_1}{100-l_1}$$

or $\frac{1}{2} = \frac{l_1}{100-l_1}$

or $100 - l_1 = 2l_1$ or $100 = 3l_1$

or $l_1 = \frac{100}{3} = 33.33$ cm

So, the null point will be obtained at 33.33 cm on the wire from left end A.

14. Here $E_1 = -13.6$ eV

$$E_3 = \frac{E_1}{3^2} = \frac{-13.6}{9} = -1.51 \text{ eV}$$

$$E_4 = \frac{E_1}{4^2} = \frac{-13.6}{16} = -0.85 \text{ eV}$$

(i) K.E. of an electron in 2nd excited state $= -E_3 = 1.51$ eV.

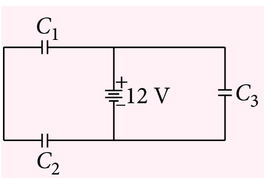
(ii) P.E. of an electron in 3rd excited state $= 2E_4 = -1.70$ eV.

(iii) $E_4 - E_1 = -0.85 - (-13.6)$
$= 12.75 \text{ eV} = 12.75 \times 1.6 \times 10^{-19}$ J

As $E_4 - E_1 = \frac{hc}{\lambda}$

$$\therefore \quad \lambda = \frac{hc}{E_4 - E_1} = \frac{6.6\times10^{-34}\times3\times10^8}{12.75\times1.6\times10^{-19}}$$

$= 970 \times 10^{-10}$ m $= 970$ Å

15. 

Here, $V = 12$ V and

$C_1 = C_2 = C_3 = 6\ \mu F = 6 \times 10^{-6}$ F

Charge on capacitor C_3 is

$Q_3 = C_3V = 6 \times 10^{-6} \times 12 = 72 \times 10^{-6}$ C $= 72\ \mu$C

Capacitors C_1 and C_2 are in series, their equivalent capacitance is

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}, C_S = 3\ \mu F$$

Charge on capacitor C_1 is

$Q_1 = C_SV = 3 \times 10^{-6} \times 12 = 36 \times 10^{-6}$ C $= 36\ \mu$C

Charge on capactior C_2 is $Q_2 = Q_1 = 36\ \mu$C

(ii) C_3 is in parallel with series combination of C_1 and C_2. Hence the equivalent capacitance of the network is $C_{eq} = C_3 + C_S = 6 + 3 = 9\ \mu$F

(iii) Energy stored $= \frac{1}{2}C_{eq}V^2$
$= \frac{1}{2} \times 9 \times 10^{-6} \times (12)^2$
$= 648 \times 10^{-6} = 648\ \mu$J

OR

(i) Capacitance of X, $C_X = \frac{\varepsilon_0 A}{d}$

Capacitance of Y, $C_Y = \varepsilon_r \frac{\varepsilon_0 A}{d} = \frac{4\varepsilon_0 A}{d}$

$\therefore \frac{C_Y}{C_X} = 4$ ... (i)

or $C_Y = 4C_X$

As X and Y are in series, therefore their equivalent capacitance is $C_{eq} = \frac{C_XC_Y}{C_X + C_Y}$

$$4\ \mu F = \frac{C_X 4C_X}{C_X + 4C_X}$$

$C_X = 5\ \mu$F $\therefore C_Y = 4C_X = 20\ \mu$F

(ii) In series, charge on each capacitor is same

$\therefore \frac{V_X}{V_Y} = \frac{C_Y}{C_X} = 4$ (Using (i))

or $V_X = 4V_Y$ ... (ii)

Also, $V_X + V_Y = 12$ ... (iii)

Solving (ii) and (iii), we get

$V_X = 9.6$ V and $V_Y = 2.4$ V

(iii) $\frac{\text{Electrostatic energy stored in } X}{\text{Electrostatic energy stored in } Y}$

$$= \frac{U_X}{U_Y} = \frac{\frac{1}{2}C_XV_X^2}{\frac{1}{2}C_YV_Y^2}$$

or $\frac{U_X}{U_Y} = \frac{C_X}{C_Y} \times \left(\frac{C_Y}{C_X}\right)^2 = \frac{C_Y}{C_X} = 4$

16. Suppose the two interfering waves have the same amplitude a.

The intensity of a bright fringe will be

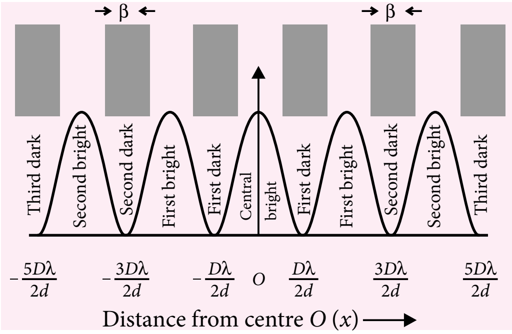
$I_{max} = k(a + a)^2 = 4ka^2 =$ constant

So all bright fringes will have the same maximum intensity.

The intensity of a dark fringe will be

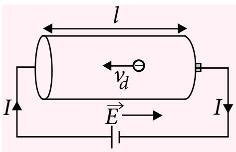
$I_{min} = k(a - a)^2 = 0$

So all dark fringes will be perfectly dark.



On plotting the intensities of bright and dark fringes against distance x from O, we get a curve as shown in the given figure. The intensity is maximum at the central point O. Then it becomes zero and maximum alternately on either side of O, depending on x is odd multiple of $\frac{D\lambda}{2d}$ and integral multiple of $\frac{D\lambda}{d}$ respectively.

17. Consider a conductor of length l and of uniform area of cross-section A.



Volume of the conductor = Al

If n is the number density of electrons (number of free electrons per unit volume) of the conductor. Then,

Total number of free electrons in the conductor = Aln.

Total charge on all the free electrons in the conductor, $q = Alne$

Let a constant potential difference V be applied across the ends of the conductor.

The electric field set up across the conductor is given by $E = \frac{V}{l}$ (in magnitude)

$\therefore$ Time taken by the free electrons to cross the conductor, $t = \frac{l}{v_d}$

where v_d is the drift speed of an electron

Current, $I = \frac{q}{t} = \frac{Alne}{(l/v_d)}$ or $I = Anev_d$

Current density, $J = \frac{I}{A} = nev_d$

Hence, $J \propto v_d$

18. Mass of nitrogen molecule,

$m = 2\,(14.0076)(1.66\times10^{-27})$ kg $= 46.5\times10^{-27}$ kg

$T = 300$ K

Average KE per molecule

$$= \frac{1}{2}mv^2 = \frac{3}{2}k_BT \text{ or } v = \sqrt{\frac{3k_BT}{m}}$$

Thus, $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{3mk_BT}}$

$$= \frac{(6.63\times10^{-34})}{\sqrt{3(46.5\times10^{-27})(1.38\times10^{-23})300}}$$

$$= \frac{6.63}{\sqrt{577.5}}\times10^{-10} = \frac{6.63}{24.03}\times10^{-10}\text{ m} = 0.28\text{ Å}$$

19. Here, $L = 0.12$ H, $C = 480$ nF $= 480\times10^{-9}$ F, $R = 23\ \Omega$, $\varepsilon_{rms} = 230$ V

(a) Current amplitude is maximum at resonant frequency (υ_r) where

$$\upsilon_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\times3.14\sqrt{0.12\left(480\times10^{-9}\right)}}$$

$$= \frac{10^4}{2\times3.14\times2.4} = 663\text{ Hz}$$

$$(I_0)_{max} = \frac{\varepsilon_0}{R} = \frac{\sqrt{2}\varepsilon_{rms}}{R} = \frac{1.414\times230}{23} = 14.14\text{ A}$$

(b) The average power (P) absorbed is given by

$$P = \frac{1}{2}I_0^2R.$$

Obviously, P is maximum (i.e., P_{max}) when I_0 is maximum, *i.e.*, $(I_0)_{max}$ and this occurs at resonant frequency, υ_r.

Thus, $P_{max} = \frac{1}{2}(I_0)^2_{max}R = \frac{1}{2}\times(14.14)^2\times23$
$= 2300$ W

20.

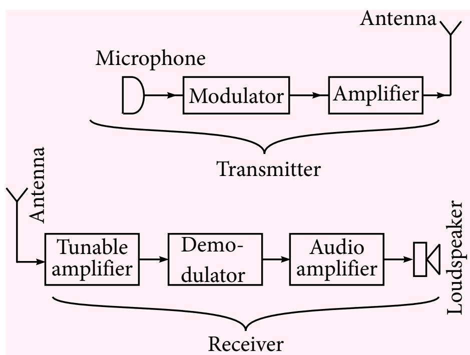


Figure shows a block diagram for an arrangement for transmitting a signal when transmitter and receiver are several thousand kilometers apart.

At the transmitting end, a microphone converts the sound into a time-varying electrical signal called the message signal. With the help of a modulator, the message signal is translated into radio frequency range. The power of the modulated wave is boosted by a suitable amplifier and then the wave is radiated into space from an antenna.

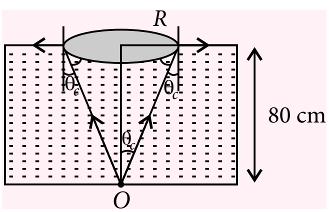
At the receiving end, another antenna picks up the waves from different transmitting stations. A tunable amplifier can selectively tune in and boost up the radio frequency wave from a particular station. The original signal is recovered by using a demodulator. After being amplified, the electrical signal is converted into sound signal by using a loudspeaker.

21. As shown in the figure, all those light rays which are incident on the surface at angle of incidence more than critical angle, does total internal reflection and are reflected back in water only. All those light rays which are incident before critical angle emerges out of surface bending away from normal. All those light beams which are incident at critical angle grazes the surface of water.

We know

$\sin\theta_c = \frac{1}{{}^a\mu_w}$

or $\sin\theta_c = \frac{3}{4}$



So, $\tan\theta_c = \frac{3}{\sqrt{7}}$

Radius of the area on the surface which allows light to emerge can be calculated.

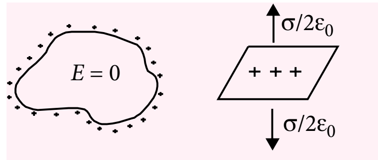
$$\tan\theta_c = \frac{R}{h}$$

$$R = h\tan\theta_c = 80 \times \frac{3}{\sqrt{7}} = 90.7 \text{ cm}$$

Area of patch of light, $A = \pi R^2$

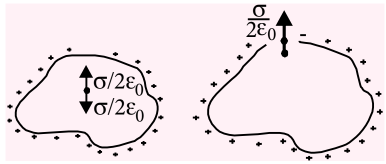
$= 3.14 \times (90.7)^2 = 25838 \text{ cm}^2 \approx 2.60 \text{ m}^2$

22. Inside a charged conductor the electric field is zero.



But a uniformly charged flat surface provide an electric field $\sigma/2\varepsilon_0$ normal to its plane

If we consider a small flat part on the surface of charged conductor, it certainly provides an electric field $\sigma/2\varepsilon_0$inside the conductor, which is nullified by an equal field due to rest of charged conductor.



Now if a hole is made in charged conductor, the field due to small flat part is absent but the field due to rest of charged conductor is present *i.e.*, equal to $\sigma/2\varepsilon_0$.

23. (a) Ankush has the qualities of responsibility, concern for the nation, cost economic attitude, promptness to use latest technology, sharing attitude.

(b) LPG is costly. It is difficult to carry heavy gas cylinders. Availability of LPG is limited. LPG is based on import, on the other hand induction stove uses cheap and easily available electricity.

(c) $\varepsilon = -L\, dI/dt = -\frac{7\times(7-10)}{9\times10^{-2}} = 233.3 \text{ V.}$

24. Refer points 9.1(5) and 9.2(2), MTG Excel in Physics

OR

Refer point 9.4(7), MTG Excel in Physics

25. Refer points 3.3(9) and 3.4(1), MTG Excel in Physics

OR

Refer point 3.8(8), MTG Excel in Physics.

26. Refer point 6.14(1, 3, 4), MTG Excel in Physics

OR

Refer points 6.10(6) and 6.11(1, 5), MTG Excel in Physics

TARGET PMTs

Useful for All National and State Level PMTs

1. The earth circles the sun once a year. The work which would have to be done on the earth to bring it to rest relative to the sun is: (Ignore the rotation of earth about its own axis. Given that mass of the earth = 6×10^{24} kg and distance between sun and earth is 1.5×10^8 km).
(a) 2.7×10^{30} J (b) 2.7×10^{31} J
(c) -2.7×10^{33} J (d) $+2.7 \times 10^{33}$ J

2. Three points masses, each of mass m, are placed at the corners of an equilateral triangle of side l. Then the moment of inertia of this system about an axis along one of the side of the triangle is
(a) $3ml^2$ (b) ml^2
(c) $\frac{3}{4}ml^2$ (d) $\frac{3}{2}ml^2$

3. A stone is dropped from the 25th storey of a multistoreyed building and it reaches the ground in 5 sec. In the first second, it passes through how many storeys of the building? (Take $g = 10$ m s^{-2})
(a) 1 (b) 2
(c) 3 (d) 5

4. A physical quantity, $y = \frac{a^4b^2}{(cd^4)^{1/3}}$ has four observables a, b, c and d. The percentage error in a, b, c and d are 2%, 3%, 4% and 5% respectively. The error in y will be
(a) 6% (b) 11% (c) 12% (d) 22%

5. The work of electric field done during the displacement of a negatively charged particle towards a fixed positively charged particle is 9 J. As a result the distance between the charges has been decreased by half. What work is done by the electric field over the first half of this distance?
(a) 3 J (b) 1.5 J (c) 6 J (d) 9 J

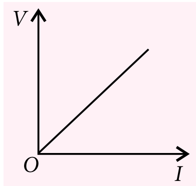
6. A variable condenser is permanently connected to a 100 V battery. If capacity is changed from 2 μF to 10 μF, then energy change is equal to
(a) 2×10^{-2} J (b) 2.5×10^{-2} J
(c) 6.5×10^{-2} J (d) 4×10^{-2} J

7. A wire of length L and cross-section A is made of material of Young's modulus Y. It is stretched by an amount x, the work done is
(a) $\frac{YxA}{2L}$ (b) $\frac{Yx^2A}{L}$
(c) $\frac{Yx^2A}{2L}$ (d) $\frac{2Yx^2A}{L}$

8. A smooth inclined plane of length L having inclination θ with the horizontal is inside a lift which is moving down with retardation a. The time taken by a body to slide down the inclined plane, from rest, will be
(a) $\sqrt{\frac{2L}{(g+a)\sin\theta}}$ (b) $\sqrt{\frac{2L}{(g-a)\sin\theta}}$
(c) $\sqrt{\frac{2L}{g\sin\theta}}$ (d) $\sqrt{\frac{2L}{a\sin\theta}}$

9. Magnetic field at the centre of a circular loop of area A is B. The magnetic moment of the loop will be
(a) $\frac{BA^2}{\mu_0\pi}$ (b) $\frac{BA^{3/2}}{\mu_0\pi}$
(c) $\frac{BA^{3/2}}{\mu_0\pi^{1/2}}$ (d) $\frac{2BA^{3/2}}{\mu_0\pi^{1/2}}$

10. The *V-I* graph for a wire of copper of length *L* and cross-section area *A* is shown in adjoining figure. The slope of the graph will be



(a) less if the experiment is repeated at a higher temperature
(b) more if a wire of silver having the same dimension is used
(c) doubled if the length of the wire is doubled
(d) same as earlier if length of the wire is halved

11. Speeds of two identical cars are u and $4u$ at a specific instant. The ratio of the respective distances at which the two cars are stopped from that instant is

(a) 1 : 1 (b) 1 : 4 (c) 1 : 8 (d) 1 : 16

12. In the formula : $X = 3YZ^2$, X and Z have dimensions of capacitance and magnetic induction respectively. What are the dimensions of Y in *MKSQ* system?

(a) $[M^{-3}L^{-1}T^3Q^4]$ (b) $[M^{-3}L^{-2}T^4Q^4]$
(c) $[M^{-2}L^{-2}T^4Q^4]$ (d) $[M^{-3}L^{-2}T^4Q^1]$

13. A body cools from 80°C to 64°C in 5 min and same body cools from 80°C to 52°C in 10 min, what is the temperature of the surrounding?

(a) 24°C (b) 28°C (c) 22°C (d) 25°C

14. Two resistances 3 Ω and 6 Ω are connected in parallel and a 4 Ω resistance is connected in series with this combination. The current through 3 Ω resistance is 0.8 A. Then the potential drop across 4 Ω resistance is

(a) 9.6 V (b) 2.6 V (c) 4.8 V (d) 1.2 V

15. A ray of light falls on a transparent glass slab with refractive index (relative to air) of 1.62. The angle of incidence for which the reflected and refracted rays are mutually perpendicular is

(a) $\tan^{-1}(1.62)$ (b) $\sin^{-1}(1.62)$
(c) $\cos^{-1}(1.62)$ (d) none of these

16. A cubical block is heated from 0°C to 100°C. The percentage increase in its length is 0.10%. What will be the percentage increase in its volume?

(a) 0.03% (b) 0.10%
(c) 0.30% (d) None of these

17. The change in potential energy when a body of mass m is raised to a height nR from earth's surface is (R = radius of the earth)

(a) $mgR\frac{n}{(n-1)}$ (b) mgR

(c) $mgR\frac{n}{(n+1)}$ (d) $mgR\frac{n^2}{(n^2+1)}$

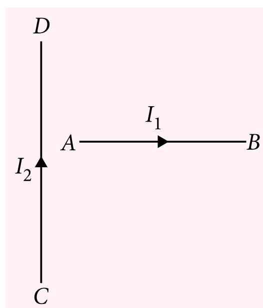
18. An isolated particle of mass m is moving in a horizontal plane (x-y), along the x-axis, at a certain height above the ground. It suddenly explodes into two fragments of masses $m/4$ and $3m/4$. An instant later, the smaller fragment is at y = + 15 cm. The larger fragment at this instant is at

(a) $y = -5$ cm (b) $y = +20$
(c) $y = +5$ cm (d) $y = -20$ cm

19. One gram of ice is mixed with one gram of steam. At thermal equilibrium the temperature of mixture is

(a) 0 °C (b) 100 °C
(c) 55 °C (d) 80 °C

20. A current I_1 carrying wire *AB* is placed near an another long wire *CD* carrying current I_2 as shown in figure. If free to move, wire *AB* will have



(a) rotational motion only
(b) translational motion only
(c) rotational as well as translational motion
(d) neither rotational nor translational motion

21. The dielectric strength of air is 3×10^6 V m^{-1}. A parallel plate capacitor has area 20 cm^2 and plate separation 0.1 mm. Find the maximum rms voltage of an ac source which can be connected.

(a) 212 V (b) 300 V
(c) 435 V (d) None of these

22. A particle undergoes uniform circular motion. About which point on the plane of the circle, will the angular momentum of the particle remain conserved?
(a) Centre of the circle
(b) On the circumference of the circle
(c) Inside the circle (d) Outside the circle

23. A block of mass 0.50 kg is moving with a speed of 2.0 m s^{-1} on a smooth surface. It strikes another stationary block of mass 1.0 kg and then move together as a single body. The energy loss during the collision is
(a) 0.16 J (b) 1.00 J (c) 0.67 J (d) 0.34 J

24. A boat crosses a river from port A to port B, which are just on the opposite side. The speed of the water is v_W and that of boat is v_B relative to water. Assume $v_B = 2v_W$. What is the time taken by the boat, if it has to cross the river directly on the AB line?
(a) $\frac{2D}{v_B\sqrt{3}}$ (b) $\frac{\sqrt{3}D}{2v_B}$
(c) $\frac{D}{v_B\sqrt{2}}$ (d) $\frac{D\sqrt{2}}{v_B}$

25. The magnitude of the magnetic field required to accelerate protons (mass = 1.67×10^{-27} kg) in a cyclotron that is operated at an oscillator frequency 12 MHz is approximately
(a) 0.8 T (b) 1.6 T (c) 2.0 T (d) 3.2 T

26. A pure inductor of 25 mH is connected to an ac source of 220 V. Given the frequency of the source as 50 Hz, the rms current in the circuit is
(a) 7 A (b) 14 A (c) 28 A (d) 42 A

27. A plane electromagnetic wave travels in free space along x-direction. If the value of $\vec{B}$ (in tesla) at a particular point in space and time is $1.2 \times 10^{-8}\hat{k}$. The value of $\vec{E}$ (in V m^{-1}) at that point is
(a) $1.2\hat{j}$ (b) $3.6\hat{k}$ (c) $1.2\hat{k}$ (d) $3.6\hat{j}$

28. A concave lens forms the image of an object such that the distance between the object and image is 10 cm and the magnification produced is 1/4. The focal length of the lens will be
(a) – 6.2 cm (b) – 4.4 cm
(c) – 8.6 cm (d) – 10 cm

29. A particle is moving three times as fast as an electron. The ratio of the de Broglie wavelength of the particle to that of the electron is 1.813×10^{-4}. The mass of the particle is
(Mass of electron = 9.1×10^{-31} kg)
(a) 1.67×10^{-27} kg (b) 1.67×10^{-31} kg
(c) 1.67×10^{-30} kg (d) 1.67×10^{-32} kg

30. A source is moving towards stationary observer with some velocity. The frequency of sound heard is 4/3 of its original frequency. Then, the velocity of source is
(Given velocity of sound is 332 m s^{-1})
(a) 50 m s^{-1} (b) 75 m s^{-1}
(c) 90 m s^{-1} (d) 83 m s^{-1}

31. A 6 volt battery is connected to the terminals of a three metre long wire of uniform thickness and resistance of 100 Ω. The difference of potential between two points on the wire separated by a distance of 50 cm will be:
(a) 2 V (b) 3 V (c) 1 V (d) 1.5 V

32. The masses of neutron, proton and deuteron in amu are 1.00893, 1.00813 and 2.01473 respectively. The packing fraction of the deuteron in amu is
(a) 11.65×10^{-4} (b) 23.5×10^{-4}
(c) 33.5×10^{-4} (d) 47.15×10^{-4}

33. A uniform wire 20 metre long and weighing 50 N hangs vertically. If g = 10 m s^{-2}, then the speed of the transverse wave at the middle point of the wire is
(a) 4 m s^{-1} (b) $10\sqrt{2}$ m s^{-1}
(c) 10 m s^{-1} (d) zero m s^{-1}

34. A large horizontal surface moves up and down in SHM with an amplitude of 1 cm. If a mass of 10 kg (which is placed on the surface) is to remain continously is in contact with it, the maximum frequency of SHM will be
(a) 5 Hz (b) 0.5 Hz
(c) 1.5 Hz (d) 10 Hz

35. An open organ pipe has a length of 5 cm. The highest harmonic of such a tube that is in the audible range (20 Hz - 20000 Hz) is (speed of sound in air is 340 m s^{-1})
(a) 4 (b) 5 (c) 6 (d) 7

36. A circular coil of 70 turns and radius 5 cm carrying a current of 8 A is suspended vertically in a uniform horizontal magnetic field of magnitude 1.5 T. The field lines make an angle of 30° with the normal of the coil then the magnitude of the counter torque that must be applied to prevent the coil from turning is
(a) 33 N m
(b) 3.3 N m
(c) 3.3×10^{-2} N m
(d) 3.3×10^{-4} N m

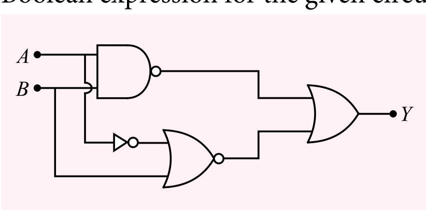
37. Speed of 2 cm radius ball in a viscous liquid is 20 cm s^{-1}. Then the speed of 1 cm radius ball in the same liquid is
(a) 5 cm s^{-1}
(b) 10 cm s^{-1}
(c) 40 cm s^{-1}
(d) 80 cm s^{-1}

38. From the following data, find the magnitude of Joule's mechanical equivalent of heat : C_P for hydrogen = 3.409 cal $g^{-1}\,°C^{-1}$; C_V for hydrogen = 2.409 cal $g^{-1}\,°C^{-1}$ and molecular weight of hydrogen = 2.
(a) $J = 2.11$ J cal^{-1}
(b) $J = 1.11$ J cal^{-1}
(c) $J = 3.11$ J cal^{-1}
(d) $J = 4.11$ J cal^{-1}

39. A particle of mass 10 g is kept on the surface of a uniform sphere of mass 100 kg and radius 10 cm. Find the work to be done against the gravitational force between them, to take the particle far away from the sphere. (Take $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2})
(a) 13.34×10^{-10} J
(b) 3.33×10^{-10} J
(c) 6.67×10^{-9} J
(d) 6.67×10^{-10} J

40. At time $t = 0$, activity of radioactive substance is 1600 Bq, at $t = 8$ s activity falls to 100 Bq. The activity at $t = 2$ s is
(a) 400 Bq
(b) 800 Bq
(c) 200 Bq
(d) 600 Bq

41. The Boolean expression for the given circuit is

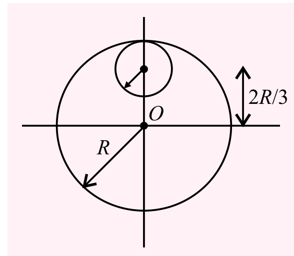


(a) $\overline{A \cdot B}$
(b) $A + B$
(c) $\bar{A} + B$
(d) $A + \bar{B}$

42. 0.1 m^3 of water at 80°C is mixed with 0.3 m^3 of water at 60°C. The final temperature of the mixture is
(a) 65°C (b) 70°C (c) 60°C (d) 75°C

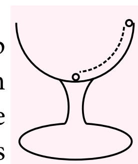
43. A spherical soap bubble of radius 1 cm is formed inside another bubble of radius 3 cm. The radius of a single soap bubble which maintains the same pressure difference as inside the smaller and outside the larger soap bubble is
(a) 0.75 cm
(b) 0.75 m
(c) 7.5 cm
(d) 7.5 m

44. From a circular disc of radius R and mass $9M$, a small disc of radius $R/3$ is removed. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through O is



(a) $4MR^2$
(b) $\frac{40}{9}MR^2$
(c) $10MR^2$
(d) $\frac{37}{9}MR^2$

45. A 2 g ball of glass is released from the edge of a hemispherical cup whose radius is 20 cm. How much work is done on the ball by the gravitational force during the ball's motion to the bottom of the cup?



(a) 1.96 mJ
(b) 3.92 mJ
(c) 4.90 mJ
(d) 5.88 mJ

SOLUTIONS

1. (c) : $\omega = \frac{2\pi}{T} = \frac{2\pi}{365 \times 24 \times 3600}$

$$= \frac{2\pi}{3.15 \times 10^7} = 1.99 \times 10^{-7} \text{ rad s}^{-1}$$

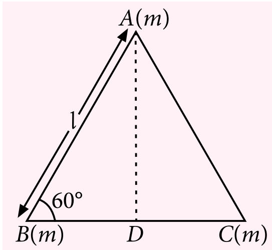
$$W = K_F - K_I = 0 - \frac{1}{2}mv^2$$

$$= -\frac{1}{2} \times 6 \times 10^{24} \times (1.5 \times 10^{11} \times 1.99 \times 10^{-7})^2$$

$(\because\ v = r\omega)$

$$= -2.7 \times 10^{33} \text{ J}$$

2. (c) : Moment of inertia of each of the point masses (m) at B and C about the side $BC = m(0)^2 + m(0)^2 = 0$. Moment of inertia of point mass m at A about the side $BC = m(AD)^2$.



Now $AD = l \sin 60° = l\dfrac{\sqrt{3}}{2}$

∴ Moment of inertia of the system about side BC

$$= 0 + 0 + m\left(\frac{l\sqrt{3}}{2}\right)^2 = \frac{3ml^2}{4}$$

3. (a) : Suppose h be the height of each storey, then

$$25h = 0 + \frac{1}{2}\times 10 \times t^2 = \frac{1}{2}\times 10\times 5^2$$

∴ $h = 5$ m

In first second, let the stone passes through n storeys. So,

$$n \times 5 = \frac{1}{2}\times 10 \times (1)^2 \quad \text{or} \quad n = 1$$

4. (d): Here $y = \dfrac{a^4 b^2}{(cd^4)^{1/3}}$

Taking log on both sides, we get

$$\log y = 4\log a + 2\log b - \frac{1}{3}\log c - \frac{4}{3}\log d$$

Differentiating both sides to calculate error,

$$\frac{\Delta y}{y}\times 100 = 4\left(\frac{\Delta a}{a}\times 100\right) + 2\left(\frac{\Delta b}{b}\times 100\right) + \frac{1}{3}\left(\frac{\Delta c}{c}\times 100\right) + \frac{4}{3}\left(\frac{\Delta d}{d}\times 100\right)$$

$= [4 \times 2\% + 2 \times 3\% + \dfrac{1}{3} \times 4\% + \dfrac{4}{3} \times 5\%]$

$= 22\%$

5. (a) : As $U_1 = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q(-q)}{r}$; $U_2 = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q(-q)}{(r/2)}$

$$U_1 - U_2 = \frac{Qq}{4\pi\varepsilon_0 r}[-1+2] = \frac{Qq}{4\pi\varepsilon_0 r} = 9 \text{ J}$$

$$U_3 = \frac{Q(-q)}{4\pi\varepsilon_0(3r/4)} = \frac{-Qq}{4\pi\varepsilon_0 r}\times\frac{4}{3}$$

$$\therefore \quad U_1 - U_3 = \frac{Qq}{4\pi\varepsilon_0 r}\times\frac{1}{3} = 9\times\frac{1}{3} = 3 \text{ J}$$

6. (d): $E_1 = \dfrac{1}{2}C_1V^2 = \dfrac{1}{2}\times 2\times 10^{-6}\times 100^2 = 0.01$ J

$E_2 = \dfrac{1}{2}C_2V^2 = \dfrac{1}{2}\times 10\times 10^{-6}\times (100)^2 = 0.05$ J

Energy change $= E_2 - E_1$

$= 0.05 - 0.01 = 0.04 \text{ J} = 4\times 10^{-2}$ J

7. (c) : Here, $\Delta l = x$; $Y = \dfrac{F/A}{\Delta l/L}$ or $F = \dfrac{YA\,\Delta l}{L}$

The work is done from 0 to x (change in length), so the average distance $\dfrac{0+\Delta l}{2} = \dfrac{\Delta l}{2}$

Work done = Force × distance

$$= \frac{YA\,\Delta l}{L}\times\frac{\Delta l}{2} = \frac{YA(\Delta l)^2}{2L} = \frac{YAx^2}{2L}$$

8. (a) : Downward retardation means upward acceleration.

$g' = g + a$

Now $t = \sqrt{\dfrac{2L}{g'\sin\theta}}$ or $t = \sqrt{\dfrac{2L}{(g+a)\sin\theta}}$

9. (d): As $B = \dfrac{\mu_0}{4\pi}\dfrac{2\pi I}{r} = \dfrac{\mu_0 I}{2r}$ or $I = \dfrac{2Br}{\mu_0}$;

Also, $A = \pi r^2$ or $r = \left(\dfrac{A}{\pi}\right)^{1/2}$

Magnetic moment, $M = IA = \dfrac{2Br}{\mu_0}A$

$$= \frac{2BA}{\mu_0}\times\left(\frac{A}{\pi}\right)^{1/2} = \frac{2BA^{3/2}}{\mu_0\pi^{1/2}}$$

10. (c) : Slope of the given graph $m = \dfrac{\Delta V}{\Delta I} = R$

On doubling the length of wire its resistance is doubled and slope of V-I graph is doubled.

11. (d) : For first car : $u = u$, $a = -a$, $v = 0$

As $v^2 - u^2 = 2as$

∴ $v^2 - u^2 = 2(-a)s_1$ or $s_1 = \dfrac{u^2}{2a}$

For second car : $u = 4u$, $a = -a$, $v = 0$

∴ $s_2 = \dfrac{(4u)^2}{2a} = \dfrac{8u^2}{a}$

Hence, $\dfrac{s_1}{s_2} = \dfrac{u^2}{2a}\cdot\dfrac{a}{8u^2} = \dfrac{1}{16} = 1:16.$

12. (b): $[X] = [C] = [M^{-1}L^{-2}T^2Q^2]$

$[Z] = [B] = [MT^{-1}Q^{-1}]$

$\therefore \quad [Y] = \frac{[X]}{[Z^2]} = \frac{[M^{-1}L^{-2}T^2Q^2]}{[MT^{-1}Q^{-1}]^2}$

$= [M^{-3}L^{-2}T^4Q^4]$

13. (a) : According to Newton's law of cooling,

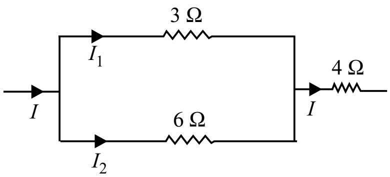
$$\frac{80-64}{5} = k\left[\frac{80+64}{2} - T\right] \quad ...(i)$$

and $$\frac{80-52}{10} = k\left[\frac{80+52}{2} - T\right] \quad ...(ii)$$

where T is the temperature of the surrounding. Solving Eqs. (i) and (ii), we get

$T = 24° \text{ C}$

14. (c) :



Here, $I_1 = 0.8$ A

As the resistances 3 Ω and 6 Ω are connected in parallel. Therefore potential drop across these resistances is same.

$\therefore \quad 3I_1 = 6I_2 \quad$ or $\quad (0.8) \times 3 = I_2 \times 6$

or $\quad I_2 = \frac{2.4}{6} = 0.4$ A

The current flowing through 4 Ω resistor is

$I = I_1 + I_2 = 0.8 \text{ A} + 0.4 \text{ A} = 1.2 \text{ A}$

Potential drop across 4 Ω resistance $= I \times 4$

$= (1.2) \times 4 = 4.8$ V

15. (a) : $\mu = \frac{\sin i}{\sin r}$ and $i + r = 90°$

or $\quad r = 90° - i$

$\mu = \frac{\sin i}{\sin(90 - i)} = \tan i$

or $\quad i = \tan^{-1}(\mu) = \tan^{-1}(1.62)$

16. (c) : As $\frac{\Delta L}{L} = 0.10\% = 0.001$ and $\Delta T = 100°$C,

hence using $\frac{\Delta L}{L} = \alpha\, \Delta T$,

we get $\alpha = \frac{0.001}{100} = 10^{-5}/°C$

$\therefore \quad \gamma = 3\alpha = 3 \times 10^{-5}/°C$

and $\frac{\Delta V}{V} = \gamma \Delta T = 3 \times 10^{-5} \times 100 = 3 \times 10^{-3} = 0.30\%$

17. (c) : Change in PE, $\Delta U = U_2 - U_1$

$$= -\frac{GMm}{(R+nR)} + \frac{GMm}{R} = -\frac{GMm}{R(n+1)} + \frac{GMm}{R}$$

$$= \frac{(R^2 g)m}{R} \times \frac{n}{(n+1)} = mgR\left(\frac{n}{n+1}\right)$$

18. (a) : Since there is no external force acting on the particle, hence

$$y_{CM} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = 0,$$

$$\therefore \quad \left(\frac{m}{4}\right) \times (+15) + \left(\frac{3m}{4}\right)(y_2) = 0$$

$\Rightarrow \quad y_2 = -5$ cm

19. (b): Heat required to melt 1 g of ice at 0°C to water at 0°C = 1 × 80 cal.

Heat required to raise temperature of 1 g of water from 0°C to 100°C = 1 × 1 × 100 = 100 cal

Total heat required for maximum temperature of 100°C = 80 + 100 = 180 cal

As one gram of steam gives 540 cal of heat when it is converted to water at 100°C, therefore, temperature of the mixture would be 100°C.

20. (c) : Since, the magnetic field, due to current through wire CD at various locations on wire AB is not uniform, therefore, the wire AB, carrying current I_1 is subjected to variable magnetic field. Due to which, neither the force nor the torque on the wire AB will be zero. As a result of which the wire AB will have both translational and rotational motion.

21. (a) : Electric field, $E = \frac{V}{d}$

$V = Ed = 3 \times 10^6 \times (10^{-4}) = 300$ V

$V_{rms} = \frac{V_0}{\sqrt{2}} = 300\,(0.707) = 212\ V$

22. (a) : In uniform circular motion, centripetal force acts towards the centre. Torque due to such a force about the centre is zero. Hence angular momentum is conserved about the centre of the circle.

23. (c) : Momentum after collision

= Momentum before collision

$(m_1 + m_2)v = m_1 u_1 + m_2 u_2$

$(0.5 + 1.0)v = 0.5 \times 2.0 + 1.0 \times 0$

$$v = \frac{1}{1.5} = \frac{2}{3} \text{ m s}^{-1}$$

Loss of energy,

$$= \frac{1}{2}m_1u_1^2 - \frac{1}{2}(m_1 + m_2)v^2$$

$$= \frac{1}{2}\times 0.5\times 2^2 - \frac{1}{2}(0.50 + 1.0)\left(\frac{2}{3}\right)^2$$

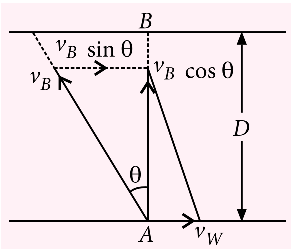
$$= 1 - \frac{1}{3} = 0.67 \text{ J.}$$

24. ( ) : To move straight along AB,

$v \sin\theta = v_W$

$$\sin\theta = \frac{v_W}{v_B} = \frac{v_W}{2v_W} = \frac{1}{2}$$

$\therefore \quad \theta = 30°$



Time taken to cross the river,

$$t = \frac{D}{v_B\cos\theta} = \frac{D}{v_B\cos 30°} = \frac{2D}{v_B\sqrt{3}}.$$

25. (a) : The oscillator frequency should be same as proton's cyclotron frequency.

Cyclotron frequency, $\upsilon_c = \dfrac{qB}{2\pi m}$

or $\quad B = \dfrac{2\pi m\upsilon_c}{q}$

$$= \frac{2\times 3.14\times 1.67\times 10^{-27}\times 12\times 10^6}{1.6\times 10^{-19}}$$

$= 78.6 \times 10^{-2}$ T $\approx$ 0.8 T

26. (c) : Here, L = 25 mH = 25 × 10^{-3} H

υ = 50 Hz, V_{rms} = 220 V

The inductive reactance is

$$X_L = 2\pi\upsilon L = 2\times\frac{22}{7}\times 50\times 25\times 10^{-3}\ \Omega$$

The rms current in the circuit is

$$I_{rms} = \frac{V_{rms}}{X_L} = \frac{220}{2\times\frac{22}{7}\times 50\times 25\times 10^{-3}}$$

$$= \frac{7\times 1000}{2\times 5\times 25}\text{ A} = 28 \text{ A}$$

27. (d): Here, $\vec{B} = 1.2\times 10^{-8}\hat{k}$ T

The magnitude of $\vec{E}$ is

$E = Bc = (1.2 \times 10^{-8}$ T$)(3 \times 10^8$ m s$^{-1})$

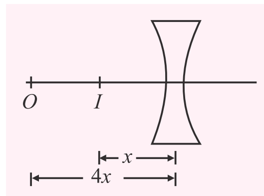
$= 3.6$ V m^{-1}

$\vec{B}$ is along Z-direction and the wave propagates along X-direction. Therefore $\vec{E}$ should be in a direction perpendicular to both X and Z axes. Using vector algebra $\vec{E}\times\vec{B}$ should be along X-direction.

Since $(+\hat{j})\times(+\hat{k}) = \hat{i}$, $\vec{E}$ is along the Y-direction.

Thus, $\vec{E} = 3.6\hat{j}$ Vm^{-1}.

28. (b): A concave lens forms virtual image I of a point object O.



As $\quad m = \dfrac{1}{4} = \dfrac{v}{u}$

$\therefore \quad u = 4v$

If $\quad v = -x$, $u = -4x$

Distance between object and image

$|OI| = 4x - x = 3x = 10$ cm,

or $\quad x = \dfrac{10}{3}$ cm

$\therefore \quad u = -\dfrac{40}{3}$ cm and $v = \dfrac{-10}{3}$ cm

Using lens formula $\dfrac{1}{f} = \dfrac{1}{v} - \dfrac{1}{u}$

$$\frac{1}{f} = -\frac{3}{10} + \frac{3}{40} = -\frac{9}{40}$$

$$f = -\frac{40}{9}\text{ cm} = -4.4\text{ cm}$$

29. (a) : de Broglie wavelength of a moving particle, having mass m and velocity v is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv} \text{ or } m = \frac{h}{\lambda v}$$

Given : $\dfrac{v}{v_e} = 3$, $\dfrac{\lambda}{\lambda_e} = 1.813\times 10^{-4}$

Mass of the particle, $m = m_e\left(\dfrac{v_e}{v}\right)\left(\dfrac{\lambda_e}{\lambda}\right)$

Substituting the values, we get

$$m = (9.1\times 10^{-31}\text{ kg})\times\left(\frac{1}{3}\right)\times\left(\frac{1}{1.813\times 10^{-4}}\right)$$

$m = 1.67 \times 10^{-27}$ kg

30. (d): According to Doppler's effect,

$$\upsilon' = \upsilon\left(\frac{v}{v - v_s}\right)$$

$$\frac{4}{3}\upsilon = \upsilon\left(\frac{v}{v - v_s}\right)$$

or $4v - 4v_s = 3v$

or $v = 4v_s$

or $v_s = \frac{v}{4} = \frac{332}{4} = 83 \text{ m s}^{-1}$

31. (c) : $E = 6$ volt, $R = 100$ W

$$I = \frac{E}{R} = \frac{6}{100} \text{ A}$$

L = Length of wire = 3 m = 300 cm

Fall of potential/length $= \frac{6 \text{ volt}}{300 \text{ cm}}$

Hence, potential difference between two points on the wire separated by a distance of 50 cm

$$= \frac{6 \text{ volt}}{300 \text{ cm}} \times 50 \text{ cm} = 1 \text{ volt}$$

32. (a) : Deuteron nucleus consists of 1 proton and 1 neutron *i.e.*, $A = 2$; $Z = 1$

Mass of neutron = 1.00893 amu

Mass of proton = 1.00813 amu

Mass of deuteron = 2.01473

Mass defect,

$\Delta m = (1.00893 + 1.00813) - 2.01473$

$= 0.00233$ amu

Packing fraction $= \frac{\Delta m}{A} = \frac{0.00233}{2}$

$= 11.65 \times 10^{-4}$ amu

33. (c) : From the question, $v = \sqrt{\frac{T}{m}}$.

Weight of wire = 50 N

Mass of wire = (50/10) = 5 kg

Length = 20 m

$\therefore$ $m = (5/20) = 0.25$ kg/m

At the middle point $T = 25$ N

$$\therefore \quad v = \sqrt{\frac{T}{m}} = \sqrt{\frac{25}{0.25}} = \sqrt{100} = 10 \text{ m s}^{-1}$$

34. (a) : Here, a = 1 cm = 0.01 m; The mass will remain in contact with surface, if

$$mg = m\omega^2 a \text{ or } \omega = \sqrt{g/a}$$

or $2\pi\upsilon = \sqrt{g/a}$ or $\upsilon = \frac{1}{2\pi}\sqrt{\frac{g}{a}}$

$$= \frac{7}{2 \times 22}\sqrt{\frac{980}{1}} = 4.9 \text{ Hz} \approx 5 \text{ Hz}$$

35. (b) : For an open pipe,

$$\upsilon = \frac{n \cdot c}{2L} = \frac{n \cdot (340 \text{ m s}^{-1})}{2 \times (5 \times 10^{-2} \text{ m})} = n \cdot (3400 \text{ Hz})$$

$3400 \cdot n < 20000$; $\therefore n < \left(\frac{20000}{3400}\right)$; $n < 5.9$

$\Rightarrow$ The greater integral value of $n = 5$.

73. (b) : $N = 70$

$r = 5$ cm $= 5 \times 10^{-2}$ m, $I = 8$ A

$B = 1.5$ T, $\theta = 30°$

The counter torque to prevent the coil from turning will be equal and opposite to the torque acting on the coil,

$\therefore \tau = NIAB \sin\theta = NI\pi r^2 B \sin 30°$

$$= 70 \times 8 \times 3.14 \times (5 \times 10^{-2})^2 \times 1.5 \times \frac{1}{2} = 3.297 \text{ N m}$$

≈ 3.3 N m

37. (a) : Terminal velocity, $v = \frac{2r^2(\rho - \rho_0)g}{9\eta}$

i.e, $v \propto r^2$.

$$\therefore \quad \frac{v_1}{v} = \frac{r_1^2}{r^2}$$

or $v_1 = v\left(\frac{r_1}{r}\right)^2 = 20\left(\frac{1}{2}\right)^2 = 5 \text{ cm s}^{-1}$

38. (d) : Here, J = ?

$C_p = 3.409$ cal g^{-1}°C^{-1}

$C_V = 2.409$ cal g^{-1}°C^{-1}, $M = 2$

$R = 8.31$ J mol^{-1} K^{-1}

As $C_p - C_V = \frac{r}{J} = \frac{R}{MJ}$

$$\therefore \quad 3.409 - 2.409 = \frac{8.31}{2J}; \; 1 = \frac{4.155}{J}$$

or $J = 4.11$ J cal^{-1}

39. (d) : $U_i = -\frac{GMm}{r}$

$$U_i = -\frac{6.67 \times 10^{-11} \times 100 \times 10^{-2}}{0.1}$$

We know

$\therefore \quad W = \Delta U = U_f - U_i$

$\therefore \quad W = -U_i = 6.67 \times 10^{-10}$ J ($\because U_f = 0$)

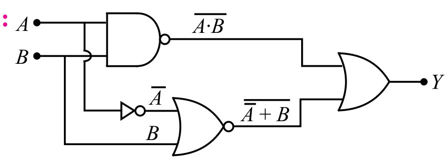
40. (b): $R = R_0\left(\frac{1}{2}\right)^n$

where n is the number of half-lives.

Given : $R = \frac{R_0}{16} \quad \therefore \quad \frac{R_0}{16} = R_0\left(\frac{1}{2}\right)^n$ or $n = 4$

Four half-lives are equivalent to 8 s. Hence, 2 s is equal to one half-life. So, in one half-life activity will fall half of 1600 Bq, *i.e.*, 800 Bq.

41. (a) :



$Y = \overline{A \cdot B} + \overline{\overline{A} + B}$

$= \overline{A} + \overline{B} + \overline{\overline{A}} \cdot \overline{B}$ (By De Morgan's theorem)

$= \overline{A} + \overline{B} + A \cdot \overline{B}$ $(\therefore \overline{\overline{A}} = A)$

$= \overline{A} + \overline{B} + \overline{B} \cdot A$ (Using $A \cdot \overline{B} = \overline{B} \cdot A$)

$= \overline{A} + \overline{B}$ (Using $1 + A = 1$)

$= \overline{A \cdot B}$

42. (a) : Density of water = 10^3 kg m^{-3}

Let the final temperature of the mixture be T.

Assuming no heat transfer to or from container.

Heat lost by water at 80°C

$= 0.1 \times 10^3 \times s_{water} \times (80 - T)$

Heat gained by water at 60°C

$= 0.3 \times 10^3 \times s_{water} \times (T - 60)$

According to principle of calorimetry

heat lost = heat gain

$0.1 \times 10^3 \times s_{water} \times (80 - T)$

$= 0.3 \times 10^3 \times s_{water} \times (T - 60)$

$1 \times (80 - T) = 3 \times (T - 60)$

$4T = 260;\ T = \frac{260}{4}°\text{C} = 65°\text{C}$

43. (a) : Pressure outside the bigger drop = P_1

Pressure inside the bigger drop = P_2

Radius of bigger drop, r_1 = 3 cm

Excess pressure $= P_2 - P_1 = \frac{4S}{r_1} = \frac{4S}{3}$

Pressure inside small drop = P_3

Excess pressure $= P_3 - P_2 = \frac{4S}{r_2} = \frac{4S}{1}$

Pressure difference between inner side of small drop and outer side of bigger drop

$$= P_3 - P_1 = \frac{4S}{3} + \frac{4S}{1} = \frac{16S}{3}$$

This pressure difference should exist in a single drop of radius r.

$\therefore \quad \frac{4S}{r} = \frac{16S}{3}$ or $r = \frac{3}{4}$ cm = 0.75 cm

44. (a) : Mass per unit area of disc $= \frac{9\,M}{\pi R^2}$

Mass of removed portion of disc

$$= \frac{9\,M}{\pi R^2} \times \pi\left(\frac{R}{3}\right)^2 = M$$

Moment of inertia of removed portion about an axis passing through centre of disc and perpendicular to the plane of disc, using theorem of parallel axis is

$$I_1 = \frac{M}{2}\left(\frac{R}{3}\right)^2 + M\left(\frac{2\,R}{3}\right)^2 = \frac{1}{2}MR^2$$

when portion of disc would not have been removed, then the moment of inertia of complete disc about the given axis is

$$I_2 = \frac{9}{2}MR^2$$

So moment of inertia of the disc with removed portion, about the given axis is

$$I = I_2 - I_1 = \frac{9}{2}MR^2 - \frac{1}{2}MR^2 = 4\,MR^2$$

45. (b): According to the conservation of energy, kinetic energy at A + potential energy at A = kinetic energy at B + potential energy at B

$$0 + mgh = \frac{1}{2}mv^2 + 0$$

or $v^2 = 2gh = 2 \times 9.8 \times 0.20$

[$\because$ h = radius = 20 cm = 0.2 m]

36. A circular coil of 70 turns and radius 5 cm carrying a current of 8 A is suspended vertically in a uniform horizontal magnetic field of magnitude 1.5 T. The field lines make an angle of 30° with the normal of the coil then the magnitude of the counter torque that must be applied to prevent the coil from turning is
(a) 33 N m (b) 3.3 N m
(c) 3.3×10^{-2} N m (d) 3.3×10^{-4} N m

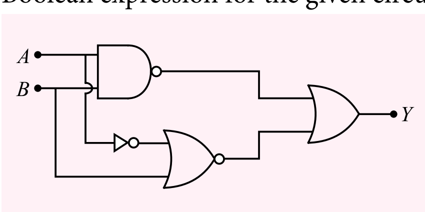
37. Speed of 2 cm radius ball in a viscous liquid is 20 cm s^{-1}. Then the speed of 1 cm radius ball in the same liquid is
(a) 5 cm s^{-1} (b) 10 cm s^{-1}
(c) 40 cm s^{-1} (d) 80 cm s^{-1}

38. From the following data, find the magnitude of Joule's mechanical equivalent of heat : C_P for hydrogen = 3.409 cal $g^{-1}\,°C^{-1}$; C_V for hydrogen = 2.409 cal $g^{-1}\,°C^{-1}$ and molecular weight of hydrogen = 2.
(a) $J = 2.11$ J cal^{-1} (b) $J = 1.11$ J cal^{-1}
(c) $J = 3.11$ J cal^{-1} (d) $J = 4.11$ J cal^{-1}

39. A particle of mass 10 g is kept on the surface of a uniform sphere of mass 100 kg and radius 10 cm. Find the work to be done against the gravitational force between them, to take the particle far away from the sphere. (Take $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2})
(a) 13.34×10^{-10} J (b) 3.33×10^{-10} J
(c) 6.67×10^{-9} J (d) 6.67×10^{-10} J

40. At time $t = 0$, activity of radioactive substance is 1600 Bq, at $t = 8$ s activity falls to 100 Bq. The activity at $t = 2$ s is
(a) 400 Bq (b) 800 Bq
(c) 200 Bq (d) 600 Bq

41. The Boolean expression for the given circuit is

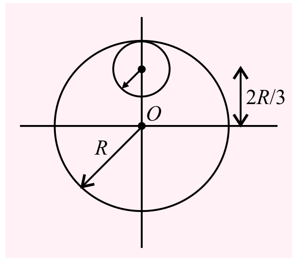


(a) $\overline{A \cdot B}$ (b) $A + B$
(c) $\bar{A} + B$ (d) $A + \bar{B}$

42. 0.1 m^3 of water at 80°C is mixed with 0.3 m^3 of water at 60°C. The final temperature of the mixture is
(a) 65°C (b) 70°C (c) 60°C (d) 75°C

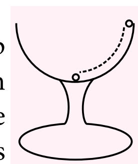
43. A spherical soap bubble of radius 1 cm is formed inside another bubble of radius 3 cm. The radius of a single soap bubble which maintains the same pressure difference as inside the smaller and outside the larger soap bubble is
(a) 0.75 cm (b) 0.75 m
(c) 7.5 cm (d) 7.5 m

44. From a circular disc of radius R and mass $9M$, a small disc of radius $R/3$ is removed. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through O is



(a) $4MR^2$ (b) $\frac{40}{9}MR^2$
(c) $10MR^2$ (d) $\frac{37}{9}MR^2$

45. A 2 g ball of glass is released from the edge of a hemispherical cup whose radius is 20 cm. How much work is done on the ball by the gravitational force during the ball's motion to the bottom of the cup?



(a) 1.96 mJ (b) 3.92 mJ
(c) 4.90 mJ (d) 5.88 mJ

SOLUTIONS

1. (c) : $\omega = \frac{2\pi}{T} = \frac{2\pi}{365 \times 24 \times 3600}$

$$= \frac{2\pi}{3.15 \times 10^7} = 1.99 \times 10^{-7} \text{ rad s}^{-1}$$

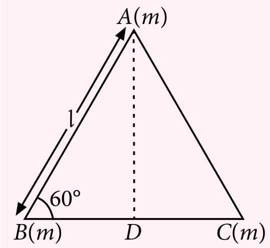
$$W = K_F - K_I = 0 - \frac{1}{2}mv^2$$

$$= -\frac{1}{2} \times 6 \times 10^{24} \times (1.5 \times 10^{11} \times 1.99 \times 10^{-7})^2$$

$(\because v = r\omega)$

$$= -2.7 \times 10^{33} \text{ J}$$

2. **(c) :** Moment of inertia of each of the point masses (m) at B and C about the side $BC = m(0)^2 + m(0)^2 = 0.$ Moment of inertia of point mass m at A about the side $BC = m(AD)^2$.



Now $AD = l \sin 60° = l\frac{\sqrt{3}}{2}$

$\therefore$ Moment of inertia of the system about side BC

$$= 0 + 0 + m\left(\frac{l\sqrt{3}}{2}\right)^2 = \frac{3ml^2}{4}$$

3. **(a) :** Suppose h be the height of each storey, then

$$25h = 0 + \frac{1}{2}\times 10 \times t^2 = \frac{1}{2}\times 10\times 5^2$$

$\therefore$ $h = 5$ m

In first second, let the stone passes through n storeys. So,

$$n\times 5 = \frac{1}{2}\times 10\times (1)^2 \quad \text{or} \quad n = 1$$

4. **(d):** Here $y = \frac{a^4b^2}{(cd^4)^{1/3}}$

Taking log on both sides, we get

$$\log\ y = 4\ \log\ a + 2\ \log b - \frac{1}{3}\log c - \frac{4}{3}\log\ d$$

Differentiating both sides to calculate error,

$$\frac{\Delta y}{y}\times 100 = 4\left(\frac{\Delta a}{a}\times 100\right) + 2\left(\frac{\Delta b}{b}\times 100\right) + \frac{1}{3}\left(\frac{\Delta c}{c}\times 100\right) + \frac{4}{3}\left(\frac{\Delta d}{d}\times 100\right)$$

$= [4 \times 2\% + 2 \times 3\% + \frac{1}{3} \times 4\% + \frac{4}{3} \times 5\%]$

$= 22\%$

5. **(a) :** As $U_1 = \frac{1}{4\pi\varepsilon_0}\frac{Q(-q)}{r}$; $U_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q(-q)}{(r/2)}$

$$U_1 - U_2 = \frac{Qq}{4\pi\varepsilon_0 r}[-1+2] = \frac{Qq}{4\pi\varepsilon_0 r} = 9\text{ J}$$

$$U_3 = \frac{Q(-q)}{4\pi\varepsilon_0(3r/4)} = \frac{-Qq}{4\pi\varepsilon_0 r}\times\frac{4}{3}$$

$$\therefore \quad U_1 - U_3 = \frac{Qq}{4\pi\varepsilon_0 r}\times\frac{1}{3} = 9\times\frac{1}{3} = 3\text{ J}$$

6. **(d):** $E_1 = \frac{1}{2}C_1V^2 = \frac{1}{2}\times 2\times 10^{-6}\times 100^2 = 0.01$ J

$E_2 = \frac{1}{2}C_2V^2 = \frac{1}{2}\times 10\times 10^{-6}\times (100)^2 = 0.05$ J

Energy change $= E_2 - E_1$

$= 0.05 - 0.01 = 0.04\text{ J} = 4\times 10^{-2}$ J

7. **(c) :** Here, $\Delta l = x$; $Y = \frac{F/A}{\Delta l/L}$ or $F = \frac{YA\ \Delta l}{L}$

The work is done from 0 to x (change in length), so the average distance $\frac{0+\Delta l}{2} = \frac{\Delta l}{2}$

Work done = Force × distance

$$= \frac{YA\,\Delta l}{L}\times\frac{\Delta l}{2} = \frac{YA\,(\Delta l)^2}{2L} = \frac{YAx^2}{2L}$$

8. **(a) :** Downward retardation means upward acceleration.

$g' = g + a$

Now $t = \sqrt{\frac{2L}{g'\sin\theta}}$ or $t = \sqrt{\frac{2L}{(g+a)\sin\theta}}$

9. **(d):** As, $B = \frac{\mu_0}{4\pi}\frac{2\pi I}{r} = \frac{\mu_0 I}{2r}$ or $I = \frac{2Br}{\mu_0}$;

Also, $A = \pi r^2$ or $r = \left(\frac{A}{\pi}\right)^{1/2}$

Magnetic moment, $M = IA = \frac{2Br}{\mu_0}A$

$$= \frac{2BA}{\mu_0}\times\left(\frac{A}{\pi}\right)^{1/2} = \frac{2BA^{3/2}}{\mu_0\pi^{1/2}}$$

10. **(c) :** Slope of the given graph $m = \frac{\Delta V}{\Delta I} = R$

On doubling the length of wire its resistance is doubled and slope of V-I graph is doubled.

11. **(d) :** For first car, $u = u$, $a = -a$, $v = 0$

As $v^2 - u^2 = 2as$

$\therefore$ $v^2 - u^2 = 2(-a)s_1$ or $s_1 = \frac{u^2}{2a}$

For second car, $u = 4u$, $a = -a$, $v = 0$

$$\therefore \quad s_2 = \frac{(4u)^2}{2a} = \frac{8u^2}{a}$$

Hence, $\frac{s_1}{s_2} = \frac{u^2}{2a}\cdot\frac{a}{8u^2} = \frac{1}{16} = 1:16.$

12. **(b):** $[X] = [C] = [M^{-1}L^{-2}T^2Q^2]$

$[Z] = [B] = [MT^{-1}Q^{-1}]$

$\therefore \quad [Y]=\frac{[X]}{[Z^2]}=\frac{[M^{-1}L^{-2}T^2Q^2]}{[MT^{-1}Q^{-1}]^2}$

$= [M^{-3}L^{-2}T^4Q^4]$

13. (a) : According to Newton's law of cooling,

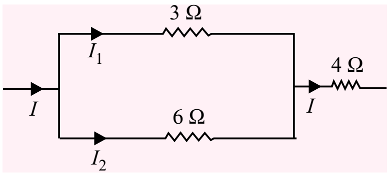
$$\frac{80-64}{5}=k\left[\frac{80+64}{2}-T\right] \quad \text{...(i)}$$

and $$\frac{80-52}{10}=k\left[\frac{80+52}{2}-T\right] \quad \text{...(ii)}$$

where T is the temperature of the surrounding. Solving Eqs. (i) and (ii), we get

$T = 24° \text{ C}$

14. (c) :



Here, $I_1 = 0.8$ A

As the resistances 3 Ω and 6 Ω are connected in parallel. Therefore potential drop across these resistances is same.

$\therefore \quad 3I_1 = 6I_2 \quad$ or $\quad (0.8) \times 3 = I_2 \times 6$

or $\quad I_2=\frac{2.4}{6}=0.4$ A

The current flowing through 4 Ω resistor is

$I = I_1 + I_2 = 0.8 \text{ A} + 0.4 \text{ A} = 1.2 \text{ A}$

Potential drop across 4 Ω resistance = $I \times 4$

$= (1.2) \times 4 = 4.8$ V

15. (a) : $\mu=\frac{\sin i}{\sin r}$ and $i+r=90°$

or $\quad r = 90° - i$

$\mu=\frac{\sin i}{\sin(90-i)}=\tan i$

or $\quad i = \tan^{-1}(\mu) = \tan^{-1}(1.62)$

16. (c) : As $\frac{\Delta L}{L}$ = 0.10% = 0.001 and ΔT =100°C,

hence using $\frac{\Delta L}{L}=\alpha\,\Delta T$,

we get $\alpha=\frac{0.001}{100}=10^{-5}/°C$

$\therefore \quad \gamma = 3\alpha = 3 \times 10^{-5}/°C$

and $\frac{\Delta V}{V}=\gamma\Delta T=3\times10^{-5}\times100=3\times10^{-3}=0.30\%$

17. (c) : Change in PE, $\Delta U = U_2 - U_1$

$$=-\frac{GMm}{(R+nR)}+\frac{GMm}{R}=-\frac{GMm}{R(n+1)}+\frac{GMm}{R}$$

$$=\frac{(R^2g)m}{R}\times\frac{n}{(n+1)}=mgR\left(\frac{n}{n+1}\right)$$

18. (a) : Since there is no external force acting on the particle, hence

$$y_{CM}=\frac{m_1y_1+m_2y_2}{m_1+m_2}=0,$$

$$\therefore \quad \left(\frac{m}{4}\right)\times(+15)+\left(\frac{3m}{4}\right)(y_2)=0$$

$\Rightarrow \quad y_2 = -5$ cm

19. (b): Heat required to melt 1 g of ice at 0°C to water at 0°C = 1 × 80 cal.

Heat required to raise temperature of 1 g of water from 0°C to 100°C = 1 × 1 × 100 = 100 cal

Total heat required for maximum temperature of 100°C = 80 + 100 = 180 cal

As one gram of steam gives 540 cal of heat when it is converted to water at 100°C, therefore, temperature of the mixture would be 100°C.

20. (c) : Since, the magnetic field, due to current through wire CD at various locations on wire AB is not uniform, therefore, the wire AB, carrying current I_1 is subjected to variable magnetic field. Due to which, neither the force nor the torque on the wire AB will be zero. As a result of which the wire AB will have both translational and rotational motion.

21. (a) : Electric field, $E=\frac{V}{d}$

$V = Ed = 3 \times 10^6 \times (10^{-4}) = 300$ V

$$V_{rms}=\frac{V_0}{\sqrt{2}}=300\ (0.707)=212 \text{ V}$$

22. (a) : In uniform circular motion, centripetal force acts towards the centre. Torque due to such a force about the centre is zero. Hence angular momentum is conserved about the centre of the circle.

23. (c) : Momentum after collision

= Momentum before collision

$(m_1 + m_2)v = m_1u_1 + m_2u_2$

$(0.5 + 1.0)v = 0.5 \times 2.0 + 1.0 \times 0$

$$v = \frac{1}{1.5} = \frac{2}{3} \text{ m s}^{-1}$$

Loss of energy,

$$= \frac{1}{2}m_1u_1^2 - \frac{1}{2}(m_1 + m_2)v^2$$

$$= \frac{1}{2}\times 0.5 \times 2^2 - \frac{1}{2}(0.50 + 1.0)\left(\frac{2}{3}\right)^2$$

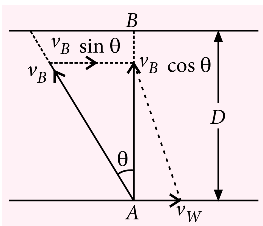
$$= 1 - \frac{1}{3} = 0.67 \text{ J.}$$

24. (a) : To move straight along AB,

$v_B \sin\theta = v_W$

$$\sin\theta = \frac{v_W}{v_B} = \frac{v_W}{2v_W} = \frac{1}{2}$$

$\therefore \quad \theta = 30°$



Time taken to cross the river,

$$t = \frac{D}{v_B\cos\theta} = \frac{D}{v_B\cos 30°} = \frac{2D}{v_B\sqrt{3}}.$$

25. (a) : The oscillator frequency should be same as proton's cyclotron frequency.

Cyclotron frequency, $\upsilon_c = \dfrac{qB}{2\pi m}$

or $\quad B = \dfrac{2\pi m\upsilon_c}{q} = \dfrac{2\times 3.14\times 1.67\times 10^{-27}\times 12\times 10^6}{1.6\times 10^{-19}}$

$= 78.6 \times 10^{-2}$ T ≈ 0.8 T

26. (c) : Here, $L = 25$ mH $= 25 \times 10^{-3}$ H

$\upsilon = 50$ Hz, $V_{rms} = 220$ V

The inductive reactance is

$$X_L = 2\pi\upsilon L = 2\times\frac{22}{7}\times 50\times 25\times 10^{-3}\ \Omega$$

The rms current in the circuit is

$$I_{rms} = \frac{V_{rms}}{X_L} = \frac{220}{2\times\frac{22}{7}\times 50\times 25\times 10^{-3}}$$

$$= \frac{7\times 1000}{2\times 5\times 25}\text{ A} = 28 \text{ A}$$

27. (d): Here, $\vec{B} = 1.2\times 10^{-8}\,\hat{k}$ T

The magnitude of $\vec{E}$ is

$E = Bc = (1.2 \times 10^{-8}$ T$)(3 \times 10^8$ m s$^{-1})$

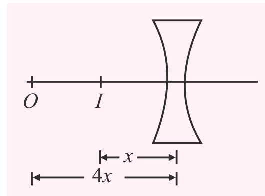
$= 3.6$ V m^{-1}

$\vec{B}$ is along z-direction and the wave propagates along x-direction. Therefore $\vec{E}$ should be in a direction perpendicular to both x and z axes. Using vector algebra $\vec{E}\times\vec{B}$ should be along x-direction.

Since $(+\hat{j})\times(+\hat{k}) = \hat{i}$, $\vec{E}$ is along the y-direction.

Thus, $\vec{E} = 3.6\,\hat{j}$ Vm^{-1}.

28. (b): A concave lens forms virtual image I of a point object O.



As $\quad m = \dfrac{1}{4} = \dfrac{v}{u}$

$\therefore \quad u = 4v$

If $\quad v = -x,\ u = -4x$

Distance between object and image

$|OI| = 4x - x = 3x = 10$ cm,

or $\quad x = \dfrac{10}{3}$ cm

$\therefore \quad u = -\dfrac{40}{3}$ cm and $v = \dfrac{-10}{3}$ cm

Using lens formula $\dfrac{1}{f} = \dfrac{1}{v} - \dfrac{1}{u}$

$$\frac{1}{f} = -\frac{3}{10} + \frac{3}{40} = -\frac{9}{40}$$

$$f = -\frac{40}{9}\text{ cm} = -4.4 \text{ cm}$$

29. (a) : de Broglie wavelength of a moving particle, having mass m and velocity v is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv} \quad\text{or}\quad m = \frac{h}{\lambda v}$$

Given : $\dfrac{v}{v_e} = 3,\ \dfrac{\lambda}{\lambda_e} = 1.813\times 10^{-4}$

Mass of the particle, $m = m_e\left(\dfrac{v_e}{v}\right)\left(\dfrac{\lambda_e}{\lambda}\right)$

Substituting the values, we get

$$m = (9.1\times 10^{-31}\text{ kg})\times\left(\frac{1}{3}\right)\times\left(\frac{1}{1.813\times 10^{-4}}\right)$$

$m = 1.67 \times 10^{-27}$ kg

30. (d): According to Doppler's effect,

$$\upsilon' = \upsilon\left(\frac{v}{v - v_s}\right);\ \frac{4}{3}\upsilon = \upsilon\left(\frac{v}{v - v_s}\right)$$

or $\quad 4v - 4v_s = 3v$ or $\quad v = 4v_s$

or $\quad v_s = \frac{v}{4} = \frac{332}{4} = 83 \text{ m s}^{-1}$

31. (c) : $V = 6$ volt, $R = 100\ \Omega$

$I = \frac{V}{R} = \frac{6}{100}$ A

L = Length of wire = 3 m = 300 cm

Fall of potential/length $= \frac{6 \text{ volt}}{300 \text{ cm}}$

Hence, potential difference between two points on the wire separated by a distance of 50 cm

$= \frac{6 \text{ volt}}{300 \text{ cm}} \times 50 \text{ cm} = 1 \text{ V}$

32. (a) : Deuteron nucleus consists of 1 proton and 1 neutron *i.e.*, $A = 2$; $Z = 1$

Mass of neutron = 1.00893 amu

Mass of proton = 1.00813 amu

Mass of deuteron = 2.01473 amu

Mass defect,

$\Delta m = (1.00893 + 1.00813) - 2.01473$

$= 0.00233$ amu

Packing fraction $= \frac{\Delta m}{A} = \frac{0.00233}{2}$

$= 11.65 \times 10^{-4}$ amu

33. (c) : From the question, $v = \sqrt{\frac{T}{m}}$.

Weight of wire = 50 N

Mass of wire = (50/10) = 5 kg

Length = 20 m

$\therefore \quad m = (5/20) = 0.25 \text{ kg m}^{-1}$

At the middle point, $T = 25$ N

$\therefore \quad v = \sqrt{\frac{T}{m}} = \sqrt{\frac{25}{0.25}} = \sqrt{100} = 10 \text{ m s}^{-1}$

34. (a) : Here, a = 1 cm = 0.01 m; The mass will remain in contact with surface, if

$mg = m\omega^2 a$ or $\omega = \sqrt{g/a}$

or $\quad 2\pi\upsilon = \sqrt{g/a}$ *or* $\upsilon = \frac{1}{2\pi}\sqrt{\frac{g}{a}}$

$= \frac{7}{2\times 22}\sqrt{\frac{980}{1}} = 4.9 \text{ Hz} \approx 5 \text{ Hz}$

35. (b): For an open pipe,

$\upsilon = \frac{n\cdot v}{2L} = \frac{n\cdot(340 \text{ m s}^{-1})}{2 \times (5 \times 10^{-2} \text{ m})} = n\cdot(3400 \text{ Hz})$

$3400\cdot n < 20000$; $\therefore\ n < \left(\frac{20000}{3400}\right)$; $n < 5.9$

$\Rightarrow$ The greatest integral value of $n = 5$.

36. (b): $N = 70$, $r = 5 \text{ cm} = 5 \times 10^{-2}$ m, $I = 8$ A

$B = 1.5$ T, $\theta = 30°$

The counter torque to prevent the coil from turning will be equal and opposite to the torque acting on the coil,

$\therefore \tau = NIAB \sin\theta = NI\pi r^2 B \sin 30°$

$= 70 \times 8 \times 3.14 \times (5\times 10^{-2})^2 \times 1.5 \times \frac{1}{2} = 3.297 \text{ N m}$

≈ 3.3 N m

37. (a) : Terminal velocity, $v = \frac{2r^2(\rho - \rho_0)g}{9\eta}$

i.e, $\quad v \propto r^2$.

$\therefore \quad \frac{v_1}{v} = \frac{r_1^2}{r^2}$

or $\quad v_1 = v\left(\frac{r_1}{r}\right)^2 = 20\left(\frac{1}{2}\right)^2 = 5 \text{ cm s}^{-1}$

38. (d): Here, J = ?

$C_P = 3.409 \text{ cal g}^{-1}\text{°C}^{-1}$

$C_V = 2.409 \text{ cal g}^{-1}\text{°C}^{-1}$, $M = 2$

$R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$

As $C_P - C_V = \frac{r}{J} = \frac{R}{MJ}$

$\therefore \quad 3.409 - 2.409 = \frac{8.31}{2J}$; $1 = \frac{4.155}{J}$

or $\quad J = 4.11 \text{ J cal}^{-1}$

39. (d): $U_i = -\frac{GMm}{r}$

$U_i = -\frac{6.67\times 10^{-11} \times 100 \times 10^{-2}}{0.1}$

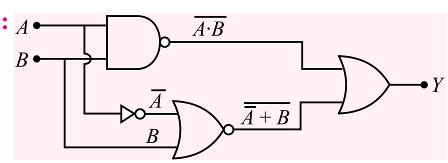
We know

$\therefore \quad W = \Delta U = U_f - U_i$

$\therefore \quad W = -U_i = 6.67 \times 10^{-10}$ J ($\because U_f = 0$)

40. (b)

41. (a) :



$Y = \overline{A\cdot B} + \overline{\overline{A}+B}$

$= \overline{A} + \overline{B} + \overline{\overline{A}}\cdot\overline{B}$ (By De Morgan's theorem)

$= \overline{A} + \overline{B} + A\cdot\overline{B}$ (Using $\overline{\overline{A}} = A$)

$= \overline{A} + \overline{B} + \overline{B} \cdot A$ (Using $A \cdot \overline{B} = \overline{B} \cdot A$)

$= \overline{A} + \overline{B}$ (Using $1 + A = 1$)

$= \overline{A \cdot B}$

42. **(a)**

43. **(a) :** Pressure outside the bigger drop = P_1

Pressure inside the bigger drop = P_2

Radius of bigger drop, r_1 = 3 cm

Excess pressure $= P_2 - P_1 = \frac{4S}{r_1} = \frac{4S}{3}$

Pressure inside small drop = P_3

Excess pressure $= P_3 - P_2 = \frac{4S}{r_2} = \frac{4S}{1}$

Pressure difference between inner side of small drop and outer side of bigger drop

$$= P_3 - P_1 = \frac{4S}{3} + \frac{4S}{1} = \frac{16S}{3}$$

This pressure difference should exist in a single drop of radius r.

$$\therefore \quad \frac{4S}{r} = \frac{16S}{3} \quad \text{or} \quad r = \frac{3}{4}\text{ cm} = 0.75\text{ cm}$$

44. **(a) :** Mass per unit area of disc $= \frac{9M}{\pi R^2}$

Mass of removed portion of disc

$$= \frac{9M}{\pi R^2} \times \pi\left(\frac{R}{3}\right)^2 = M$$

Moment of inertia of removed portion about an axis passing through centre of disc and perpendicular to the plane of disc, using theorem of parallel axis is

$$I_1 = \frac{M}{2}\left(\frac{R}{3}\right)^2 + M\left(\frac{2R}{3}\right)^2 = \frac{1}{2}MR^2$$

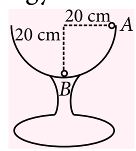
when portion of disc would not have been removed, then the moment of inertia of complete disc about the given axis is

$$I_2 = \frac{9}{2}MR^2$$

So moment of inertia of the disc with removed portion, about the given axis is

$$I = I_2 - I_1 = \frac{9}{2}MR^2 - \frac{1}{2}MR^2 = 4\,MR^2$$

45. **(b) :** According to the conservation of energy, kinetic energy at A + potential energy at A = kinetic energy at B + potential energy at B



$$0 + mgh = \frac{1}{2}mv^2 + 0$$

or $\quad v^2 = 2gh = 2 \times 9.8 \times 0.20$

[$\because$ h = radius = 20 cm = 0.2 m]

According to work energy theorem workdone on the ball

= change in kinetic energy

$$= \frac{1}{2} \times \frac{2}{1000} \times 2 \times 9.8 \times 0.2$$

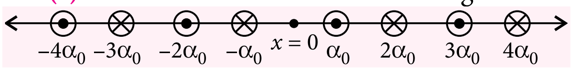
$= 3.92 \times 10^{-3}$ J = 3.92 mJ

PHYSICS MUSING

SOLUTION SET-18

1. **(c):** For $0 < \omega < \omega_0$, impedance decreases and current increases, where ω_0 is resonant frequency.
At $\omega = \omega_0$, impedance is minimum and current is maximum.
For $\omega > \omega_0$, impedance increases and current decreases.

2. **(c):** Given situation is shown in the figure



Magnitude of magnetic field at $x = 0$ is

$$B = \frac{2\mu_0 I}{2\alpha_0 \pi}\left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + ...\right) = \frac{\mu_0 I \ln(2)}{\pi\alpha_0}$$

3. **(c):** Here, $E_x = 8x$, $E_y = -4y$, $E_z = -4z$
For $z = 0$, $E_z = 0$

$$\frac{-dV}{dx} = 8x, \frac{-dV}{dy} = -4y, \frac{dy}{dx} = -\frac{2x}{y}$$

$$\therefore \quad \frac{y^2}{2} = \frac{-2x^2}{2} + \text{constant}$$

$\Rightarrow \quad y^2 + 2x^2 = \text{constant}$

4. **(b) :** Area vector, $\vec{A} = \pi R^2(-\sin\theta\,\hat{i} + \cos\theta\,\hat{k})$

$$\text{Torque} = I\vec{A} \times (-B\,\hat{i})$$

$$= -(BI)(\pi R^2)\cos\theta\,\hat{j}$$

Net torque about the origin O is zero.

$$mgR\cos\theta - 2RT_2\cos\theta - BIR^2\pi\cos\theta = 0$$

$$\text{or} \quad mg - 2T_2 - \frac{mg}{4} = 0$$

$$\text{or} \quad T_2 = \frac{3mg}{8}, T_1 = mg - T_2 = \frac{5mg}{8}$$

$$\therefore \quad \frac{T_1}{T_2} = \frac{5}{3}$$

5. **(b) :** As $I = \frac{dq}{dt} = \frac{V}{R} \quad \Rightarrow \quad \frac{dq}{dR}\cdot\frac{dR}{dt} = \frac{V}{R}$

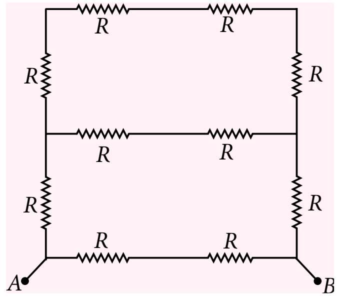
$$\text{or} \quad \frac{dq}{dR} \times \left(\frac{5}{60}\right) = \frac{V}{R}$$

$$\text{or} \quad dq = 12V\frac{dR}{R} \quad \text{or} \quad q = 12V\int_{20}^{40}\frac{dR}{R}$$

($\because$ $R_i = 20\ \Omega$, $R_f = 20 + 5 \times 4 = 40\ \Omega$)

$\therefore \quad q = 12\ V \ln 2 = 120 \ln 2$ C

6. **(a) :** Points C, D and E are at same potential. So the circuit can be redrawn as



Equivalent resistance, $R_{eq} = \frac{5R}{4}$

7. **(a) :** Here, $u = 20$ m s^{-1}, $\theta = 45°$, $q = 2\ \mu$C,

$$\vec{E} = 2 \times 10^7\,\hat{i}\ \text{V m}^{-1}, m = 1\ \text{kg}$$

$$a_x = \frac{qE}{m} = 40\ \text{m s}^{-2} \text{ and } a_y = g = -10\ \text{m s}^{-2}$$

$$T = \frac{2u\sin\theta}{g} = 2\sqrt{2}\ \text{s}$$

$$\therefore \quad R = u\cos\theta\,T + \frac{1}{2}a_x T^2 = 40 + 160 = 200\ \text{m}$$

8. **(b) :** Initially, current through external resistance R,

$$I = \frac{N\varepsilon}{R + Nr} \qquad ...(i)$$

When n cells are reversed in polarity, then current through R,

$$\frac{I}{3} = \frac{(N - 2n)\varepsilon}{R + Nr} \qquad ...(ii)$$

From eqn, (i) and (ii)

$$3 = \frac{N}{(N - 2n)} \quad \Rightarrow \quad n = \frac{N}{3}$$

9. **(d) :** As $I = \upsilon e$ ...(i)

$$\upsilon = \frac{\omega}{2\pi} = \frac{v}{2\pi r} = \frac{0.6\pi \times 10^6}{2\pi \times 5 \times 10^{-11}} = 6 \times 10^{15}\ \text{rad s}^{-1}$$

$I = 6 \times 10^{15} \times 1.6 \times 10^{-19}$ (Using (i))
$= 0.96 \times 10^{-3}$ A $= 0.96$ mA

10. **(a) :** As, $I = \frac{V}{R} = \frac{(20 - 10)}{(7.5 + 0.5 + 1 + 1)} = 1$ A

$V_{PS} = 7.5 \times 1 = 7.5$ V, $V_{QR} = 1 \times 1 = 1$ V

G is connected to earth, hence potentials of R and S is zero. Direction of current is from P to S.
$\Rightarrow$ V_P is higher than V_S
$\Rightarrow$ $V_P = 7.5$ V
Similarly $V_Q = -1$ V

PRACTICE PAPER 2015

JEE Advanced

Exam on 24^{th} May

PAPER-1

SECTION-1

One or More Than One Options Correct Type

This section contains 10 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which ONE or MORE THAN ONE are correct.

1. A particle of mass m (starting from rest) moves vertically upwards from the surface of earth under an external force $\vec{F}$ which varies with height z as, $\vec{F} = (2 - \alpha z)m\vec{g}$, where α is a positive constant. If H is the maximum height to which particle rises, then

   (a) $H = \frac{1}{\alpha}$

   (b) $H = \frac{2}{\alpha}$

   (c) work done by $\vec{F}$ during motion upto $\frac{H}{2}$ is $\frac{3mg}{2\alpha}$

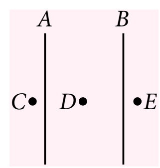
   (d) velocity of particle at $\frac{H}{2}$ is $\sqrt{\frac{g}{\alpha}}$

2. A ray of light from a denser medium strikes the plane boundary of a rarer medium at an angle of incidence i. The angle of refraction is r. If the reflected and the refracted rays are mutually perpendicular, the critical angle of the medium is

   (a) $\sin^{-1}(\cot i)$ (b) $\sin^{-1}(\tan i)$

   (c) $\tan^{-1}(\sin i)$ (d) $\sin^{-1}(\cot r)$

3. Two infinite plane sheets A and B are shown in the figure. The surface charge densities on A and B are $(2/\pi) \times 10^{-9}$ C m^{-2} and $(-1/\pi) \times 10^{-9}$ C m^{-2} respectively. C, D, E are three points where electric fields (in N C^{-1}) are E_C, E_D and E_E respectively.



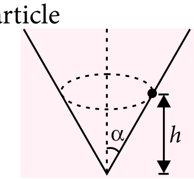
   (a) $E_C = 18$, towards right

   (b) $E_D = 54$, towards right

   (c) $E_D = 18$, towards right

   (d) $E_E = 18$, towards right

4. A particle is describing circular motion in a horizontal plane in contact with the smooth inside surface of a fixed right circular cone with its axis vertical and vertex down. The height of the plane of motion above the vertex is h and the semivertical angle of the cone is α. The period of revolution of the particle



   (a) increases as h increases

   (b) decreases as h increases

   (c) increases as α increases

   (d) decreases as α increases

5. A closed vessel contains a mixture of two diatomic gases A and B. Molar mass of A is 16 times that of B and mass of gas A, contained in the vessel is 2 times that of B. Then

   (a) Average kinetic energy per molecule of gas A is equal to that of gas B.

   (b) Root mean square value of translational velocity of gas B is four times that of A.

(c) Pressure exerted by gas B is eight times of that exerted by gas A.

(d) Number of molecules of gas B in the cylinder is eight times that of gas A.

6. A particle of charge q and mass m moves rectilinearly under the action of an electric field $E = \alpha - \beta x$. Here, α and β are positive constants and x is the distance from the point where the particle was initially at rest. Then, the

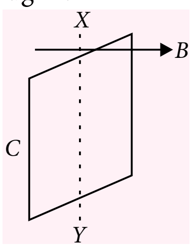
(a) motion of the particle is oscillatory

(b) amplitude of the particle is (α/β)

(c) mean position of the particle is at $x = (\alpha/\beta)$

(d) maximum acceleration of the particle is $\frac{q\alpha}{m}$.

7. A flat coil, C of n turns, area A and resistance R is placed in a uniform magnetic field of magnitude B, as shown in the figure. The plane of the coil is initially perpendicular to B. If the coil is rotated by an angle θ about the axis XY, charge of amount Q flows through it



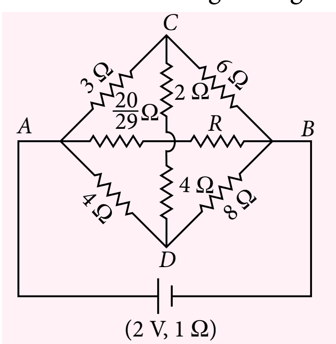
(a) if $\theta = 90°$, $Q = \frac{BAn}{R}$

(b) if $\theta = 180°$, $Q = \frac{2BAn}{R}$

(c) if $\theta = 180°$, $Q = 0$

(d) if $\theta = 360°$, $Q = 0$

8. A battery of emf 2 V and internal resistance 1 Ω is connected across terminals A and B of the circuit as shown in the given figure.



(a) Thermal power generated in external circuit will be maximum possible, when $R = \frac{16}{29}\ \Omega$.

(b) Maximum possible thermal power generated in external circuit is equal to 4 W.

(c) The ratio of current through 6 Ω to that through 4 Ω is independent of R.

(d) None of these.

9. The position of a particle varies according to the expression $x = t(t - 1)(t - 2)$, then

(a) velocity will be zero at $t = \left(1 - \frac{1}{\sqrt{3}}\right)$ s and $t = \left(1 + \frac{1}{\sqrt{3}}\right)$ s

(b) acceleration changes its direction between $t = 0$ and $t = 2$ s

(c) acceleration remains constant in direction between $t = 0$ and $t = 2$ s

(d) None of these

10. Three SHMs in the same direction having the same amplitude A and same period are superposed. If each differs in phase from the next by 45°, then

(a) the resultant amplitude is $A(1+\sqrt{2})$.

(b) the phase of the resultant motion relative to the first is 90°.

(c) the energy associated with the resulting motion is $(3+2\sqrt{2})$ times the energy associated with any single motion.

(d) the resultant motion is not simple harmonic.

SECTION-2

One Integer Value Correct Type

This section contains 10 questions. Each question, when worked out will result in one integer from 0 to 9 (both inclusive).

11. Two particles of mass m and $3m$ are initially at rest at infinite distance apart. Both the particles start moving due to gravitational attraction. At any instant their relative velocity of approach is $\sqrt{\frac{\eta G m}{d}}$, where d is their separation at that instant. Find η.

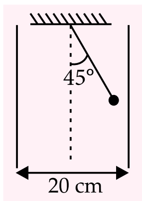
12. A radioactive substance is being produced at a constant rate of 200 nuclei per second. The

decay constant of the substance is 1 s^{-1}. The number of radioactive nuclei will become 100 after the time ln(p) s. Initially, there are no nuclei present. Find the value of p.

13. Two spherical bodies A (radius 6 cm) and B (radius 18 cm) are at temperatures T_A and T_B, respectively. The maximum intensity in the emission spectrum of A is at 500 nm and in that of B is at 1500 nm. Considering them to be black bodies, what will be the ratio of the rate of total energy radiated by A to that of B?

14. A string of length 70 cm is fixed at its ends to two points 50 cm apart. The string passes through a smooth ring which rotates in a horizontal circle as a result of which the two sections of the string make a right angle. If the speed of ring is v m s^{-1}, find v (to nearest integer). (Take $g = 10$ m s^{-2})

15. A small sphere of mass 2.0 g and having charge 0.5 mC is suspended by a string between the plates of a parallel plate capacitor as shown in the figure.



What potential difference (in volt) between the plates (separation 20 cm) should be applied so that the string makes an angle of 45° with the vertical? (Take $g = 10$ m s^{-2})

16. A flat thin circular disc has a radius 4 cm and a circular hole of radius $\frac{1}{2}$ cm is made in it with its centre at a distance of 1 cm from the centre of the disc.
The mass of the disc is 10 kg. If the moment of inertia of the system about an axis passing through the centre of the hole is $N \times 10^{-3}$ kg m^2, find the value of N.

17. An air filled parallel plate capacitor has a capacitance 1 pF. The separation between the plates is doubled and wax is inserted between the plates and the capacitance becomes 2 pF. What is the dielectric constant of the wax?

18. A container completely filled with water ($\mu = 4/3$) has a scratch at its bottom. An observer looks at the scratch from a height 1 m above the surface of water. A small hole is now made in the wall of the container and very close to its bottom so that water flows out. When the level of water reduces to half, the hole is closed. It appears to the observer that the scratch has moved away by a distance 25 cm relative to its initial position. Find the initial level (height) of water (in m) in the container.

19. An excited He^+ ion emits two photons in succession with wavelength 1026.7 Å and 304 Å while making a transition to its ground state. Taking value of $R = 1.097 \times 10^7$ m^{-1}, find the quantum number n of excited state of He^+ ion.

20. P–V diagram of a diatomic gas is a straight line passing through origin. The molar heat capacity of the gas in the process will be yR. What is the value of y?

PAPER-2

SECTION-1

Only One Option Correct Type

This section contains 10 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which ONLY ONE is correct.

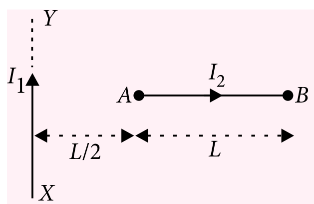
1. A conductor AB of length L carrying a current I_2, is placed perpendicular to a long straight conductor XY carrying current I_1, as shown in figure. The force on AB in magnitude is

(a) $\dfrac{3\mu_0 I_1 I_2}{2\pi}$

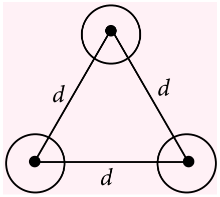
(b) $\dfrac{\mu_0 I_1 I_2}{2\pi} \log_e 2$

(c) $\dfrac{\mu_0 I_1 I_2}{2\pi} \log_e 3$

(d) $\dfrac{2\mu_0 I_1 I_2}{3\pi}$



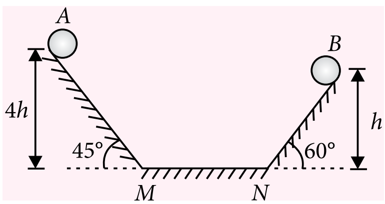
2. Three solid spheres each of mass m and radius R are released from the position shown in figure. The speed of any one sphere at the time of collision would be



(a) $\sqrt{Gm\left(\frac{1}{d}-\frac{3}{R}\right)}$ (b) $\sqrt{Gm\left(\frac{3}{d}-\frac{1}{R}\right)}$

(c) $\sqrt{Gm\left(\frac{2}{R}-\frac{1}{d}\right)}$ (d) $\sqrt{Gm\left(\frac{1}{R}-\frac{2}{d}\right)}$

3. Two identical balls A and B are released from the positions shown in the figure. They collide elastically on horizontal portion MN. The ratio of heights attained by A and B after collision will be (neglect friction)



(a) 1 : 4 (b) 2 : 1 (c) 4 : 13 (d) 2 : 11

4. A particle of charge q and mass m moves in a circular orbit of radius r with angular speed ω. The ratio of the magnitude of its magnetic moment to that of its angular momentum depends on

(a) ω and q (b) ω, q and m

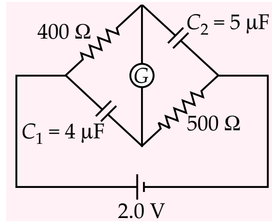
(c) q and m (d) ω and m

5. In a stationary wave that forms as a result of reflection of waves from an obstacle, the ratio of the amplitude at an antinode to the amplitude at node is n. The fraction of energy reflected is

(a) $\left(\frac{n}{n-1}\right)^2$ (b) $\left(\frac{1}{n}\right)^2$

(c) $\left(\frac{n-1}{n}\right)^2$ (d) $\left(\frac{n-1}{n+1}\right)^2$

6. In the circuit shown, if the resistance of the galvanometer is 100 Ω, then the potential differences across C_1 and C_2 are respectively



(a) 1 V, 1 V (b) 1 V, 1.2 V

(c) 2 V, 1 V (d) 1 V, 2 V

7. The density of a solid at normal pressure is ρ. When the solid is subjected to an excess pressure P, the density changes to ρ'. If the bulk modulus of the solid is K, then the ratio $\frac{\rho'}{\rho}$ is

(a) $1+\frac{P}{K}$ (b) $1+\frac{K}{P}$ (c) $\frac{P}{P+K}$ (d) $\frac{K}{P+K}$

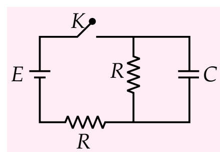
8. A rocket is launched vertically from the surface of the earth with an initial velocity v. How far above the surface of earth it will go? Neglect the air resistance.

(a) $R\left(\frac{2gR}{v^2}-1\right)^{-1/2}$ (b) $R\left(\frac{2gR}{v^2}-1\right)$

(c) $R\left(\frac{2gR}{v^2}-1\right)^{-1}$ (d) $R\left(\frac{2gR}{v^2}-1\right)^{2}$

(where R is the radius of the earth)

9. In the circuit shown in the figure, E is the emf of the cell, connected to two resistances, each of resistance R and a capacitor of capacitance C as shown in the figure. If the key K is connected at time $t = 0$, growth of potential V across the capacitor will be given by



(a) $V(t)=E\left[1-\exp\left(-\frac{t}{RC}\right)\right]$

(b) $V(t)=\frac{E}{2}\left[1-\exp\left(-\frac{2t}{RC}\right)\right]$

(c) $V(t)=E\left[1-\exp\left(-\frac{2t}{RC}\right)\right]$

(d) $V(t)=\frac{E}{2}\left[1-\exp\left(-\frac{t}{RC}\right)\right]$

10. Two bodies of masses 3 kg and 2 kg collide head-on. Their relative velocities before and after collision are 15 m s^{-1} and 5 m s^{-1} respectively. The loss of kinetic energy of the system is

(a) 120 J (b) 100 J (c) 80 J (d) 240 J

SECTION-2

Comprehension Type (Only One Option Correct)

This section contains 3 paragraphs, each describing theory, experiments, data etc. Six questions relate to the three paragraphs with two questions on each paragraph. Each question has only one correct answer among the four given options (a), (b), (c) and (d).

Paragraph for questions 11 and 12

A stone is projected from level ground with speed u and at an angle θ with horizontal. Some how the acceleration due to gravity (g) becomes double (that is $2g$) immediately after the stone reaches the maximum height and remains same thereafter. Assume direction of acceleration due to gravity always vertically downwards.

11. The horizontal range of particle is

(a) $\frac{3}{4}\frac{u^2 \sin 2\theta}{g}$ (b) $\frac{u^2 \sin 2\theta}{2g}\left(1+\frac{1}{\sqrt{2}}\right)$

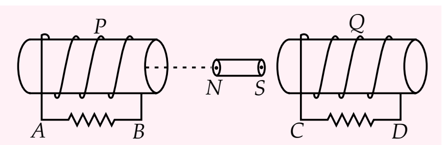
(c) $\frac{u^2}{g}\sin 2\theta$ (d) $\frac{u^2 \sin 2\theta}{2g}\left(2+\frac{1}{\sqrt{2}}\right)$

12. The angle ϕ which the velocity vector of stone makes with horizontal just before hitting the ground is given by

(a) $\tan\phi = 2\tan\theta$ (b) $\tan\phi = 2\cot\theta$

(c) $\tan\phi = \sqrt{2}\tan\theta$ (d) $\tan\phi = \sqrt{2}\cot\theta$

Paragraph for questions 13 and 14



In the above figure, NS is a permanent magnet with its poles N and S as shown. P and Q are two coils with iron core, connected to resistors AB (for P) and CD (for Q) respectively lying on the common axis of the magnet to its left and right respectively as in the figure. The windings on the coil are similar as shown.

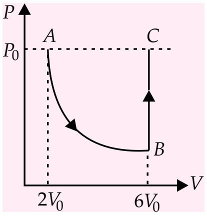
13. If the magnet is now moved to the left, then during its motion, a current will flow from

(a) A to B in P and from C to D in Q.
(b) A to B in P and from D to C in Q.
(c) B to A in P and from C to D in Q.
(d) B to A in P and from D to C in Q.

14. If the magnet is kept fixed, and the coil P is moved to the left with coil Q fixed, then during its motion, a current will flow from

(a) B to A in P and no current will flow in Q.
(b) B to A in P and from C to D in Q.
(c) B to A in P and from D to C in Q.
(d) A to B in P and no current will flow in Q.

Paragraph for questions 15 and 16



n moles of a monoatomic ideal gas undergo the process ABC as shown on the adjacent P-V diagram. The process AB is isothermal and BC is isochoric. The temperature of the gas at A is T_0. Total heat given to the gas during the process ABC is measured to be Q.

15. Temperature of the gas at C is equal to

(a) T_0 (b) $3T_0$ (c) $6T_0$ (d) $2T_0$

16. Heat absorbed by the gas in the process BC is

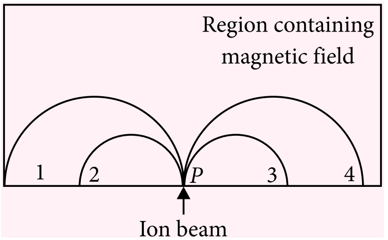
(a) $3nRT_0$ (b) nRT_0 (c) $2nRT_0$ (d) $6nRT_0$

SECTION-3

Matching List Type (Only One Option Correct)

This section contains four questions, each having two matching lists. Choices for the correct combination of elements from List-I and List-II are given as options (a), (b), (c) and (d), out of which one is correct.

17. A beam consisting of four types of ions A, B, C and D enters a region at P that contains a uniform magnetic field as shown in figure. The field is perpendicular to the plane of the paper, but its precise direction is not given. All ions in the beam travel with the same speed. The following table shows the masses and charges of the ions.



Ion	Mass	Charge
A	$2m$	e
B	$4m$	$-e$
C	$2m$	$-e$
D	m	$+e$

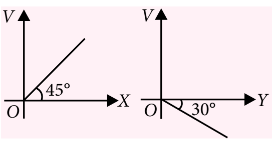
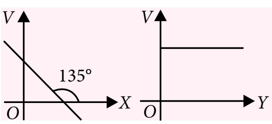
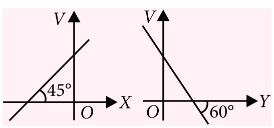
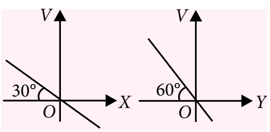
The ions fall at different positions 1, 2, 3 and 4 as shown. Correctly match the ions with their respective positions of fall.

List I		List II	
P.	A	1.	1
Q.	B	2.	2
R.	C	3.	3
S.	D	4.	4

Code :

(a) P - 4, Q - 1, R - 2, S - 3
(b) P - 1, Q - 3, R - 4, S - 3
(c) P - 2, Q - 3, R - 2, S - 4
(d) P - 3, Q - 2, R - 2, S - 1

18. List-I shows graphs of electric potential V versus X and Y in a certain region for four situations. List-II gives angle which the electric field vector makes with positive X-direction.

List I		List II	
P.		1.	0°
Q.		2.	$\tan^{-1}(3)$
R.		3.	120°
S.		4.	150°

Code :

(a) P - 3, Q - 1, R - 4, S - 2
(b) P - 4, Q - 1, R - 3, S - 2
(c) P - 2, Q - 3, R - 2, S - 1
(d) P - 1, Q - 4, R - 2, S - 4

19. A frictionless piston contains 5 mole of gas.

Given $R = C_P - C_V, \gamma = \dfrac{C_P}{C_V}$

List I		List II	
P.	$\dfrac{\Delta Q}{\Delta W}$, For diatomic gas in isobaric process	1.	1.5
Q.	$\dfrac{\Delta Q}{\Delta W}$, For diatomic gas in isothermal process	2.	3.5
R.	$\dfrac{\Delta Q}{\Delta U}$, For polyatomic gas in isochoric process	3.	1.0
S.	$\dfrac{\Delta W}{\Delta U}$, For monoatomic gas in adiabatic process	4.	2.5

Code :

(a) P - 2, Q - 3, R - 3, S - 3
(b) P - 2, Q - 2, R - 4, S - 3
(c) P - 3, Q - 2, R - 1, S - 4
(d) P - 1, Q - 2, R - 1, S - 3

20. A piece of metal of density ρ_1 floats on mercury of density ρ_2. The coefficients of expansion of the metal and mercury are γ_1 and γ_2, respectively. The temperature of both mercury and metal are increased by ΔT. Then match the following.

List I		List II	
P.	If $\gamma_2 > \gamma_1$	1.	no effect on submergence
Q.	$\gamma_2 = \gamma_1$	2.	fraction of the volume of metal submerged in mercury
R.	If $\gamma_2 < \gamma_1$	3.	the solid sinks
S.	$(\gamma_2 - \gamma_1)\,\Delta T$	4.	the solid lifts up

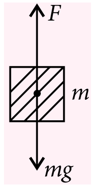
Code :

(a) P - 2, Q - 3, R - 2, S - 4
(b) P - 4, Q - 3, R - 1, S - 2
(c) P - 3, Q - 1, R - 4, S - 2
(d) P - 1, Q - 3, R - 1, S - 4

SOLUTIONS

PAPER-1

1. (b, c, d) :



Resultant force on particle = $F - mg$

$=(2 - \alpha z)mg - mg = (1 - \alpha z)mg$

$$mv\frac{dv}{dz} = (1-\alpha z)mg$$

$$\Rightarrow \frac{v^2}{2} = \left(z - \frac{\alpha z^2}{2}\right)g$$

For maximum height, $v = 0 \Rightarrow \quad z = H = \frac{2}{\alpha}$

Velocity at $\frac{H}{2} = \left(\frac{1}{\alpha}\right)$

$$v = \sqrt{2\left(z - \frac{\alpha z^2}{2}\right)g} = \sqrt{2\left(\frac{1}{\alpha} - \frac{1}{2\alpha}\right)g} = \sqrt{\frac{g}{\alpha}}$$

$$W = \int_0^{1/\alpha}(2-\alpha z)mgdz$$

$$= mg\left[2z - \frac{\alpha z^2}{2}\right]_0^{1/\alpha} = mg\left[\frac{2}{\alpha} - \frac{1}{2\alpha}\right] = \frac{3mg}{2\alpha}$$

2. (b, d) : PQ = incident ray, QS = reflected ray

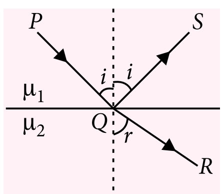
QR = refracted ray

$\therefore \quad \angle SQR = 180° - (i + r) = 90°$

$\therefore \quad i + r = 90°$ or $r = 90° - i$

$$\frac{\mu_2}{\mu_1} = \frac{\sin i}{\sin r} = \frac{\sin i}{\sin(90° - i)} = \tan i$$

If C is the critical angle



$$\frac{\mu_1}{\mu_2} = \frac{1}{\sin C}$$

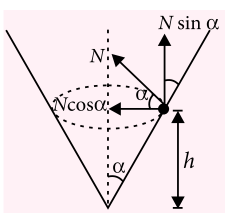
$\therefore \quad \sin C = \tan i$

$C = \sin^{-1}(\tan i)$

$= \sin^{-1}(\tan(90° - r)) = \sin^{-1}(\cot r)$

3. (b, d)

4. (a, c) : As $N \sin\alpha = mg$



$N\cos\alpha = m\omega^2 r$

$\therefore \quad \tan\alpha = \frac{g}{\omega^2 r}$

or $\quad T^2 \propto \tan\alpha$

$\therefore \quad$ when α increases, T also increases.

Also $T^2 \propto r\tan\alpha$

but $r = h\tan\alpha \quad \therefore \quad T^2 \propto h\tan^2\alpha$

for constant α; $T^2 \propto h$

Thus when h increases, T also increases.

5. (a, b, c, d) : Average KE per molecule = $\frac{5}{2}kT$

It depends only on temperature.

$$v_{\text{rms }A} = \sqrt{\frac{3RT}{M_A}},$$

$$v_{\text{rms }B} = \sqrt{\frac{3RT}{M_B}} = \sqrt{\frac{3RT}{M_A/16}} = 4v_{\text{rms }A}$$

$$n_A = \frac{m_A}{M_A},\ n_B = \frac{m_B}{M_B} = \frac{m_A/2}{(M_A/16)} = 8n_A$$

$$P_B = \left(\frac{n_B}{n_A + n_B}\right)P_0,\ P_A = \left(\frac{n_A}{n_A + n_B}\right)P_0$$

6. (a, b, c, d): Acceleration of particle,

$$a = \frac{F}{m} = \frac{qE}{m} = \frac{q}{m}(\alpha - \beta x) \qquad \text{... (i)}$$

$a = 0$ at $x = \frac{\alpha}{\beta}$

i.e., force on the particle is zero at $x = \frac{\alpha}{\beta}$

So, mean position of the particle is at $x = \frac{\alpha}{\beta}$

Equation (i) can be written as

$$v\frac{dv}{dx} = \frac{q}{m}(\alpha - \beta x)$$

$$\therefore \quad \int_0^v vdv = \frac{q}{m}\int_0^x(\alpha - \beta x)dx$$

$$\therefore \quad v = \sqrt{\frac{2qx}{m}\left(\alpha - \frac{\beta}{2}x\right)}$$

$v = 0$ at $x = 0$ and $x = \frac{2\alpha}{\beta}$

So, the particle oscillates between $x = 0$ and $x = \frac{2\alpha}{\beta}$ with mean position at $x = \frac{\alpha}{\beta}$

Maximum acceleration of the particle is at extreme position (at $x = 0$ or $x = 2\alpha/\beta$) and

$a_{max} = q\alpha/m$ [from equation (i)]

7. (a, b, d) : Charge flowing in the circuit, $Q = \frac{d\phi}{R}$ where R is resistance of the circuit and

$d\phi = \phi_{final} - \phi_{initial}$ = change in magnetic flux

Initially, plane of coil is perpendicular to B. Therefore, $\theta = 0°$

When $\theta = 90°$, $d\phi = n\,AB\cos 90° - nAB\cos 0°$
$= -n\,AB$

If $\theta = 180°$, $d\phi = n\,AB\cos 180° - n\,AB\cos 0°$
$= -2\,n\,AB$

If $\theta = 360°$, $d\phi = n\,AB\cos 360° - n\,AB\cos 0°$
= zero

8. (a,b,c): Here $ABCD$ forms a balanced Wheatstone bridge, hence, resistance (2 Ω + 4 Ω) in arm CD becomes ineffective. So in circuit we have $(3+6)\,\Omega$, $\left(\frac{20}{29}+R\right)\Omega$ and $(4+8)\,\Omega$ in parallel across A and B.

Thermal power generated in external circuit is maximum if total resistance across A and B is equal to internal resistance of cell = 1 Ω.

$$\text{So } \frac{1}{R_{eff}} = \frac{1}{9} + \frac{1}{\left(\frac{20}{29}+R\right)} + \frac{1}{12} = \frac{1}{1}.$$

On solving, we get $R = \frac{16}{29}\,\Omega$

Maximum power generated, $\frac{V^2}{R_{eff}} = \frac{2^2}{1} = 4$ W

The ratio of current through 6 Ω and 4 Ω is independent of current of R.

9. (a, b) : $x = t^3 - 3t^2 + 2t$

$$v = \frac{dx}{dt} = 3t^2 - 6t + 2 = 0$$

$$\Rightarrow\ t = \frac{6 \pm \sqrt{36-24}}{6} = \frac{6 \pm \sqrt{12}}{6} = \left(1 \pm \frac{1}{\sqrt{3}}\right) \text{s}$$

$$a = \frac{dv}{dt} = 6t - 6$$

when $t = 1$ s, $a = 0$

when $0 < t < 1$ s, a is negative

when 1 s $< t <$ 2 s, a is positive.

10. (a, c): According to the principle of superposition, the resultant displacement is

$x = x_1 + x_2 + x_3$

$$= A\sin\omega t + A\sin\left(\omega t + \frac{\pi}{4}\right) + A\sin\left(\omega t + \frac{\pi}{2}\right)$$

$$= A\left[\left\{\sin\omega t + \sin\left(\omega t + \frac{\pi}{2}\right)\right\} + \sin\left(\omega t + \frac{\pi}{4}\right)\right]$$

$$= A\left[2\sin\left(\omega t + \frac{\pi}{4}\right)\cos\frac{\pi}{4} + \sin\left(\omega t + \frac{\pi}{4}\right)\right]$$

$$= A\left[\frac{2}{\sqrt{2}}\sin\left(\omega t + \frac{\pi}{4}\right) + \sin\left(\omega t + \frac{\pi}{4}\right)\right]$$

$$= A(\sqrt{2}+1)\sin\left(\omega t + \frac{\pi}{4}\right)$$

Thus, resultant motion is SHM with an amplitude $A(\sqrt{2}+1)$.

Again energy of resultant motion

$$= \frac{1}{2}m\omega^2\left[A(\sqrt{2}+1)\right]^2 = \frac{1}{2}m\omega^2A^2(3+2\sqrt{2})$$

$= (3+2\sqrt{2})$ times energy of any one motion.

11. (8) : Using energy conservation principle,

$$\frac{-Gm \times 3m}{d} + \frac{1}{2}\mu v_{rel}^2 = 0$$

$$\frac{3Gm^2}{d} = \frac{1}{2} \times \frac{3m^2}{4m} \times v_{rel}^2,\quad v_{rel}^2 = \frac{8Gm}{d}$$

$$v_{rel} = \sqrt{\frac{8GM}{d}} \quad \therefore \eta = 8$$

12. (2) : Let N be the number of nuclei at any time t, then

$$\frac{dN}{dt} = 200 - \lambda N \quad \text{or} \quad \int_0^N \frac{dN}{(200-\lambda N)} = \int_0^t dt$$

$$-\frac{1}{\lambda}[\log(200 - \lambda N)]_0^N = t$$

$$\log(200 - \lambda N) - \log 200 = -\lambda t$$

$$\log\frac{200-\lambda N}{200} = -\lambda t,\quad \frac{200-\lambda N}{200} = e^{-\lambda t}$$

$$1 - \frac{\lambda N}{200} = e^{-\lambda t},\ N = \frac{200}{\lambda}(1 - e^{-\lambda t})$$

As $N = 100$ and $\lambda = 1\ \text{s}^{-1}$.

$\therefore\ 100 = 200\,(1 - e^{-t})$

or $e^{-t} = \frac{1}{2}$ or $t = \ln(2)$ s

$\therefore\ p = 2$

13. (9) : According to Wien's displacement law,
$\lambda_m T$ = constant

$\therefore \quad (\lambda_m)_A T_A = (\lambda_m)_B (T_B)$

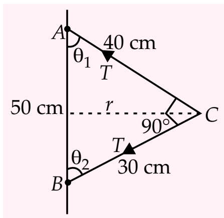
or $\dfrac{T_A}{T_B} = \dfrac{(\lambda_m)_B}{(\lambda_m)_A} = \dfrac{1500 \text{ nm}}{500 \text{ nm}}$ or $\dfrac{T_A}{T_B} = 3$

According to Stefan Boltzmann law, rate of energy radiated by a black body

$E = \sigma A T^4 = \sigma 4\pi R^2 T^4$ [Here, $A = 4\pi R^2$]

$$\therefore \quad \frac{E_A}{E_B} = \left(\frac{R_A}{R_B}\right)^2 \left(\frac{T_A}{T_B}\right)^4 = \left(\frac{6 \text{ cm}}{18 \text{ cm}}\right)^2 (3)^4 = 9$$

14. (4) : $r = 40\sin\theta_1 = 40\left(\dfrac{3}{5}\right) = 24$ cm



$$\frac{mv^2}{r} = T(\sin\theta_1 + \sin\theta_2)$$

$$= T\left(\frac{3}{5} + \frac{4}{5}\right) = T\left(\frac{7}{5}\right) \quad ...(i)$$

$$mg = T(\cos\theta_1 - \cos\theta_2)$$

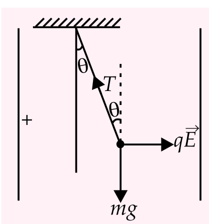
$$= T\left(\frac{4}{5} - \frac{3}{5}\right) = \frac{T}{5} \quad ...(ii)$$

Dividing (i) by (ii), we get

$$\frac{v^2}{rg} = \frac{\left(\frac{7T}{5}\right)}{\left(\frac{T}{5}\right)} = 7$$

$$v^2 = 7 \times \left(\frac{24}{100}\right) \times 10 = 16.8 \quad v \approx 4 \text{ m s}^{-1}$$

15. (8) :



In equilibrium,

$T\sin\theta = qE$ ...(i)

$T\cos\theta = mg$ ...(ii)

Divide (i) by (ii), we get

$$\tan\theta = \frac{qE}{mg}$$

Here, $\theta = 45°$

$$\therefore \quad \tan 45° = \frac{qE}{mg} \quad \text{or} \quad 1 = \frac{qE}{mg}$$

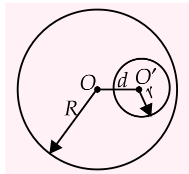
Hence electric field, $E = \dfrac{mg}{q}$

$\therefore$ P.D. between the plates $V = Ed = \dfrac{mgd}{q}$

Substituting the given values, we get

$$V = \frac{2 \times 10^{-3} \times 10 \times 20 \times 10^{-2}}{0.5 \times 10^{-3}} = \frac{4}{0.5} = 8 \text{ V}$$

16. (9) :



Let O be the centre of the disc of radius R and O' be centre of hole of radius r at a distance d from the centre of the disc.

The moment of inertia of the whole disc about an axis through O' is

$$I_1 = \frac{1}{2}MR^2 + Md^2$$

The moment of inertia of the disc (removed portion) about an axis through O' is

$$I_2 = \frac{1}{2}mr^2 = \frac{1}{2}M\frac{r^2}{R^2}r^2 \left(\because \; m = \frac{M}{\pi R^2}\pi r^2\right)$$

$$= \frac{1}{2}\frac{Mr^4}{R^2}$$

The moment of inertia of the system about the given axis is

$$I = I_1 - I_2 = M\left[\frac{R^2}{2} + d^2 - \frac{r^4}{2R^2}\right]$$

Here, $M = 10$ kg, $R = 4$ cm $= 4 \times 10^{-2}$ m,

$$r = \frac{1}{2} \text{ cm} = \frac{1}{2} \times 10^{-2} \text{ m}, \; d = 1 \text{ cm} = 10^{-2} \text{ m}$$

$$\therefore I = 10\left[\frac{16}{2} \times 10^{-4} + 1 \times 10^{-4} - \frac{10^{-8}}{2 \times 16 \times 16 \times 10^{-4}}\right]$$

$= 10(8 \times 10^{-4} + 10^{-4} - 0.002 \times 10^{-4})$

$= 9 \times 10^{-3}$ kg m^2 $\quad \therefore \quad N = 9$

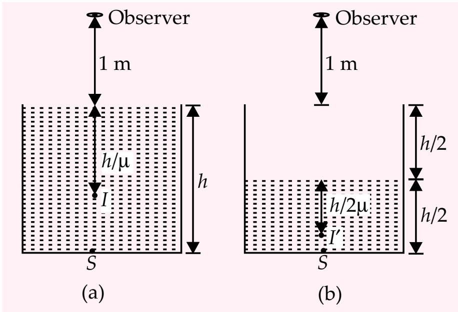
17. (4) : Capacitance of parallel plate capacitor $= \dfrac{K\varepsilon_0 A}{d}$

In the first case $K = 1$, $\dfrac{\varepsilon_0 A}{d} = 1$

In the second case, $\dfrac{K\varepsilon_0 A}{2d} = 2$

$\therefore \quad K = 4$

18. (2) :



Initially, let water be filled to a height h m. Actual depth of scratch from the surface is h. However, due to refraction at water-air boundary, it will appear to be $\frac{h}{\mu}$ so that the observer finds the scratch to be at a distance $1+\frac{h}{\mu}$ (figure (a)).

When the container is half filled, as shown in figure (b) actual depth of scratch below the surface of water is $\frac{h}{2}$ but, due to refraction, it appears to be $\frac{h}{2\mu}$ to the observer located in air.

In this case, the scratch appears to the observer to be at distance $1+\frac{h}{2}+\frac{h}{2\mu}$.

According to the given condition,

$$1+\frac{h}{2}+\frac{h}{2\mu}=1+\frac{h}{\mu}+0.25$$

$$\therefore \quad \frac{h}{2}-\frac{h}{2\mu}=0.25$$

$$\therefore \quad \frac{h}{2}-\frac{3h}{8}=0.25 \text{ or } h=2\text{ m}$$

19. (6) : According to conservation of energy,

$$\frac{hc}{\lambda_1}+\frac{hc}{\lambda_2}=RhcZ^2\left[\frac{1}{1^2}-\frac{1}{n^2}\right]$$

$$\frac{1}{\lambda_1}+\frac{1}{\lambda_2}=RZ^2\left[1-\frac{1}{n^2}\right]$$

$$\frac{1}{n^2}=1-\frac{\lambda_1+\lambda_2}{\lambda_1\lambda_2}\times\frac{1}{RZ^2}$$

$$\frac{1}{n^2}=1-\frac{1330.7\times10^{-10}}{1026.7\times304\times10^{-20}\times1.097\times10^{7}\times4}$$

$$\frac{1}{n^2}=0.0284 \quad \text{or} \quad n=5.93\approx6$$

20. (3) : As $P-V$ diagram is a straight line passing through origin, therefore,

$P\propto V$ or $PV^{-1}=$ constant

In the process, PV^x = constant, molar heat capacity is given by

$$C=\frac{R}{\gamma-1}+\frac{R}{1-x}$$

where $x=-1$ here and $\gamma=1.4$ for diatomic gas.

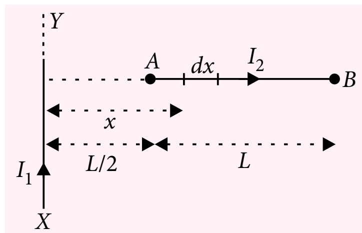
$$\therefore \quad C=\frac{R}{1.4-1}+\frac{R}{1-(-1)}=\frac{5}{2}R+\frac{R}{2}$$

$C=3R$

Hence, $y=3$

PAPER-2

1. (c):



Consider an element of length dx on AB at a distance x from XY. Force on the element is

$$dF=\frac{\mu_0}{4\pi}\frac{2I_1}{x}\times I_2\times dx=\frac{\mu_0 I_1I_2}{2\pi}\frac{dx}{x}$$

Total force on AB is

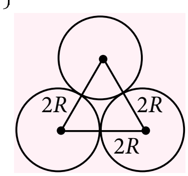
$$F=\frac{\mu_0}{2\pi}I_1I_2\int_{L/2}^{(3L/2)}\frac{dx}{x}=\frac{\mu_0 I_1I_2}{2\pi}\log_e 3$$

2. (d) : From conservation of mechanical energy

$$3\left\{\frac{1}{2}mv^2\right\}=3\left\{\frac{Gm^2}{2R}-\frac{Gm^2}{d}\right\}$$

$$v^2=Gm\left\{\frac{1}{R}-\frac{2}{d}\right\}$$

$$v=\sqrt{Gm\left\{\frac{1}{R}-\frac{2}{d}\right\}}$$

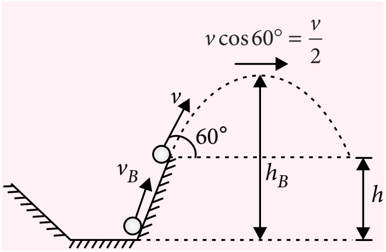


3. (c): After collision balls exchange their velocities

i.e., $v_A=\sqrt{2gh}$

and $v_B = \sqrt{2g(4h)} = 2\sqrt{2gh}$

Height attained by A will be $h_A = \frac{v_A^2}{2g} = h$



But path of B will be first straight line and then parabolic as shown in figure.

Using energy conservation for ball B,

$$\frac{1}{2}mv_B^2 = \frac{1}{2}mv^2 + mgh \text{ or } v = \sqrt{6gh}$$

$$\therefore \quad h_B = h + \frac{v^2 \sin^2 60°}{2g} = h + \frac{9h}{4} = \frac{13h}{4}$$

Hence, $\frac{h_A}{h_B} = \frac{4}{13}$

4. **(c):** Ratio of magnetic moment and angular momentum is given by

$$\frac{M}{L} = \frac{q}{2m}$$

which is a function of q and m only. This can be derived as follows

$M = IA = (q\upsilon)(\pi r^2)$

$$= (q)\left(\frac{\omega}{2\pi}\right)(\pi r^2) = \frac{q\omega r^2}{2}$$

and $L = I\omega = (mr^2\omega)$ $\therefore \frac{M}{L} = \frac{q\omega r^2/2}{mr^2\omega} = \frac{q}{2m}$

5. **(d) :** Let A_i = amplitude of incident wave,

A_r = amplitude of the reflected wave.

$\therefore$ Amplitude at antinode = $A_i + A_r$

and amplitude at node = $A_i - A_r$

$$\frac{A_i + A_r}{A_i - A_r} = \frac{n}{1} \qquad \text{(Given)}$$

$$\therefore \quad \frac{A_r}{A_i} = \frac{n-1}{n+1}$$

Fraction of energy reflected is

$$\frac{E_r}{E_i} = \left(\frac{A_r}{A_i}\right)^2 = \left(\frac{n-1}{n+1}\right)^2$$

6. **(b) :** The total current in the circuit flows through the two resistors and the galvanometer only and it is equal to 2 mA. This produces a drop of 1 V across C_1 and a drop of 1.2 V across C_2.

7. **(a) :** Density, $\rho = \frac{m}{V}$

$$\therefore \quad \frac{d\rho}{\rho} = -\frac{dV}{V} \qquad \text{...(i)}$$

Bulk modulus, $K = -\frac{P}{dV/V}$

Substituting (i) in the expression of the bulk modulus we get

$$K = \frac{P}{d\rho/\rho} \text{ or } \frac{d\rho}{\rho} = \frac{P}{K} \qquad \text{...(ii)}$$

Since increase of pressure increases the density,

$$\therefore \quad d\rho = \rho' - \rho \qquad \text{...(iii)}$$

From (ii) and (iii), we get

$$\frac{\rho' - \rho}{\rho} = \frac{P}{K} \text{ or } \frac{\rho'}{\rho} = 1 + \frac{P}{K}$$

8. **(c):** On the surface of earth,

Total energy = Kinetic energy + Potential energy

$$= \frac{1}{2}mv^2 - \frac{GmM}{R}$$

At the highest point, $v = 0$,

Potential energy $= -\frac{GmM}{(R+h)}$

where h is the maximum height.

According to the law of conservation of mechanical energy, we get

$$\frac{1}{2}mv^2 - \frac{GmM}{R} = -\frac{GmM}{R+h}, \quad \frac{1}{2}v^2 = \frac{GMh}{(R)(R+h)}$$

$$\frac{1}{2}v^2 = \frac{gRh}{R+h} \qquad \left(\because g = \frac{GM}{R^2}\right)$$

$$\text{or} \quad \frac{R+h}{h} = \frac{2gR}{v^2}, \quad h = R\left(\frac{2gR}{v^2} - 1\right)^{-1}$$

9. **(b) :** In steady state, the resistances R and R are in series with the battery, the potential difference across R (which charges the capacitor) will have maximum value $E/2$. Also, if the battery is absent, and the (charged) capacitor were to discharge, the two resistors

R and R will connect in parallel *i.e.* effective resistance across the capacitor is $(R/2)$ and hence the time-constant in the circuit is $\tau = RC/2$. Thus the growth of potential in the capacitor will be given by

$$V(t) = \frac{E}{2}\left[1 - \exp\left(-\frac{t}{\tau}\right)\right] = \frac{E}{2}\left[1 - \exp\left(-\frac{2t}{RC}\right)\right]$$

10. (a) : Here $m_1 = 3$ kg, $m_2 = 2$ kg

The reduced mass of the system is

$$\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{(3)(2)}{3+2} = \frac{6}{5}\ \text{kg}$$

Kinetic energy of the system before collision is

$$K_i = \frac{1}{2}\mu u^2_{\text{relative}} = \frac{1}{2} \times \frac{6}{5} \times (15)^2 = 135\ \text{J}$$

Kinetic energy of the system after collision is

$$K_f = \frac{1}{2}\mu v^2_{\text{relative}} = \frac{1}{2} \times \frac{6}{5} \times 5^2 = 15\ \text{J}$$

Loss of kinetic energy of the system is

$\Delta K = K_i - K_f = 120$ J

11. (b) : The time taken to reach maximum height and maximum height are

$$t = \frac{u \sin\theta}{g} \text{ and } H = \frac{u^2 \sin^2\theta}{2g}$$

For remaining half, the time of flight is

$$t' = \sqrt{\frac{2H}{(2g)}} = \sqrt{\frac{u^2 \sin^2\theta}{2g^2}} = \frac{t}{\sqrt{2}}$$

$\therefore$ Total time of flight is $t + t' = t\left(1 + \frac{1}{\sqrt{2}}\right)$

$$T = \frac{u \sin\theta}{g}\left(1 + \frac{1}{\sqrt{2}}\right)$$

So horizontal range is $u \cos\theta \times T$

$$= \frac{u^2 \sin 2\theta}{2g}\left(1 + \frac{1}{\sqrt{2}}\right)$$

12. (c): Let u_y and v_y be initial and final vertical components of velocity respectively.

$\therefore$ $u_y^2 = 2gH$ and $v_y^2 = 4gH$ $\therefore$ $v_y = \sqrt{2}u_y$

Final velocity makes angle (ϕ) with horizontal.

$$\therefore \quad \tan\phi = \frac{v_y}{u_x} = \sqrt{2}\,\frac{u_y}{u_x} = \sqrt{2}\,\tan\theta$$

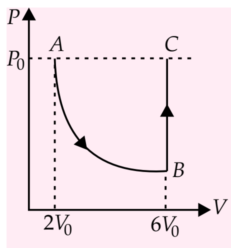
13. (b) : The magnet moving to the left increases the flux entering into coil P and decreases the flux leaving coil Q. Hence the near end of coil P must develop north pole to compensate the increasing flux (Lenz law). Similarly the near end of coil Q must develop south pole to compensate the decreasing flux. Hence current must flow from A to B in P and D to C in Q.

14. (a) : Both magnet and coil Q are fixed. Hence flux through Q does not change. Hence no induced current will flow in Q. When P is moved away, the flux entering it decreases, hence the nearer face must develop south pole. Hence current must flow from B to A in P.

15. (b) : AB is an isothermal process. Then

$P_0 2V_0 = P_B 6V_0$

or $P_B = \frac{P_0}{3}$



BC is an isochoric process. Then

$$\frac{P_B}{T_B} = \frac{P_C}{T_C}$$

$$\frac{P_0}{3T_0} = \frac{P_0}{T_C},\ T_C = 3T_0$$

16. (a) : Heat absorbed during BC is given by

$$Q = nC_V\Delta T = n \times \frac{3R}{2} \times (T_C - T_B)$$

$$= n \times \frac{3R}{2} \times (3T_0 - T_0) = 3nRT_0$$

17. (a) **18. (b)** **19. (a)** **20. (c)**

VIT University Chancellor Dr. G. Viswanathan met the Hon'ble Minister for Human Resource and Development, Mrs. Smriti Zubin Irani in New Delhi recently and honoured her on assuming as Union Minister.

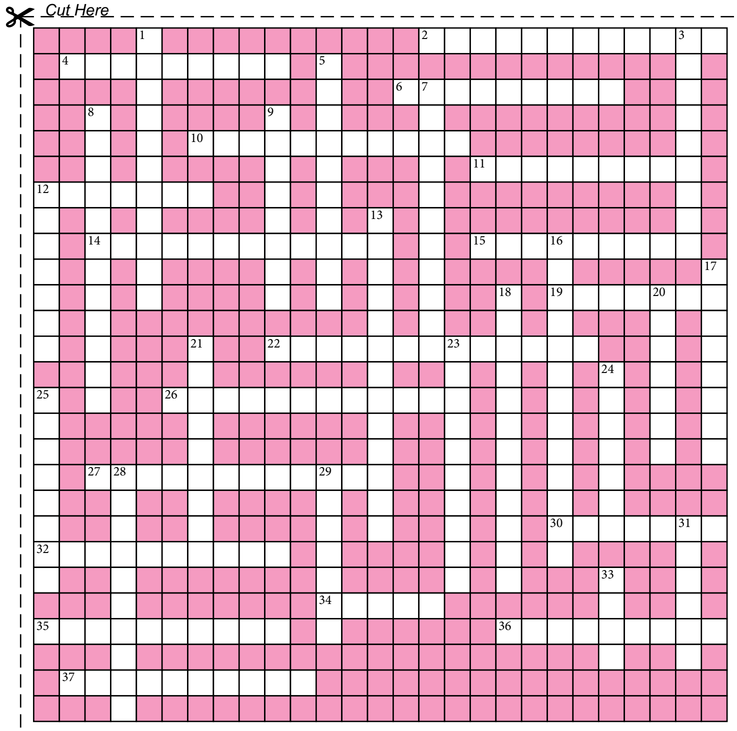
25 MUST KNOW FACTS

Work, Energy and Power

- The work done by a force can be calculated sometimes even if the exact nature of the force is not known.
- Work done by the friction or viscous force on a moving body is negative.
- Work energy theorem is a scalar form of Newton's second law of motion.
- Area under force-displacement curve must be added with proper sign to obtain work done by the force.
- Work done by centripetal force is zero.
- A person walking in a horizontal road with a load on his head does no work against gravity.
- Concept of work provides a link between force and energy.
- When force and displacement are in same direction, kinetic energy of body increases, and when force and displacement are in opposite direction, kinetic energy of body decreases.
- Total mechanical energy of a freely falling body is conserved.
- Internal energy of a body is sometimes called the microscopic mechanical energy of the body.
- When two bodies of different masses are moving with same kinetic energy, and same retarding force is applied on each, both the bodies will stop after travelling the same distance.
- In 1774, Lavoisier stated that matter can neither be created nor destroyed. This is called the law of conservation of mass.
- According to Einstein, law of conservation of mass and law of conservation of energy are not different or independent from each other, which is now referred as the law of conservation of mass-energy.
- Every force in mechanics does not have an associated potential energy.
- Work done on a body may also be stored in the body in the form of potential energy.
- Kinetic energy can never be negative but potential energy may be positive or negative.
- A body possessing kinetic energy possesses momentum and vice-versa.
- The word kinetic has its origin in the Greek word 'kinetikos' which means 'to move'.
- In elastic and inelastic collision, total energy and linear momentum are conserved, kinetic energy is conserved in elastic collision and not conserved in an inelastic collision.
- Conservative forces are called potential forces and non-conservative forces are called non-potential forces.
- Work done in a conservative field is independent of the path followed.
- Conservative forces are also called potential forces and non-conservative forces are called non-potential forces.
- Energy is the product of power and time, power is the flow of energy at any one time.
- The area under power-time graph represents the work done by a force.
- James Watt introduced the unit of power, 'horse power'. According to him, a powerful horse could lift 550 lb through a distance of 1 ft in 1s and work steadily at this rate during the working day.

CROSSWORD

Readers can send their answer with complete address by 15th of every month to win exciting prizes.
Winners' name with their valuable feedback will be published in next issue.



DOWN

1. Inventor of first ac motor. (6, 5)
3. A highly dense region in space whose escape velocity exceeds the speed of light. (5, 4)
5. Phenomenon of flotation in air, defying gravity. (10)
7. The interval of time following excitation during which light is emitted from the screen of a cathode ray tube. (11)
8. Commercial unit of electrical energy. (8, 4)
9. The positive or negative state in which a body reacts to field. (8)
12. Medium with highest optical density. (7)
13. Phenomenon of scattering of light by colloidal particles. (7, 6)
16. A temporary strong magnet. (13)
17. Quantum of the gravitational field. (8)
18. Rate of flow of the electric field through a given area. (8, 4)
20. Equipment for plasma confinement in fusion power reactors. (7)
21. Distribution of audio or video content via radio or television. (9)
23. Type of motion of an object when it travels unequal distances in equal intervals of time. (3, 7)
24. A unit of frequency equal to 10^{12} Hz. (7)
25. Element which acts like an unknown variable resistor. (8)
28. A total reflection prism. (5, 5)
29. Self induced emf. (4, 3)
31. A device used for changing a circuit's operating conditions. (6)
33. A musical sound of specific frequency. (4)

ACROSS

2. A tube designed to demonstrate the luminous effects of an electrical discharge through a rarefied gas. (8, 4)
4. The measure of difference between an observation and its true value. (9)
6. A point on earth's surface that is directly above the focus, point where an earthquake originate. (9)
10. Process of measuring the heat of physical changes. (11)
11. Difference between upper and lower frequency of a band. (9)
12. Unit of power of a spectacle lens. (7)
14. A measure of the ability of a substance to absorb radiation. (12)
15. Complex ratio of voltage to current in an ac circuit. (9)
19. An imaginary line that divides earth into northern and southern hemisphere. (7)
22. Belts of energetic charged particles lying across the equatorial plane. (3, 5, 5)
26. Minimum magnetic moment an electron can have. (4, 8)
27. Communication technology based on total internal reflection. (7, 5)
30. Partial or total blocking of light of one celestial object by another. (7)
32. Failure of a lens or mirror to produce a good image. (10)
34. Mixture found beneath the surface of the earth. (5)
35. Instrument for detecting sounds under water. (10)
36. Reciprocal of capacitance. (9)
37. Property of a material related to the existence of magnetic domains in it. (10)